Wavica Users Guide

1. Introduction

Welcome to *Wavica*, the first optical design package to utilize analytic solutions for optical systems. With its revolutionary approach to optical design, *Wavica* taps into the symbolic powers of *Mathematica*. Once loaded, *Wavica* fully integrates its capablities into the *Rayica* design system.

Loading Wavica

The *Wavica* package (the Wavica directory) should be located in the same directory as the *Ray*ica package (the Rayica and RayicaTools directories). *Wavica* is loaded with the following command. *Wavica* automatically loads the *Rayica* package before loading itself.

```
In[1]:= Needs["Wavica`Wavica`"]
```

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2. Symbolic Calculations: SymbolicTrace

2.1 Overview

Introduction

The principle function responsible for symbolic calculations is **SymbolicTrace**. **SymbolicTrace** creates the symbolic solution to an optical system. In its output, **SymbolicTrace** returns a host of symbolic parameters in the form of rules. Each parameter has rule name identified with it, and the most important ones are: **SymbolicRayEnd**, **SymbolicRayTilt**, **SymbolicIntensity**, **SymbolicSurfaceCoordinates**, and **SymbolicOpticalLength**. Nearly all important functions in the Wavica package depend on **SymbolicTrace** in some fashion. Functions like **GaussianTrace** and **FindABCDMatrix** depend entirely on first order **SymbolicTrace** results. Therefore, it is best to first study **SymbolicTrace** in order to acquire a good understanding of these two functions. **FindLensParameters** uses **SymbolicTrace** for pupil calculations and finding paraxial information such as the lens principle points, magnification, and the lens focal length. Other functions such as **PointSpreadFunction** (**PSF**) and **OpticalTransferFunction** (**MTF**) use **SymbolicTrace** indirectly through **FindLensParameters**. However, it is not necessary to study **SymbolicTrace** before learning about these latter functions.

```
In[2]:= ?SymbolicTrace

        SymbolicTrace[opticalsystem, assumptions, options] is a function that finds the
          symbolic solution of one or more light sources passing through optical system.

        SymbolicTrace can also take the symbolic Taylor-series expansion
          of one or more specified parameters, as given in assumptions, in
          order to reduce the overall complexity of the resulting solution.
```

How to Enter Symbolic Parameters

The basic *Ray*ica system permits symbolic entry of most system parameters. These parameters may then be used to construct symbolic variables within the system. In many cases, the symbolic parameter "piggy-backs" together with a numerical value in the form of an ordered list. In general, *Ray*ica requires that all symbolic parameters are also accompanied by a numerical value. Here is an example of symbolic parameter entry within **PlanoConvexLens**.

```
In[3]:= symbolicLens = PlanoConvexLens[{f,100},{a,50},{t,10}]

Out[3]= PlanoConvexLens[{f, 100}, {a, 50}, {t, 10}]
```

In other cases, the symbolic parameter is given in the form of an option. This occurs in particular for numerical option parameters. However, in this case, the symbolic value does not piggy-back with the numeric option, but instead gets its own option name that is prefixed with the word "**Symbolic**" in front of the corresponding numerical option name. Two such options are **SymbolicWaveLength** and **SymbolicIntensity**. An exception is **NumberOfRays**, which must be a numeric at all times.

```
In[4]:= symbolicSource = LineOfRays[40, SymbolicWaveLength->λ, SymbolicIntensity->i,
        NumberOfRays->4]

Out[4]= LineOfRays[40, NumberOfRays → 4, SymbolicIntensity → i, SymbolicWaveLength → λ]
```

Most types of light source functions cannot accept symbolic values for its built-in parameters. The **LineOfRays** source given above is one such example. Here, the *linewidth* parameter must be a numeric value. In error message is generated otherwise.

```
In[5]:= LineOfRays[{s,10}]

        Source::symbolic : LineOfRays cannot take symbolic parameters LineOfRays.

Out[5]= $Aborted
```

An exception is the **GaussianBeam** source, which can take direct symbolic parameters.

```
In[6]:= symbolicGaussianSource = GaussianBeam[{w,10},{div,.001}, SymbolicWaveLength->λ,
        NumberOfRays->4]

Out[6]= GaussianBeam[{w, 10}, {div, 0.001}, NumberOfRays → 4, SymbolicWaveLength → λ]
```

The **Move** function accepts symbolic parameters as long as a numeric value is also given with each one. Angles may also be specified in a similar fashion. Remember, that any angular information will use units of Degrees for *Rayica*'s **Move** functions. This is in contrast with *Mathematica*'s built-in trigonometric functions, which takes radians by default.

```
In[7]:= symbolicLensSystem =
            {Move[LineOfRays[40],{0,{y,0}}],
            Move[symbolicLens, {d,50}],
            Move[Screen[50],{x,200},{α,10}]}

Out[7]= {Move[LineOfRays[40], {0, {y, 0}}],
         Move[PlanoConvexLens[{f, 100}, {a, 50}, {t, 10}], {d, 50.}],
         Move[Screen[50], {{x, 200.}, 0}, {α, 10.}]}
```

Once a system has been defined with symbolic parameters, you can use Wavica'*s* **SymbolicTrace** function to calculate the symbolic solution to the optical system.

```
In[8]:= TableForm[SymbolicTrace[symbolicLensSystem,{{f>0},{y,0,2}}, FinalFormat->False]]
```

Out[8]//TableForm=

SymbolicOpticalLength → $1.00027 \left(d + \frac{28766183\ y^2}{29843208\ f}\right) + 1.51947 \left(-\frac{0.963911\ y^2}{f} + t \left(1. + \frac{0.216966\ y^2}{f^2}\right)\right) + 1.0002$

SymbolicRayEnd → $\left\{\frac{2\ f^2\ x + f\ (-2.00132\ d - 2.\ f - 0.683851\ t + 2.00132\ x)\ y\ \mathrm{Tan}[°\ \alpha] + (-2.00264\ d - 2.00132\ f - 0.684303\ t + 2.00264\ x)\ y^2\ \mathrm{Tan}[°\ \alpha]^2}{2\ f^2}\right.$

SymbolicRayTilt → $\left\{1. - \frac{0.500661\ y^2}{f^2},\ -\frac{1.00066\ y}{f},\ 0\right\}$

SymbolicSurfaceCoordinates → $\left\{\frac{y\ \mathrm{Sec}[°\ \alpha]\ (f\ (2.00132\ d + 2.\ f + 0.683851\ t - 2.00132\ x) + (2.00264\ d + 2.00132\ f + 0.684303\ t - 2.00264\ x)}{2\ f^2}\right.$

SymbolicIntensity → $100\ e^{-\frac{4\ y^2}{a^2} - \frac{4\ \left(y - \frac{0.658736\ t\ y}{f}\right)^2}{a^2} - \frac{y^2\ \mathrm{Sec}[°\ \alpha]^2\ (f\ (2.00132\ d + 2.\ f + 0.683851\ t - 2.00132\ x) + (2.00264\ d + 2.00132\ f + 0.684303\ t - 2.002}{2500\ f^4}}$

SymbolicSurfaceBoundary → {50, 50}

SymbolicOffAxis → {0, 0}

SymbolicTranslationVector → {x, 0, 0}

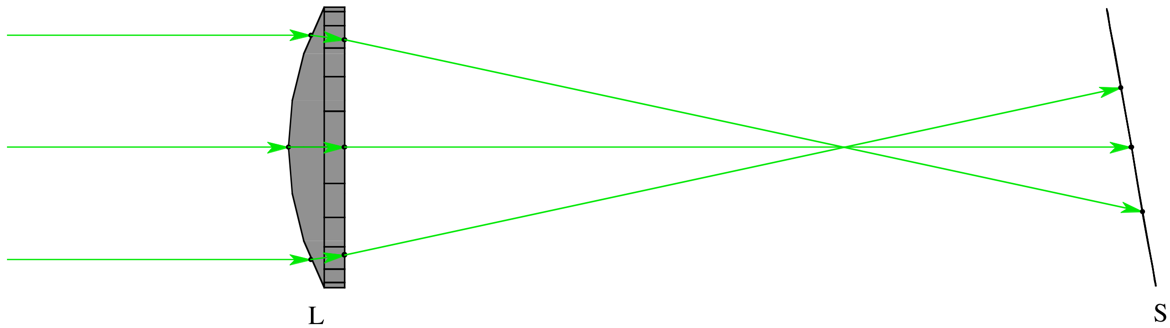
SymbolicRotationMatrix → $\left\{\left\{\frac{\mathrm{Cos}[°\ \alpha]}{\sqrt{\mathrm{Cos}[°\ \alpha]^2 + \mathrm{Sin}[°\ \alpha]^2}},\ \frac{\mathrm{Sin}[°\ \alpha]}{\sqrt{\mathrm{Cos}[°\ \alpha]^2 + \mathrm{Sin}[°\ \alpha]^2}},\ 0\right\},\ \left\{-\frac{\mathrm{Sin}[°\ \alpha]}{\sqrt{\mathrm{Cos}[°\ \alpha]^2 + \mathrm{Sin}[°\ \alpha]^2}},\ \frac{\mathrm{Cos}[°\ \alpha}{\sqrt{\mathrm{Cos}[°\ \alpha]^2 + \mathrm{Si}}}\right.\right.$

SymbolicWaveLength → $\frac{133}{250}$

SymbolicSurfaceNormalFunction → {1, 0, 0}

At the same time, you can use **AnalyzeSystem** to conduct a numeric ray-trace of the system.

```
In[9]:= AnalyzeSystem[symbolicLensSystem,PlotType->TopView]
```




```
AnalyzeSystem[{Move[LineOfRays[40], {0, {y, 0}}],
  Move[PlanoConvexLens[{f, 100}, {a, 50}, {t, 10}], {d, 50.}],
  Move[Screen[50], {{x, 200.}, 0}, {α, 10.}]}, PlotType → TopView]

Out[9]= -9 ray/surface intersections-
```

When you include a Gaussian-beam light source in a system, you can use **FindABCDMatrix** to calculate the symbolic ABCD matrix of the system.

```
In[10]:= symbolicGaussianSystem =
           {symbolicGaussianSource,
           Move[symbolicLens, {d,50}],
           Move[Screen[50],{{x,200},{y,0}}]}

Out[10]= {GaussianBeam[{w, 10}, {div, 0.001}, NumberOfRays → 4, SymbolicWaveLength → λ],
          Move[PlanoConvexLens[{f, 100}, {a, 50}, {t, 10}], {d, 50.}],
          Move[Screen[50], {{x, 200.}, {y, 0}}]}

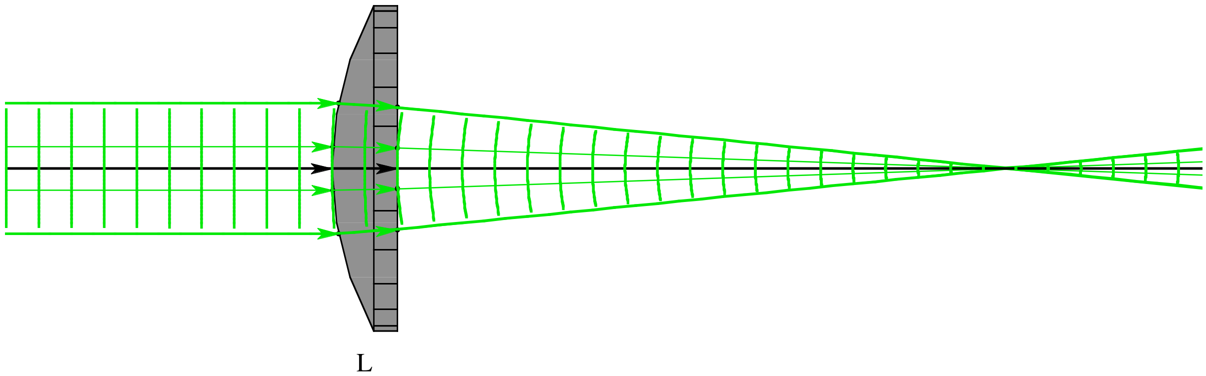
In[11]:= FindABCDMatrix[symbolicGaussianSystem,{f>0}, MatrixForm->True]
```

Out[11]//MatrixForm=

$$\begin{pmatrix} \frac{-1.00132\, d^2 + f\,(f + 0.341925\, t - 1.00066\, x) + d\,(-0.342151\, t + 1.00132\, x)}{f\sqrt{1.00132\, d^2 - 2.00132\, d\, f + f^2}} & \frac{-1.00132\, d^3 + d\, f\,(0.683851\, t - 2.00132\, x) + f^2\,(-0.3417\, t + x) + d^2\,(1.00066\, f}{f\sqrt{1.00132\, d^2 - 2.00132\, d\, f + f^2}} \\ \frac{1.00132\, d - 1.00066\, f}{f\sqrt{1.00132\, d^2 - 2.00132\, d\, f + f^2}} & \frac{\sqrt{1.00132\, d^2 - 2.00132\, d\, f + f^2}}{f} \end{pmatrix}$$

If you use the **ShowGaussian->True** option with **AnalyzeSystem**, you can see the Gaussian wave-front behavior superimposed on the normal numeric ray-trace.

```
In[12]:= AnalyzeSystem[symbolicGaussianSystem,PlotType->TopView,ShowGaussian->True]
```




```
AnalyzeSystem[
 {GaussianBeam[{w, 10}, {div, 0.001}, NumberOfRays → 4, SymbolicWaveLength → λ],
  Move[PlanoConvexLens[{f, 100}, {a, 50}, {t, 10}], {d, 50.}],
  Move[Screen[50], {{x, 200.}, {y, 0}}]}, PlotType → TopView, ShowGaussian → True]
```

```
Out[12]= -12 ray/surface intersections-
```

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How SymbolicTrace works

SymbolicTrace initially works by tracing a single chief ray through the optical system and constructing symbolic expressions that represent light propagation through the optical surfaces in the system. First, **SymbolicTrace** uses *Rayica* to make a numerical trace of the chief ray through the optical system. From this chief ray, the surface sequence is determined as well as positions and directions of the chief ray at each surface. In addition to the numerical results, *Rayica* also provides any user-specified symbolic parameter information about the system to **SymbolicTrace**. **SymbolicTrace** then uses the numerical values of the initial ray trace as boundary conditions to determine the symbolic solution required in symbolic calculations. In particular, many surface functions have more than one intercept solution. In the case of a spherical surface, for example, there are two intercept solutions for the same spherical function, but **SymbolicTrace** must choose to work with only one of the two possible intercept solutions. In such an event, **SymbolicTrace** uses the solution determined from the numerical trace (where only numbers are present and no variables are present) to dictate its choice of symbolic ray-surface intercept solution.

SymbolicTrace performs its calculations in two stages. In the first stage, **SymbolicTrace** determines the symbolic solution at each optical surface. In this stage, the surface functions are calculated in the local coordinate system of the particular surface. After each local surface result is calculated, it is then transformed in the global coordinate system before progressing to the second stage of operation. Finally in the second stage of operation, **SymbolicTrace** constructs a single, nested solution of the optical system by cascading together the symbolic results of each individual surface.

In addition to constructing a symbolic solution, **SymbolicTrace** can use symbolic Taylor's series expansions during the course of its calculations in order to reduce the size and complexity of the resulting solution. For example, **SymbolicTrace** can give the paraxial solution to system by taking the first order Taylors series expansion of the result. Such a solution is useful for determining the abcd matrix representation of the system and for calculating Gaussian beam propagation through the system. However, in other cases, **SymbolicTrace** can use a higher order Taylor series expansion in order to maintain a higher degree of accuracy in the final results. In some cases, **SymbolicTrace** may return a fully unexpanded symbolic solution that completely models the system with machine-precision, or even infinite precision. In cases where Taylor's series expansions are used, the numerical surface intercepts from the ray-trace solution provide the fixed points for the symbolic Taylor's series calculations.

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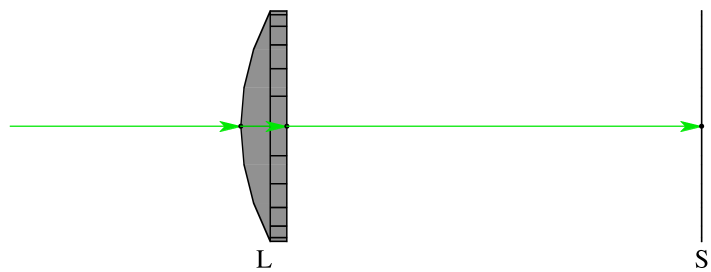
Light Sources and SymbolicTrace

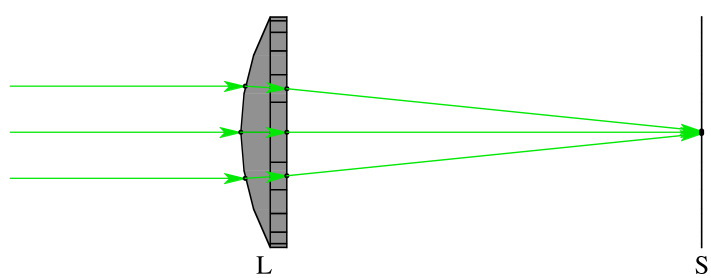
In order to function, **SymbolicTrace** requires the presence of at least one light source as well as one or more optical surfaces. However, **SymbolicTrace** does not use light sources in the same way as **PropagateSystem**. In particular, only the chief ray of the light source is utilized and any other rays from the light source are ignored by **SymbolicTrace**. To illustrate this point, consider the following three examples.

```
In[13]:= raySystem = {
             Move[SingleRay[],{-50,{y,0}}],
             PlanoConvexLens[{f,100},50,10],
             Move[Screen[50],100]};
         AnalyzeSystem[raySystem,PlotType->TopView];
         SymbolicOpticalLength/.SymbolicTrace[raySystem,{{y,0,3},{f>0}}]
```



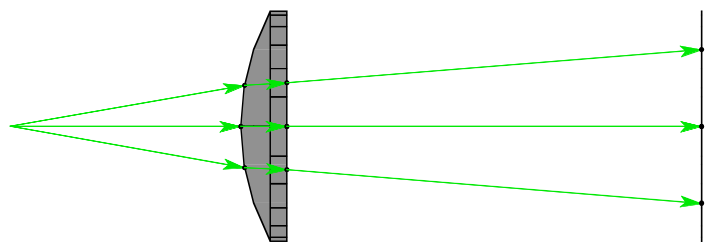
Out[15]= $155.232 + \frac{48.3684\, y^2}{f^2} - \frac{0.500465\, y^2}{f}$

```
In[16]:= linearSystem = {
             Move[LineOfRays[20],{-50,{y,0}}],
             PlanoConvexLens[{f,100},50,10],
             Move[Screen[50],100]};
         AnalyzeSystem[linearSystem,PlotType->TopView];
         SymbolicOpticalLength/.SymbolicTrace[linearSystem,{{y,0,3},{f>0}}]
```



Out[18]= $155.232 + \frac{48.3684\, y^2}{f^2} - \frac{0.500465\, y^2}{f}$

```
In[19]:= wedgeSystem = {
             Move[WedgeOfRays[20],{-50,{y,0}}],
             PlanoConvexLens[{f,100},50,10],
             Move[Screen[50],100]};
         AnalyzeSystem[wedgeSystem,PlotType->TopView];
         SymbolicOpticalLength/.SymbolicTrace[wedgeSystem,{{y,0,3},{f>0}}]
```



Out[21]= $155.232 + \frac{48.3684\, y^2}{f^2} - \frac{0.500465\, y^2}{f}$

In all three cases, even though the three different light sources have different characteristics, the resulting symbolic solution is exactly the same. This is because the chief ray from each source follows the same trajectory through the optics. Therefore, when **SymbolicTrace** is involved, the type of light source is not as important as the starting position and direction of its chief ray.

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Options of SymbolicTrace

SymbolicTrace has many special option settings, many of which are seldom changed by the user. However, one of the best ways to understand **SymbolicTrace** is to study its options. These many options are present in order to provide complete user control over the symbolic solution creation process. See the Help Browser listing for **SymbolicTrace** to learn more about its options. In addition to **SymbolicTrace**, many of these options are shared by other functions in Wavica including: **FindABCDMatrix**, **GaussianTrace**, and **GaussianPlot**.

```
In[22]:= Options[SymbolicTrace]

Out[22]= {Simplify → Automatic, ParentFunction → SymbolicTrace, SeriesOrder → 1,
          LocalCoordinateExpansions → {{SCx, Hold[SurfaceCoordinates⟦1⟧], SeriesOrder}, {SCy,
             Hold[SurfaceCoordinates⟦2⟧], SeriesOrder}, {RSy, Hold[RayStart⟦2⟧], SeriesOrder},
            {RSz, Hold[RayStart⟦3⟧], SeriesOrder}, {RTx, Hold[RayTilt⟦1⟧], SeriesOrder},
            {RTy, Hold[RayTilt⟦2⟧], SeriesOrder}, {RTz, Hold[RayTilt⟦3⟧], SeriesOrder}},
          SeriesExpand → Automatic, SymbolicRayEnd → SeriesOrder,
          SymbolicRayTilt → SeriesOrder, SymbolicRayLength → SeriesOrder,
          SymbolicSurfaceFunction → SeriesOrder,
          SymbolicSurfaceNormalFunction → SeriesOrder,
          SymbolicSurfaceCoordinates → SeriesOrder, SymbolicAperture → ∞,
          SymbolicIntensity → ∞, ApertureOrder → SeriesOrder,
          SymbolicRefractiveModels → Automatic, NormalizeSurfaceNormal → True,
          MaxByteCount → 20000, ByteCountLimit → 20000000, ExpandAlways → False,
          EliminateCrossTerms → False, FinalFormat → True, NestSurfaceSolutions → True,
          RunningCommentary → False, GlobalSurfaceCoordinates → True,
          ReportedSurfaces → {-1}, ReportedParameters → All, NumericalResults → False,
          TaylorSeries → Automatic, TimeConstraint → 10, SkipSimplify → {},
          FilterTrace → False, MakeFloatingPoint → {RefractiveIndex}, Precision → 16}
```

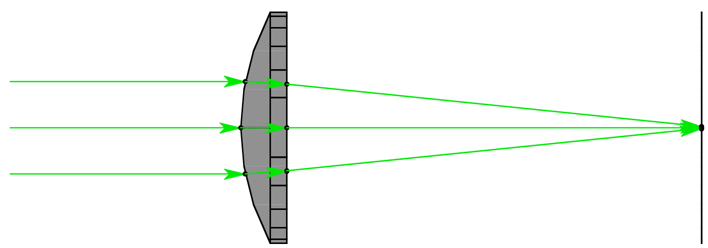
The **SeriesOrder** option is one of the most important ways of controlling the form of the resulting symbolic solution. **SymbolicTrace** uses **SeriesOrder->1** as its default option setting. **SeriesOrder->1** returns the first order Taylor's series symbolic solution to the optical system. **SeriesOrder->Infinity** returns the complete, unapproximated symbolic solution to the system. Sometimes, however, **SeriesOrder->Infinity** may result in a solution that is too large for your computer system's memory, possibly consuming hundreds of megabytes of space. Therefore, **SeriesOrder** must be used with some care and respect.

```
In[23]:= ?SeriesOrder

   SeriesOrder is a option that specifies Taylor
     series expansion order used for symbolic calculations.

   See also: SeriesExpand and LocalCoordinateExpansions.
```

```
In[24]:= opticalsystem = {
   Move[LineOfRays[20], {-50,{y,0}}],
   PlanoConvexLens[100,50,10,DesignWaveLength->.532],
   Move[Screen[50],{d,100}]};
   AnalyzeSystem[opticalsystem,PlotType->TopView];
   SymbolicTrace[opticalsystem] (*SeriesOrder->1*)
   ByteCount[%]
```



Out[26]= {SymbolicOpticalLength → 55.2055 + 1.00027 d,
SymbolicRayEnd → {d, 1.03414 y - 0.00999212 d y, 0},
SymbolicRayTilt → {1, -0.00999212 y, 0},
SymbolicSurfaceCoordinates → {1.03414 y - 0.00999212 d y, 0},
SymbolicIntensity → $100\, e^{-0.00299643\, y^2 - \frac{1}{625}(1.03414 - 0.00999212\, d)^2\, y^2}$,
SymbolicSurfaceBoundary → {50, 50}, SymbolicOffAxis → {0, 0},
SymbolicTranslationVector → {d, 0, 0},
SymbolicRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}},
SymbolicWaveLength → $\frac{133}{250}$, SymbolicSurfaceNormalFunction → {1, 0, 0}}

Out[27]= 1656

In[28]:=
```
SymbolicTrace[opticalsystem,SeriesOrder->2]
ByteCount[%]
```

```
Simplify::time :
 Time spent on a transformation exceeded 10 seconds, and the transformation
   was aborted. Increasing the value of TimeConstraint option
   may improve the result of simplification.

   $SimplifyByteConstraint has been reset to 4164 Bytes.
```

Out[28]=

$$\{\text{SymbolicOpticalLength} \to 55.2055 + 1.00027\,d - 0.00516803\,y^2 + 0.0000499347\,d\,y^2 - 3.41277\times10^{-7}\,y^4 + 3.19906\times10^{-9}\,d\,y^4,\ \text{SymbolicRayEnd} \to \{d,\ 1.03414\,y - 0.00999212\,d\,y + 0.0000668776\,y^3 - 4.98819\times10^{-7}\,d\,y^3 + 1.65847\times10^{-9}\,y^5,\ 0\},$$

$$\text{SymbolicRayTilt} \to \{1. - 0.0000499212\,y^2 - 7.06071\times10^{-10}\,y^4,\ -0.00999212\,y,\ 0\},$$

$$\text{SymbolicSurfaceCoordinates} \to \{1.03414\,y - 0.00999212\,d\,y + 0.0000668776\,y^3 - 4.98819\times10^{-7}\,d\,y^3 + 1.65847\times10^{-9}\,y^5,\ 0\},$$

$$\text{SymbolicIntensity} \to 100\,e^{-\frac{y^4}{390625} - \frac{(0.934222\,y + 0.0000618894\,y^3 + 1.33891\times10^{-9}\,y^5)^4}{390625} - \frac{y^4\,(1.03414 + 0.0000668776\,y^2 + 1.65847\times10^{-9}\,y^4 + d\,(-0.00999212 - 4.98819\times10^{-7}\,y^2))^4}{390625}},$$

$$\text{SymbolicSurfaceBoundary} \to \{50, 50\},\ \text{SymbolicOffAxis} \to \{0, 0\},$$

$$\text{SymbolicTranslationVector} \to \{d, 0, 0\},$$

$$\text{SymbolicRotationMatrix} \to \{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\},$$

$$\text{SymbolicWaveLength} \to \frac{133}{250},\ \text{SymbolicSurfaceNormalFunction} \to \{1, 0, 0\}\}$$

Out[29]= 3280

In[30]:=
```
SymbolicTrace[opticalsystem, SeriesOrder->Infinity, MakeFloatingPoint->True]
```

In[31]:=
```
ByteCount[%]
```

Out[31]= 23232

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Working with SymbolicTrace

In addition to using the **SeriesOrder** option, the power expansion of a particular user-specified variable may be directly given. In such a case, the resulting solution order corresponds directly with the specified order. This is because the expansion takes place after the surface solutions are combined. If desired, the Taylor series expansion process can be closely monitered with the **RunningCommentary->All** option setting.

In[32]:=
```
SymbolicTrace[opticalsystem, {{y,0,2}}, RunningCommentary->All]
```

```
Starting SymbolicTrace

Construct surface equations.

Calling TurboTrace to get symbolic system information for each wavefront.

Reducing surface variables into x-y planar geometry.

Surface {1, 1}

Building SymbolicSurfaceIntersectionFunction for SphericalShape.

Creating SymbolicRayTilt {Refraction, {IntrinsicMedium, BK7},
  Intensity → {IntrinsicMedium, BK7}, FresnelReflections → False, Theory → Automatic}

0.03 Seconds consumed at Surface.

7576 Bytes consumed at Surface.
```

```
Surface {1, 2}

Building SymbolicSurfaceIntersectionFunction for PlanarShape.

Creating SymbolicRayTilt {Refraction, {IntrinsicMedium, BK7},
  Intensity → {IntrinsicMedium, BK7}, FresnelReflections → False, Theory → Automatic}

0.03 Seconds consumed at Surface.

5044 Bytes consumed at Surface.

Surface {1, 3}

Building SymbolicSurfaceIntersectionFunction for PlanarShape.

Creating SymbolicRayTilt {Transmission, 100}

0.02 Seconds consumed at Surface.

3024 Bytes consumed at Surface.

0.47 Seconds consumed constructing surface equations.

Begin nesting of surface equations.

Wavefront #1

Nesting Surface #1

Taylor series expand SymbolicRayLength with 348 Bytes: {{y, 0, 2}}

Simplify SymbolicRayLength with 144 Bytes: $SimplifyID = 28.

Simplified SymbolicRayLength now contains 144 Bytes.

Taylor series expand SymbolicSurfaceCoordinates with 36 Bytes: {{y, 0, 2}}

Simplify SymbolicSurfaceCoordinates with 36 Bytes: $SimplifyID = 29.

Simplified SymbolicSurfaceCoordinates now contains 36 Bytes.

Taylor series expand SymbolicSurfaceNormalFunction with 1068 Bytes: {{y, 0, 2}}

Simplify SymbolicSurfaceNormalFunction with 280 Bytes: $SimplifyID = 30.

Simplified SymbolicSurfaceNormalFunction now contains 280 Bytes.

Taylor series expand SymbolicRayEnd with 388 Bytes: {{y, 0, 2}}

Simplify SymbolicRayEnd with 148 Bytes: $SimplifyID = 31.

Simplified SymbolicRayEnd now contains 148 Bytes.

Taylor series expand SymbolicRayTilt with 1748 Bytes: {{y, 0, 2}}

Simplify SymbolicRayTilt with 196 Bytes: $SimplifyID = 32.

Simplified SymbolicRayTilt now contains 196 Bytes.

Nesting Surface #2

Taylor series expand SymbolicRayLength with 336 Bytes: {{y, 0, 2}}

Simplify SymbolicRayLength with 116 Bytes: $SimplifyID = 34.

Simplified SymbolicRayLength now contains 116 Bytes.

Taylor series expand SymbolicSurfaceCoordinates with 404 Bytes: {{y, 0, 2}}

Simplify SymbolicSurfaceCoordinates with 76 Bytes: $SimplifyID = 35.
```

```
Simplified SymbolicSurfaceCoordinates now contains 76 Bytes.
Simplify SymbolicSurfaceNormalFunction with 64 Bytes: $SimplifyID = 36.
Simplified SymbolicSurfaceNormalFunction now contains 64 Bytes.
Taylor series expand SymbolicRayEnd with 92 Bytes: {{y, 0, 2}}
Simplify SymbolicRayEnd with 92 Bytes: $SimplifyID = 37.
Simplified SymbolicRayEnd now contains 92 Bytes.
Taylor series expand SymbolicRayTilt with 632 Bytes: {{y, 0, 2}}
Simplify SymbolicRayTilt with 196 Bytes: $SimplifyID = 38.
Simplified SymbolicRayTilt now contains 196 Bytes.
Nesting Surface #3
Taylor series expand SymbolicRayLength with 264 Bytes: {{y, 0, 2}}
Simplify SymbolicRayLength with 288 Bytes: $SimplifyID = 40.
Simplified SymbolicRayLength now contains 260 Bytes.
Taylor series expand SymbolicSurfaceCoordinates with 372 Bytes: {{y, 0, 2}}
Simplify SymbolicSurfaceCoordinates with 212 Bytes: $SimplifyID = 41.
Simplified SymbolicSurfaceCoordinates now contains 140 Bytes.
Taylor series expand SymbolicRayEnd with 144 Bytes: {{y, 0, 2}}
Simplify SymbolicRayEnd with 144 Bytes: $SimplifyID = 42.
Simplified SymbolicRayEnd now contains 144 Bytes.
Taylor series expand SymbolicRayTilt with 196 Bytes: {{y, 0, 2}}
Simplify SymbolicRayTilt with 196 Bytes: $SimplifyID = 43.
Simplified SymbolicRayTilt now contains 196 Bytes.
A final formatting of SymbolicSurfaceCoordinates with 140 Bytes: $SimplifyID = 45.
Formated SymbolicSurfaceCoordinates finally contains 144 Bytes.
A final formatting of SymbolicOpticalLength with 668 Bytes: $SimplifyID = 46.
Formated SymbolicOpticalLength finally contains 244 Bytes.
A final formatting of SymbolicRayEnd with 144 Bytes: $SimplifyID = 47.
Formated SymbolicRayEnd finally contains 148 Bytes.
A final formatting of SymbolicRayTilt with 196 Bytes: $SimplifyID = 48.
Formated SymbolicRayTilt finally contains 196 Bytes.
0.16 total Seconds consumed for wavefront segment.
1944 total Bytes consumed for wavefront segment.
Ending SymbolicTrace
```

Out[32]=

$$\{\text{SymbolicOpticalLength} \to 55.2055 + 1.00027\,d - 0.00516803\,y^2 + 0.0000499347\,d\,y^2,$$
$$\text{SymbolicRayEnd} \to \{d,\ 1.03414\,y - 0.00999212\,d\,y,\ 0\},$$
$$\text{SymbolicRayTilt} \to \{1. - 0.0000499212\,y^2,\ -0.00999212\,y,\ 0\},$$
$$\text{SymbolicSurfaceCoordinates} \to \{1.03414\,y - 0.00999212\,d\,y,\ 0\},$$
$$\text{SymbolicIntensity} \to 100\,e^{-0.00299643\,y^2 - \frac{1}{625}(1.03414-0.00999212\,d)^2\,y^2},$$
$$\text{SymbolicSurfaceBoundary} \to \{50,\ 50\},\ \text{SymbolicOffAxis} \to \{0,\ 0\},$$
$$\text{SymbolicTranslationVector} \to \{d,\ 0,\ 0\},$$
$$\text{SymbolicRotationMatrix} \to \{\{1,0,0\},\{0,1,0\},\{0,0,1\}\},$$
$$\text{SymbolicWaveLength} \to \frac{133}{250},\ \text{SymbolicSurfaceNormalFunction} \to \{1,\ 0,\ 0\}\}$$

In[33]:= **ByteCount[%]**

Out[33]= 1924

SymbolicTrace can take, within its second argument, a list of assumptions that get passed internally to the **Simplify** operations. These assumptions are used to further simplify the resulting expression. Here is the **SymbolicTrace** operation without any assumptions given.

In[34]:=
```
opticalsystem = {
Move[LineOfRays[20], {-50,{y,0}}],
PlanoConvexLens[{f,100},50,10,DesignWaveLength->.532],
Move[Screen[50],{d,100}]};
SymbolicTrace[opticalsystem]
```

Out[35]=

$$\{\text{SymbolicOpticalLength} \to$$
$$55.2055 + 1.00027\,d - 0.269712\,f - 0.519612\sqrt{f^2} + 0.789324\sqrt{f^2}\,\text{Sign}[f],$$
$$\text{SymbolicRayEnd} \to \Big\{d,\ y + \frac{3.4143\sqrt{f^2}\,y\,\text{Sign}[f]}{f^2} - \frac{0.999212\,d\sqrt{f^2}\,y\,\text{Sign}[f]}{f^2} +$$
$$\frac{0.3417\sqrt{f^2}\,y\,\text{Sign}[f]}{f} - 0.3417\,y\,\text{Sign}[f]^2,\ 0\Big\},$$
$$\text{SymbolicRayTilt} \to \Big\{1,\ -\frac{0.999212\sqrt{f^2}\,y\,\text{Sign}[f]}{f^2},\ 0\Big\},$$
$$\text{SymbolicSurfaceCoordinates} \to \Big\{y + \frac{3.4143\sqrt{f^2}\,y\,\text{Sign}[f]}{f^2} - \frac{0.999212\,d\sqrt{f^2}\,y\,\text{Sign}[f]}{f^2} +$$
$$\frac{0.3417\sqrt{f^2}\,y\,\text{Sign}[f]}{f} - 0.3417\,y\,\text{Sign}[f]^2,\ 0\Big\},\ \text{SymbolicIntensity} \to$$
$$100\,e^{-\frac{y^2}{625} - \frac{y^2\left(f^2+(-6.57782+0.3417\,f)\sqrt{f^2}\,\text{Sign}[f]-0.3417\,f^2\,\text{Sign}[f]^2\right)^2}{625\,f^4} - \frac{y^2\left(f^2+(3.4143-0.999212\,d+0.3417\,f)\sqrt{f^2}\,\text{Sign}[f]-0.3417\,f^2\,\text{Sign}[f]^2\right)^2}{625\,f^4}},$$
$$\text{SymbolicSurfaceBoundary} \to \{50,\ 50\},\ \text{SymbolicOffAxis} \to \{0,\ 0\},$$
$$\text{SymbolicTranslationVector} \to \{d,\ 0,\ 0\},$$
$$\text{SymbolicRotationMatrix} \to \{\{1,0,0\},\{0,1,0\},\{0,0,1\}\},$$
$$\text{SymbolicWaveLength} \to \frac{133}{250},\ \text{SymbolicSurfaceNormalFunction} \to \{1,\ 0,\ 0\}\}$$

Here is the same operation with some assumptions included about the user variables. The **RunningCommentary->Simplify** setting is used here to moniter the internal simplification steps.

In[36]:= **SymbolicTrace[opticalsystem, {f>0, d∈Reals}, RunningCommentary->Simplify]**

```
Starting SymbolicTrace

Simplify SymbolicSurfaceFunction with 316 Bytes: $SimplifyID = 1.

Simplified SymbolicSurfaceFunction now contains 52 Bytes.

Simplify SymbolicSurfaceIntersectionFunction with 1508 Bytes: $SimplifyID = 2.
```

```
Simplified SymbolicSurfaceIntersectionFunction now contains 1508 Bytes.
Simplify SymbolicSurfaceCoordinates with 1388 Bytes: $SimplifyID = 3.
Simplified SymbolicSurfaceCoordinates now contains 356 Bytes.
Simplify SymbolicSurfaceNormalFunction with 236 Bytes: $SimplifyID = 4.
Simplified SymbolicSurfaceNormalFunction now contains 164 Bytes.
Simplify SymbolicSurfaceNormalFunction with 484 Bytes: $SimplifyID = 5.
Simplified SymbolicSurfaceNormalFunction now contains 444 Bytes.
Simplify SymbolicRayTilt with 2788 Bytes: $SimplifyID = 6.
Simplified SymbolicRayTilt now contains 1892 Bytes.
Simplify SymbolicRayEnd with 412 Bytes: $SimplifyID = 7.
Simplified SymbolicRayEnd now contains 372 Bytes.
Simplify SymbolicRayLength with 3072 Bytes: $SimplifyID = 8.
Simplified SymbolicRayLength now contains 808 Bytes.
Simplify SymbolicAperture with 264 Bytes: $SimplifyID = 9.
Simplified SymbolicAperture now contains 264 Bytes.
Simplify inverse input-coordinate transformation with 184 Bytes: $SimplifyID = 10.
Simplified inverse input-coordinate transformation now contains 184 Bytes.
Simplify SymbolicSurfaceFunction with 52 Bytes: $SimplifyID = 11.
Simplified SymbolicSurfaceFunction now contains 52 Bytes.
Simplify SymbolicSurfaceIntersectionFunction with 220 Bytes: $SimplifyID = 12.
Simplified SymbolicSurfaceIntersectionFunction now contains 220 Bytes.
Simplify SymbolicSurfaceCoordinates with 204 Bytes: $SimplifyID = 13.
Simplified SymbolicSurfaceCoordinates now contains 124 Bytes.
Simplify SymbolicSurfaceNormalFunction with 64 Bytes: $SimplifyID = 14.
Simplified SymbolicSurfaceNormalFunction now contains 64 Bytes.
Simplify SymbolicSurfaceNormalFunction with 64 Bytes: $SimplifyID = 15.
Simplified SymbolicSurfaceNormalFunction now contains 64 Bytes.
Simplify SymbolicRayTilt with 276 Bytes: $SimplifyID = 16.
Simplified SymbolicRayTilt now contains 276 Bytes.
Simplify SymbolicRayEnd with 220 Bytes: $SimplifyID = 17.
Simplified SymbolicRayEnd now contains 140 Bytes.
Simplify SymbolicRayLength with 120 Bytes: $SimplifyID = 18.
Simplified SymbolicRayLength now contains 60 Bytes.
Simplify SymbolicAperture with 264 Bytes: $SimplifyID = 19.
Simplified SymbolicAperture now contains 264 Bytes.
Simplify inverse input-coordinate transformation with 220 Bytes: $SimplifyID = 20.
```

```
Simplified inverse input-coordinate transformation now contains 220 Bytes.
Simplify SymbolicSurfaceFunction with 52 Bytes: $SimplifyID = 21.
Simplified SymbolicSurfaceFunction now contains 52 Bytes.
Simplify SymbolicSurfaceIntersectionFunction with 220 Bytes: $SimplifyID = 22.
Simplified SymbolicSurfaceIntersectionFunction now contains 220 Bytes.
Simplify SymbolicSurfaceCoordinates with 204 Bytes: $SimplifyID = 23.
Simplified SymbolicSurfaceCoordinates now contains 124 Bytes.
Simplify SymbolicSurfaceNormalFunction with 64 Bytes: $SimplifyID = 24.
Simplified SymbolicSurfaceNormalFunction now contains 64 Bytes.
Simplify SymbolicSurfaceNormalFunction with 64 Bytes: $SimplifyID = 25.
Simplified SymbolicSurfaceNormalFunction now contains 64 Bytes.
Simplify SymbolicRayTilt with 40 Bytes: $SimplifyID = 26.
Simplified SymbolicRayTilt now contains 40 Bytes.
Simplify SymbolicRayEnd with 220 Bytes: $SimplifyID = 27.
Simplified SymbolicRayEnd now contains 140 Bytes.
Simplify SymbolicRayLength with 120 Bytes: $SimplifyID = 28.
Simplified SymbolicRayLength now contains 60 Bytes.
Simplify SymbolicAperture with 264 Bytes: $SimplifyID = 29.
Simplified SymbolicAperture now contains 264 Bytes.
Simplify inverse input-coordinate transformation with 244 Bytes: $SimplifyID = 30.
Simplified inverse input-coordinate transformation now contains 244 Bytes.
Simplify SymbolicRayLength with 12 Bytes: $SimplifyID = 31.
Simplified SymbolicRayLength now contains 12 Bytes.
Simplify SymbolicSurfaceCoordinates with 36 Bytes: $SimplifyID = 32.
Simplified SymbolicSurfaceCoordinates now contains 36 Bytes.
Simplify SymbolicRayEnd with 52 Bytes: $SimplifyID = 33.
Simplified SymbolicRayEnd now contains 52 Bytes.
Simplify SymbolicRayTilt with 132 Bytes: $SimplifyID = 34.
Simplified SymbolicRayTilt now contains 132 Bytes.
Simplify SymbolicRayLength with 12 Bytes: $SimplifyID = 36.
Simplified SymbolicRayLength now contains 12 Bytes.
Simplify SymbolicSurfaceCoordinates with 140 Bytes: $SimplifyID = 37.
Simplified SymbolicSurfaceCoordinates now contains 140 Bytes.
Simplify SymbolicRayEnd with 156 Bytes: $SimplifyID = 38.
Simplified SymbolicRayEnd now contains 156 Bytes.
Simplify SymbolicRayTilt with 132 Bytes: $SimplifyID = 39.
```

```
Simplified SymbolicRayTilt now contains 132 Bytes.
Simplify SymbolicRayLength with 36 Bytes: $SimplifyID = 41.
Simplified SymbolicRayLength now contains 36 Bytes.
Simplify SymbolicSurfaceCoordinates with 300 Bytes: $SimplifyID = 42.
Simplified SymbolicSurfaceCoordinates now contains 184 Bytes.
Simplify SymbolicRayEnd with 304 Bytes: $SimplifyID = 43.
Simplified SymbolicRayEnd now contains 188 Bytes.
Simplify SymbolicRayTilt with 132 Bytes: $SimplifyID = 44.
Simplified SymbolicRayTilt now contains 132 Bytes.
A final formatting of SymbolicSurfaceCoordinates with 184 Bytes: $SimplifyID = 46.
Formated SymbolicSurfaceCoordinates finally contains 228 Bytes.
A final formatting of SymbolicOpticalLength with 116 Bytes: $SimplifyID = 47.
Formated SymbolicOpticalLength finally contains 80 Bytes.
A final formatting of SymbolicRayEnd with 188 Bytes: $SimplifyID = 48.
Formated SymbolicRayEnd finally contains 232 Bytes.
A final formatting of SymbolicRayTilt with 132 Bytes: $SimplifyID = 49.
Formated SymbolicRayTilt finally contains 132 Bytes.
Ending SymbolicTrace
```

Out[36]=

$$\left\{\text{SymbolicOpticalLength} \to 55.2055 + 1.00027\,d,\ \text{SymbolicRayEnd} \to \left\{d,\ y + \frac{3.4143\,y}{f} - \frac{0.999212\,d\,y}{f},\ 0\right\},\ \text{SymbolicRayTilt} \to \left\{1,\ -\frac{0.999212\,y}{f},\ 0\right\},\ \text{SymbolicSurfaceCoordinates} \to \left\{y + \frac{3.4143\,y}{f} - \frac{0.999212\,d\,y}{f},\ 0\right\},\ \text{SymbolicIntensity} \to 100\,e^{-\frac{y^2}{625} - \frac{(3.4143 - 0.999212\,d + f)^2\,y^2}{625\,f^2} - \frac{1}{625}\left(y - \frac{6.57782\,y}{f}\right)^2},\ \text{SymbolicSurfaceBoundary} \to \{50,\ 50\},\ \text{SymbolicOffAxis} \to \{0,\ 0\},\ \text{SymbolicTranslationVector} \to \{d,\ 0,\ 0\},\ \text{SymbolicRotationMatrix} \to \{\{1,\ 0,\ 0\},\ \{0,\ 1,\ 0\},\ \{0,\ 0,\ 1\}\},\ \text{SymbolicWaveLength} \to \frac{133}{250},\ \text{SymbolicSurfaceNormalFunction} \to \{1,\ 0,\ 0\}\right\}$$

In some cases, the second argument of **`SymbolicTrace`** may include both assumptions as well as specifications for Taylor series expansion of symbolic system variables.

```
In[37]:= SymbolicTrace[opticalsystem, {{y,0,2},{f,100,1},{f>0, d∈Reals}}]
```

```
Out[37]= {SymbolicOpticalLength → 55.2055 + 1.00027 d -
            0.0105067 y^2 + 0.000149804 d y^2 + 0.0000533866 f y^2 - 9.98694 × 10^-7 d f y^2,
          SymbolicRayEnd → {d, 1.06829 y - 0.0199842 d y - 0.00034143 f y + 0.0000999212 d f y, 0},
          SymbolicRayTilt → {1. - 0.000149764 y^2 + 9.98425 × 10^-7 f y^2,
            -0.0199842 y + 0.0000999212 f y, 0}, SymbolicSurfaceCoordinates →
           {1.06829 y - 0.0199842 d y - 0.00034143 f y + 0.0000999212 d f y, 0}, SymbolicIntensity →
```

$$100\, e^{-\frac{y^2}{625}-\frac{1}{625}(1.06829+d(-0.0199842+0.0000999212\, f)-0.00034143\, f)^2 y^2-\frac{1}{625}(0.868444+0.000657782\, f)^2 y^2},$$

```
          SymbolicSurfaceBoundary → {50, 50}, SymbolicOffAxis → {0, 0},
          SymbolicTranslationVector → {d, 0, 0},
          SymbolicRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}},
          SymbolicWaveLength → 133/250, SymbolicSurfaceNormalFunction → {1, 0, 0}}
```

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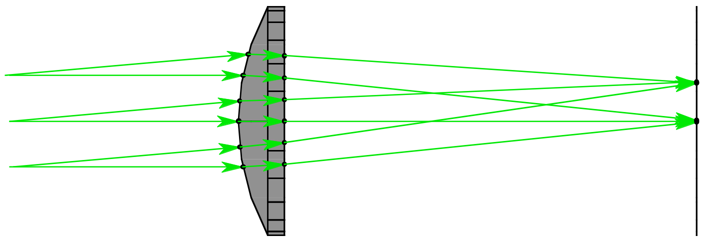
Multiple Light Paths

In addition to using the **`SymbolicTrace`** with linearly ordered systems, it is also possible to work with systems involving multiple light paths. Each light path produces a different solution in the output. Here is an example that uses two light sources. In general, every distinct light source produces a distinct light path. In this event, the two light sources produce two distinct symbolic solutions.

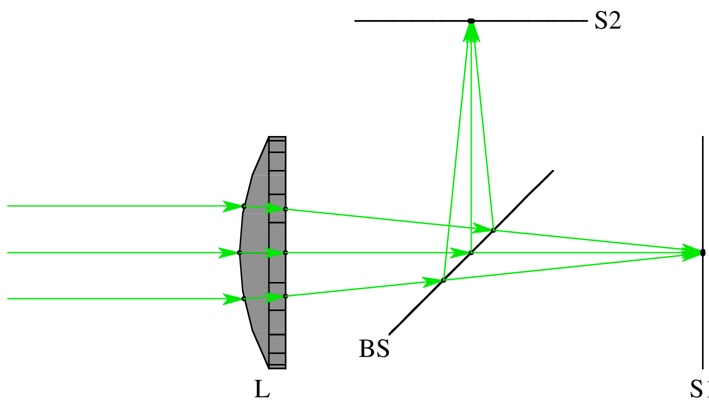
```
In[38]:= twoSourceSystem = {
             Move[LineOfRays[20],{-50,{y,0}}],
             Move[LineOfRays[20],{-50,{y,0}},5],
             PlanoConvexLens[100,50,10],
             Move[Screen[50],{d,100}]};
         AnalyzeSystem[twoSourceSystem,PlotType->TopView];
         SymbolicTrace[twoSourceSystem]
```




```
Out[40]= {{SymbolicOpticalLength → 55.2055 + 1.00027 d,
   SymbolicRayEnd → {d, 1.03419 y - 0.0100066 d y, 0}, SymbolicRayTilt → {1, -0.0100066 y, 0},
   SymbolicSurfaceCoordinates → {1.03419 y - 0.0100066 d y, 0},
   SymbolicIntensity → 100 e^(-0.00299615 y^2 - 1/625 (1.03419-0.0100066 d)^2 y^2),
   SymbolicSurfaceBoundary → {50, 50}, SymbolicOffAxis → {0, 0},
   SymbolicTranslationVector → {d, 0, 0},
   SymbolicRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, SymbolicWaveLength → 133/250,
   SymbolicSurfaceNormalFunction → {1, 0, 0}}, {SymbolicOpticalLength →
    55.2974 + 1.00119 d - 0.0425954 y - 0.000441132 d y + 0.0000248153 y^2,
   SymbolicRayEnd → {d, 4.23898 + 0.0428284 d + 1.04166 y - 0.0103066 d y +
      0.000547129 y^2 + 4.53288 × 10^-6 d y^2 - 1.10503 × 10^-7 y^3, 0},
   SymbolicRayTilt → {0.999084 + 0.000440206 y, 0.0427892 - 0.0102783 y, 0},
   SymbolicSurfaceCoordinates → {4.23898 + 0.0428284 d + 1.04166 y - 0.0103066 d y +
      0.000547129 y^2 + 4.53288 × 10^-6 d y^2 - 1.10503 × 10^-7 y^3, 0}, SymbolicIntensity → 100
     e^(-1/625 (4.39072+1.00749 y)^2 - 1/625 (4.53288×10^-6 d (-2269.59+y) (-4.16305+y)-1.10503×10^-7 (-6420.4+y) (4.07819+y) (1465.07+y))^2),
   SymbolicSurfaceBoundary → {50, 50}, SymbolicOffAxis → {0, 0},
   SymbolicTranslationVector → {d, 0, 0},
   SymbolicRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}},
   SymbolicWaveLength → 133/250, SymbolicSurfaceNormalFunction → {1, 0, 0}}}
```

In some cases, the optical arrangement may produce multiple light paths from a single light source. Here is an example that uses a beam splitter to create two light paths.

```
In[41]:= beamSplitterSystem = {
             Move[LineOfRays[20],{-50,{y,0}}],
             PlanoConvexLens[100,50,10],
             Move[BeamSplitter[{50,50},50],50,-45],
             Move[Screen[50],{d1,100}],
             Move[Screen[50],{50,{d2,50}},90]
             };
         AnalyzeSystem[beamSplitterSystem,PlotType->TopView];
         SymbolicTrace[beamSplitterSystem]
```




```
Out[43]= {{SymbolicOpticalLength → 55.2055 + 1.00027 d1 - 0.00200318 y^2 - 0.0000467807 y^3,
   SymbolicRayEnd →
    {d1, 1.03419 y - 0.0100066 d1 y - 0.00200264 y^2 - 0.0000267284 y^3 + 4.6799 × 10^-7 y^4, 0},
   SymbolicRayTilt → {1, -0.0100066 y, 0}, SymbolicSurfaceCoordinates →
    {1.03419 y - 0.0100066 d1 y - 0.00200264 y^2 - 0.0000267284 y^3 + 4.6799 × 10^-7 y^4, 0},
   SymbolicIntensity → 50
     e^(-0.00299615 y^2 - 1/625 y^2 (0.754995-0.0103871 y+0.00006614 y^2)^2 - 1/625 y^2 (-0.0100066 d1+4.6799×10^-7 (123.204+y) (17936.6-180.317 y+y
     , SymbolicSurfaceBoundary → {50, 50}, SymbolicOffAxis → {0, 0},
   SymbolicTranslationVector → {d1, 0, 0},
   SymbolicRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}},
   SymbolicWaveLength → 133/250, SymbolicSurfaceNormalFunction → {1, 0, 0}},
  {SymbolicOpticalLength → 105.219 + 1.00027 d2 - 0.00200318 y^2 - 0.0000467807 y^3,
   SymbolicRayEnd → {50. + 0.533862 y - 0.0100066 d2 y -
      0.00200264 y^2 - 0.0000267284 y^3 + 4.6799 × 10^-7 y^4, d2, 0},
   SymbolicRayTilt → {-0.0100066 y, 1., 0}, SymbolicSurfaceCoordinates →
    {-0.533862 y + 0.0100066 d2 y + 0.00200264 y^2 + 0.0000267284 y^3 - 4.6799 × 10^-7 y^4, 0},
   SymbolicIntensity → 50
     e^(-0.00299615 y^2 - 1/625 y^2 (0.754995+(-0.0103871+0.00006614 y) y)^2 - 1/625 y^2 (0.0100066 d2-4.6799×10^-7 (99.934+y) (11415.1+(-157.047+y
     , SymbolicSurfaceBoundary → {50, 50}, SymbolicOffAxis → {0, 0},
   SymbolicTranslationVector → {50, d2, 0},
   SymbolicRotationMatrix → {{0, 1, 0}, {-1, 0, 0}, {0, 0, 1}},
   SymbolicWaveLength → 133/250, SymbolicSurfaceNormalFunction → {1, 0, 0}}}
```

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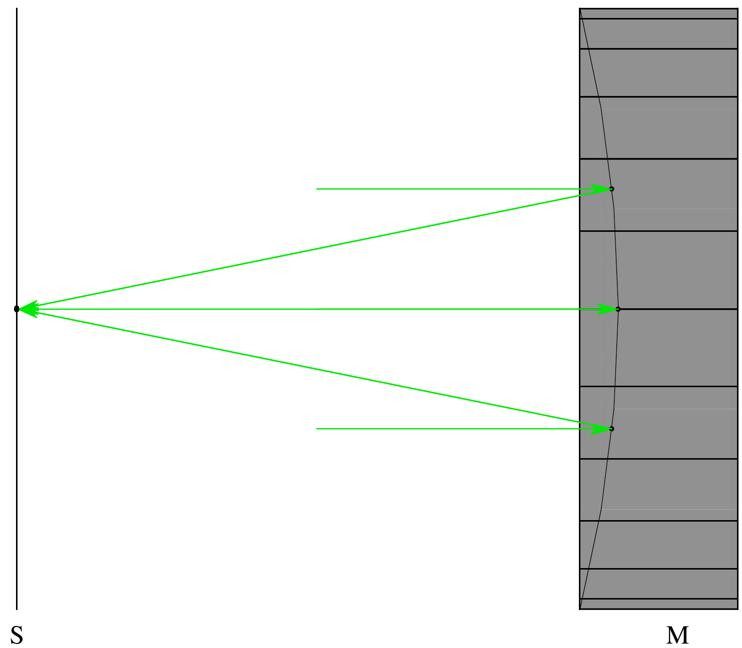
Infinite Precision Calculations

`SymbolicTrace` is also capable of producing solutions with infinite precision. This happens automatically when the input parameters are all pure integers, integer fractions, or symbols. Here is an example of a reflective spherical mirror calculation.

In[105]:=

```
opticalSystem = {
    Move[LineOfRays[20],{0,{y,0}}],
    Move[SphericalMirror[{r,-100},50,10],25],
    Move[Screen[50],{d,-25}]};
AnalyzeSystem[opticalSystem,PlotType->TopView];
SymbolicTrace[opticalSystem, {{y,0,5},{r<0}},SymbolicRefractiveModels->{Air->1}]
```



Out[107]=

$$\{\text{SymbolicOpticalLength} \to 50 - d + \frac{50\,y^2}{r^2} - \frac{2\,d\,y^2}{r^2} + \frac{y^2}{r} + \frac{100\,y^4}{r^4} - \frac{4\,d\,y^4}{r^4} + \frac{5\,y^4}{4\,r^3},$$

$$\text{SymbolicRayEnd} \to \left\{d,\ y + \frac{50\,y}{r} - \frac{2\,d\,y}{r} + \frac{75\,y^3}{r^3} - \frac{3\,d\,y^3}{r^3} + \frac{y^3}{r^2} + \frac{575\,y^5}{4\,r^5} - \frac{23\,d\,y^5}{4\,r^5} + \frac{7\,y^5}{4\,r^4},\ 0\right\},$$

$$\text{SymbolicRayTilt} \to \left\{-1 + \frac{2\,y^2}{r^2},\ \frac{2\,y}{r} - \frac{y^3}{r^3} - \frac{y^5}{4\,r^5},\ 0\right\},\ \text{SymbolicSurfaceCoordinates} \to$$

$$\left\{y + \frac{50\,y}{r} - \frac{2\,d\,y}{r} + \frac{75\,y^3}{r^3} - \frac{3\,d\,y^3}{r^3} + \frac{y^3}{r^2} + \frac{575\,y^5}{4\,r^5} - \frac{23\,d\,y^5}{4\,r^5} + \frac{7\,y^5}{4\,r^4},\ 0\right\},$$

$$\text{SymbolicIntensity} \to 100\,e^{-\frac{y^2}{625} - \frac{y^2\,\left(-8\,(-25+d)\,r^4 + 4\,r^5 - 12\,(-25+d)\,r^2\,y^2 + 4\,r^3\,y^2 - 23\,(-25+d)\,y^4 + 7\,r\,y^4\right)^2}{10000\,r^{10}}},$$

$$\text{SymbolicSurfaceBoundary} \to \{50,\ 50\},\ \text{SymbolicOffAxis} \to \{0,\ 0\},$$

$$\text{SymbolicTranslationVector} \to \{d,\ 0,\ 0\},$$

$$\text{SymbolicRotationMatrix} \to \{\{1,\ 0,\ 0\},\ \{0,\ 1,\ 0\},\ \{0,\ 0,\ 1\}\},$$

$$\text{SymbolicWaveLength} \to \frac{133}{250},\ \text{SymbolicSurfaceNormalFunction} \to \{-1,\ 0,\ 0\}\}$$

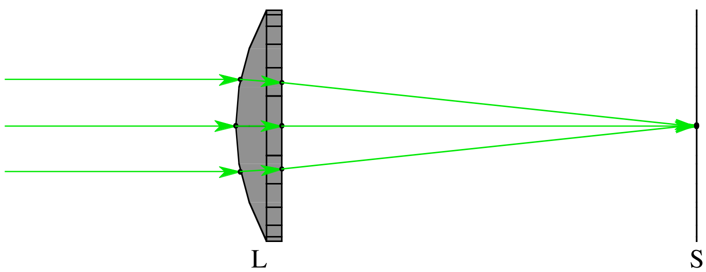
When refraction is involved, the index model needs to be substituted for a pure fraction of integer numbers within **`SymbolicTrace`** in order to maintain infinite precision. This example uses 3/2 for the **`BK7`** refractive index and 1 as the value of **`Air`**. For this, **`SymbolicTrace`** takes the **`SymbolicRefractiveModels`** as an option.

```
In[47]:= ?SymbolicRefractiveModels
```

SymbolicRefractiveModels -> {opticalmedium->symbolicexpression,...} is an option of TurboTrace and SymbolicTrace that provides a mechanism to replace pre-existing refractive index models with new models that contain user-defined symbolic parameters.

In this, each specified symbolicexpression holds the symbolic refractive index model to be exchanged with the pre-existing model identified by opticalmedium. See also: OpticalMedium.

```
In[99]:= opticalSystem = {
             Move[LineOfRays[20],{-50,{y,0}}],
             SphericalLens[{r,50},Infinity,50,10],
             Move[Screen[50],{d,100}]};
         AnalyzeSystem[opticalSystem,PlotType->TopView];
         SymbolicTrace[opticalSystem,
         {{y,0,5},{r>0}},SymbolicRefractiveModels->{BK7->3/2,Air->1}]
```



Out[101]=

$$\Big\{\text{SymbolicOpticalLength} \to 55 + d - \frac{5\,y^2}{12\,r^2} + \frac{d\,y^2}{8\,r^2} - \frac{y^2}{4\,r} - \frac{85\,y^4}{192\,r^4} + \frac{41\,d\,y^4}{384\,r^4} - \frac{5\,y^4}{48\,r^3},$$

$$\text{SymbolicRayEnd} \to \Big\{d,\ y + \frac{5\,y}{3\,r} - \frac{d\,y}{2\,r} + \frac{215\,y^3}{216\,r^3} - \frac{11\,d\,y^3}{48\,r^3} + \frac{y^3}{6\,r^2} + \frac{2885\,y^5}{3456\,r^5} - \frac{1121\,d\,y^5}{6912\,r^5} + \frac{23\,y^5}{216\,r^4},\ 0\Big\},$$

$$\text{SymbolicRayTilt} \to \Big\{1 - \frac{y^2}{8\,r^2} - \frac{35\,y^4}{384\,r^4},\ -\frac{y}{2\,r} - \frac{y^3}{6\,r^3} - \frac{19\,y^5}{216\,r^5},\ 0\Big\},$$

$$\text{SymbolicSurfaceCoordinates} \to$$

$$\Big\{y + \frac{5\,y}{3\,r} - \frac{d\,y}{2\,r} + \frac{215\,y^3}{216\,r^3} - \frac{11\,d\,y^3}{48\,r^3} + \frac{y^3}{6\,r^2} + \frac{2885\,y^5}{3456\,r^5} - \frac{1121\,d\,y^5}{6912\,r^5} + \frac{23\,y^5}{216\,r^4},\ 0\Big\},$$

$$\text{SymbolicIntensity} \to 100$$

$$e^{-\frac{y^2}{625} - \frac{y^2\,(-720\,r^4 + 216\,r^5 - 280\,r^2\,y^2 + 36\,r^3\,y^2 - 170\,y^4 + 23\,r\,y^4)^2}{29160000\,r^{10}} - \frac{y^2\,(-1152\,(-10+3\,d)\,r^4 + 6912\,r^5 - 16\,(-430+99\,d)\,r^2\,y^2 + 1152\,r^3\,y^2 + (5770-1121\,d)\,y^4 + 736\,r\,y^4)^2}{29859840000\,r^{10}}},$$

$$\text{SymbolicSurfaceBoundary} \to \{50,\ 50\},\ \text{SymbolicOffAxis} \to \{0,\ 0\},$$

$$\text{SymbolicTranslationVector} \to \{d,\ 0,\ 0\},$$

$$\text{SymbolicRotationMatrix} \to \{\{1,\ 0,\ 0\},\ \{0,\ 1,\ 0\},\ \{0,\ 0,\ 1\}\},$$

$$\text{SymbolicWaveLength} \to \frac{133}{250},\ \text{SymbolicSurfaceNormalFunction} \to \{1,\ 0,\ 0\}\Big\}$$

In this example, the original component medium, **`BK7`**, is replaced with **`3/2`**, and surrounding intrinsic medium, **`Air`**, is replaced with **`1`**. We can also specify the level of precision with the **`Precision`** option of **`SymbolicTrace`**.

Go to list of topics

2.2 Symbolic Optimization

In this section, we will consider how to use Wavica for symbolic optimization purposes. In particular, we will examine several different examples that illustrate some ways that symbolic optimization can be carried out with Wavica. Unlike numeric optimization procedures, symbolic raises the possibility of the finding the global minimum automatically.

Infinite-Conjugate Spherical Lens Design

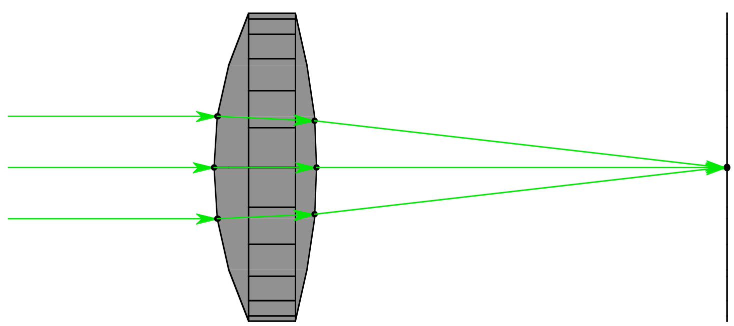
In this example, we find the optimal parameters for spherical lens system that focusses a plane wave onto a point at a surface. We first use SymbolicTrace to calculate a symbolic solution of a symbolic ray transversing through the system for different lens curvatures (represented by symbolic parameters **c1** and **c2**) and different transverse **y** positions of ray entry into the lens. First we define the optical system. In order to make the solution more clean, we assign integer values to the refractive index parameters of the system.

```
In[51]:= Clear[y,c1,c2];
```

```
In[52]:= opticalsystem = {
             Move[LineOfRays[10, IntrinsicMedium->1],{-20,{y,0}}],
             SphericalLens[{1/c1,35},{1/c2,-55},30,10,ComponentMedium->3/2],
             Move[Screen[30],50]};
```

Here is a numeric ray-trace of the system. Note that we have the trace is not optimized but instead uses the numeric assignments made to the optical system during its initial definition.

```
In[53]:= plot = TurboPlot[opticalsystem,PlotType->TopView, ShowArrows->True,
         EmbedRays->False];
```



Next, we use **SymbolicTrace** function to calculate the third order symbolic solution to the optical system. Although **SymbolicTrace** calculates several types of symbolic results, we will only use the **SymbolicSurfaceCoordinates** for optimization purposes. Here, **MakeFloatingPoint** retains the pure integer values of the numeric system parameters.

```
In[54]:= Py = (SymbolicSurfaceCoordinates/.
             SymbolicTrace[opticalsystem,{{y,0,3}, {c1>0,c2<0,y∈Reals}},
                 MakeFloatingPoint->False])[[1]]
```

Out[54]=
$$y-\frac{70\,c1\,y}{3}+20\,c2\,y-\frac{200\,c1\,c2\,y}{3}+\frac{c1^2\,y^3}{6}-\frac{565\,c1^3\,y^3}{54}+\frac{1}{12}\,c1\,c2\,y^3+\frac{215}{18}\,c1^2\,c2\,y^3-\frac{500}{9}\,c1^3\,c2\,y^3-\frac{c2^2\,y^3}{4}-\frac{55}{3}\,c1\,c2^2\,y^3+\frac{1175}{9}\,c1^2\,c2^2\,y^3-\frac{2000}{9}\,c1^3\,c2^2\,y^3+\frac{35\,c2^3\,y^3}{2}-175\,c1\,c2^3\,y^3+\frac{1750}{3}\,c1^2\,c2^3\,y^3-\frac{17500}{27}\,c1^3\,c2^3\,y^3$$

Now we are ready to calculate a symbolic figure of merit for the system performance. In particular, we want to base the performance of the system on every **y** value of the input ray across a finite aperture runs between -5 and 5. This is accomplished by integrating the square of the focal intercept solution given by **Py**, above.

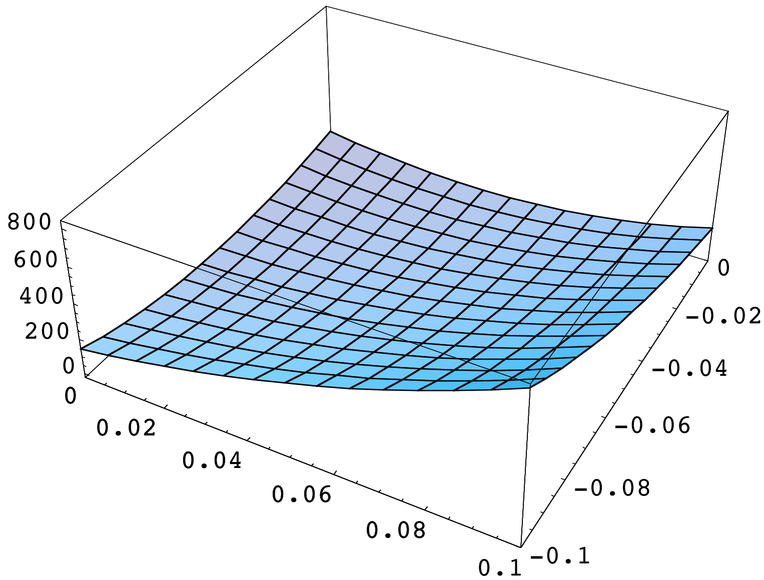
```
In[55]:= meritFunction = Expand[Integrate[Expand[Py^2],{y,-5,5}]]
```

Out[55]=

$$\frac{250}{3}-\frac{35000\,c1}{9}+\frac{1236250\,c1^2}{27}-\frac{968750\,c1^3}{27}+\frac{692828125\,c1^4}{1134}-\frac{44140625\,c1^5}{567}+\frac{24939453125\,c1^6}{10206}+\frac{10000\,c2}{3}-\frac{798125\,c1\,c2}{9}+\frac{7900000\,c1^2\,c2}{27}-\frac{523890625\,c1^3\,c2}{378}+\frac{1903046875\,c1^4\,c2}{378}-\frac{10193359375\,c1^5\,c2}{1701}+\frac{44140625000\,c1^6\,c2}{1701}+\frac{98125\,c2^2}{3}-\frac{2243750\,c1\,c2^2}{9}+\frac{3549921875\,c1^2\,c2^2}{1512}-\frac{2440937500\,c1^3\,c2^2}{189}+\frac{39389453125\,c1^4\,c2^2}{1134}-\frac{156933593750\,c1^5\,c2^2}{1701}+\frac{293750000000\,c1^6\,c2^2}{1701}+31250\,c2^3-\frac{65359375\,c1\,c2^3}{28}+\frac{2668515625\,c1^2\,c2^3}{126}-\frac{16379687500\,c1^3\,c2^3}{189}+\frac{156031250000\,c1^4\,c2^3}{567}-\frac{408007812500\,c1^5\,c2^3}{567}+\frac{4357421875000\,c1^6\,c2^3}{5103}+\frac{49078125\,c2^4}{56}-\frac{478671875\,c1\,c2^4}{42}+\frac{4602734375\,c1^2\,c2^4}{63}-\frac{23218750000\,c1^3\,c2^4}{63}+\frac{801191406250\,c1^4\,c2^4}{567}-\frac{5251953125000\,c1^5\,c2^4}{1701}+\frac{4609375000000\,c1^6\,c2^4}{1701}-\frac{390625\,c2^5}{2}-\frac{37109375\,c1\,c2^5}{3}+\frac{2148437500\,c1^2\,c2^5}{9}-\frac{44921875000\,c1^3\,c2^5}{27}+\frac{458984375000\,c1^4\,c2^5}{81}-\frac{2324218750000\,c1^5\,c2^5}{243}+\frac{1562500000000\,c1^6\,c2^5}{243}+\frac{13671875\,c2^6}{2}-136718750\,c1\,c2^6+\frac{3417968750\,c1^2\,c2^6}{3}-\frac{136718750000\,c1^3\,c2^6}{27}+\frac{341796875000\,c1^4\,c2^6}{27}-\frac{1367187500000\,c1^5\,c2^6}{81}+\frac{6835937500000\,c1^6\,c2^6}{729}$$

Here is a plot of the resulting merit function for diffferent lens curvatures.

```
In[56]:= Plot3D[meritFunction,{c1,0,.1},{c2,-.1,0}];
```



Using minimization techniques in Calculus, we can find the symbolic global optimization for this lens configuration:

```
In[57]:= Dc1 = D[meritFunction,c1]
```

Out[57]=

$$-\frac{35000}{9}+\frac{2472500\,c1}{27}-\frac{968750\,c1^{2}}{9}+\frac{1385656250\,c1^{3}}{567}-\frac{220703125\,c1^{4}}{567}+\frac{24939453125\,c1^{5}}{1701}-\frac{798125\,c2}{9}+\frac{15800000\,c1\,c2}{27}-\frac{523890625\,c1^{2}\,c2}{126}+\frac{3806093750\,c1^{3}\,c2}{189}-\frac{50966796875\,c1^{4}\,c2}{1701}+\frac{88281250000\,c1^{5}\,c2}{567}-\frac{2243750\,c2^{2}}{9}+\frac{3549921875\,c1\,c2^{2}}{756}-\frac{2440937500\,c1^{2}\,c2^{2}}{63}+\frac{78778906250\,c1^{3}\,c2^{2}}{567}-\frac{784667968750\,c1^{4}\,c2^{2}}{1701}+\frac{587500000000\,c1^{5}\,c2^{2}}{567}-\frac{65359375\,c2^{3}}{28}+\frac{2668515625\,c1\,c2^{3}}{63}-\frac{16379687500\,c1^{2}\,c2^{3}}{63}+\frac{624125000000\,c1^{3}\,c2^{3}}{567}-\frac{2040039062500\,c1^{4}\,c2^{3}}{567}+\frac{8714843750000\,c1^{5}\,c2^{3}}{1701}-\frac{478671875\,c2^{4}}{42}+\frac{9205468750\,c1\,c2^{4}}{63}-\frac{23218750000\,c1^{2}\,c2^{4}}{21}+\frac{3204765625000\,c1^{3}\,c2^{4}}{567}-\frac{26259765625000\,c1^{4}\,c2^{4}}{1701}+\frac{9218750000000\,c1^{5}\,c2^{4}}{567}-\frac{37109375\,c2^{5}}{3}+\frac{4296875000\,c1\,c2^{5}}{9}-\frac{44921875000\,c1^{2}\,c2^{5}}{9}+\frac{1835937500000\,c1^{3}\,c2^{5}}{81}-\frac{11621093750000\,c1^{4}\,c2^{5}}{243}+\frac{3125000000000\,c1^{5}\,c2^{5}}{81}-136718750\,c2^{6}+\frac{6835937500\,c1\,c2^{6}}{3}-\frac{136718750000\,c1^{2}\,c2^{6}}{9}+\frac{1367187500000\,c1^{3}\,c2^{6}}{27}-\frac{6835937500000\,c1^{4}\,c2^{6}}{81}+\frac{13671875000000\,c1^{5}\,c2^{6}}{243}$$

```
In[58]:= Dc2 = D[meritFunction,c2]
```

Out[58]=

$$\frac{10000}{3} - \frac{798125\,c1}{9} + \frac{7900000\,c1^2}{27} - \frac{523890625\,c1^3}{378} + \frac{1903046875\,c1^4}{378} - \frac{10193359375\,c1^5}{1701} + \frac{44140625000\,c1^6}{1701} + \frac{196250\,c2}{3} - \frac{4487500\,c1\,c2}{9} + \frac{3549921875\,c1^2\,c2}{756} - \frac{4881875000\,c1^3\,c2}{189} + \frac{39389453125\,c1^4\,c2}{567} - \frac{313867187500\,c1^5\,c2}{1701} + \frac{587500000000\,c1^6\,c2}{1701} + 93750\,c2^2 - \frac{196078125\,c1\,c2^2}{28} + \frac{2668515625\,c1^2\,c2^2}{42} - \frac{16379687500\,c1^3\,c2^2}{63} + \frac{156031250000\,c1^4\,c2^2}{189} - \frac{408007812500\,c1^5\,c2^2}{189} + \frac{4357421875000\,c1^6\,c2^2}{1701} + \frac{49078125\,c2^3}{14} - \frac{957343750\,c1\,c2^3}{21} + \frac{18410937500\,c1^2\,c2^3}{63} - \frac{92875000000\,c1^3\,c2^3}{63} + \frac{3204765625000\,c1^4\,c2^3}{567} - \frac{21007812500000\,c1^5\,c2^3}{1701} + \frac{18437500000000\,c1^6\,c2^3}{1701} - \frac{1953125\,c2^4}{2} - \frac{185546875\,c1\,c2^4}{3} + \frac{10742187500\,c1^2\,c2^4}{9} - \frac{224609375000\,c1^3\,c2^4}{27} + \frac{2294921875000\,c1^4\,c2^4}{81} - \frac{11621093750000\,c1^5\,c2^4}{243} + \frac{7812500000000\,c1^6\,c2^4}{243} + 41015625\,c2^5 - 820312500\,c1\,c2^5 + 6835937500\,c1^2\,c2^5 - \frac{273437500000\,c1^3\,c2^5}{9} + \frac{683593750000\,c1^4\,c2^5}{9} - \frac{2734375000000\,c1^5\,c2^5}{27} + \frac{13671875000000\,c1^6\,c2^5}{243}$$

Next we use **NSolve** to find the roots of the these two equations. These roots contain all possible solutions to third order system model.

```
In[59]:= extrema = NSolve[{Dc1==0,Dc2==0},{c1,c2}]
```

```
Out[59]= {{c1 → 0.358475 + 0.0501567 i, c2 → 1.01004 + 1.7221 i},
  {c1 → 0.358475 - 0.0501567 i, c2 → 1.01004 - 1.7221 i}, {c1 → 0.436262, c2 → -0.593664},
  {c1 → 0.382622 + 0.050184 i, c2 → 0.0616245 + 0.949032 i},
  {c1 → 0.382622 - 0.050184 i, c2 → 0.0616245 - 0.949032 i},
  {c1 → 0.255498 + 0.0885354 i, c2 → -0.642836 + 0.458605 i},
  {c1 → 0.255498 - 0.0885354 i, c2 → -0.642836 - 0.458605 i},
  {c1 → 0.218606 + 0.109023 i, c2 → -0.247463 + 0.496542 i},
  {c1 → 0.218606 - 0.109023 i, c2 → -0.247463 - 0.496542 i}, {c1 → 0.267608, c2 → -0.32648},
  {c1 → -0.01292 - 0.270887 i, c2 → 0.0677989 + 0.0553778 i},
  {c1 → -0.01292 + 0.270887 i, c2 → 0.0677989 - 0.0553778 i},
  {c1 → -0.0254064 - 0.378963 i, c2 → -0.159192 - 0.0892886 i},
  {c1 → -0.0254064 + 0.378963 i, c2 → -0.159192 + 0.0892886 i},
  {c1 → 0.143328 + 0.173272 i, c2 → -0.0916045 + 0.285773 i},
  {c1 → 0.143328 - 0.173272 i, c2 → -0.0916045 - 0.285773 i},
  {c1 → 0.0854071 - 0.298016 i, c2 → -0.157108 - 0.134021 i},
  {c1 → 0.0854071 + 0.298016 i, c2 → -0.157108 + 0.134021 i},
  {c1 → 0.00790555 - 0.18933 i, c2 → 0.0459484 + 0.0564512 i},
  {c1 → 0.00790555 + 0.18933 i, c2 → 0.0459484 - 0.0564512 i},
  {c1 → 0.127188 + 0.116232 i, c2 → -0.0083885 + 0.255923 i},
  {c1 → 0.127188 - 0.116232 i, c2 → -0.0083885 - 0.255923 i},
  {c1 → 0.0377632 + 0.0179285 i, c2 → -0.00839548 + 0.0233546 i},
  {c1 → 0.0377632 - 0.0179285 i, c2 → -0.00839548 - 0.0233546 i},
  {c1 → 0.0377511, c2 → -0.00640935}}
```

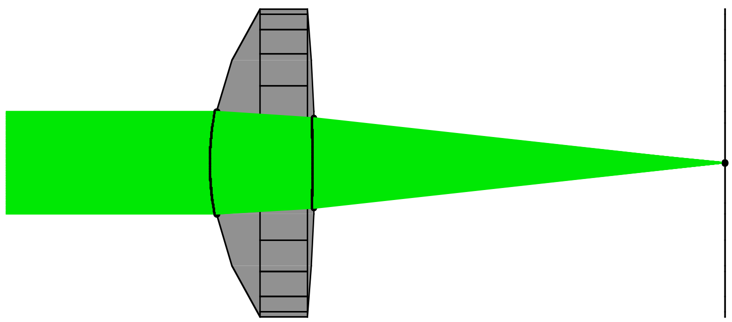
Note that many of the roots contain imaginary components. In addition, some of the solutions are local minima and do not represent the global solution to the system. However, by shifting through the different roots and comparing their results we can determine the global solution to the system.

```
In[60]:= finalextrema = Select[Union[Chop[Transpose[{meritFunction/.extrema,extrema}]]],
              (#[[1]]==Re[#[[1]]]&&#[[1]]<1&&#[[1]]>0)&]
```

Out[60]= {{0.00078945, {c1 → 0.0377511, c2 → -0.00640935}}}

Finally, we can verify the resulting solution by numerically tracing the resulting system and calculating the resulting focal point and spot size with **FindFocusFast**.

```
In[61]:= FindFocusFast[TurboPlot[
              Move[LineOfRays[10,NumberOfRays->201,IntrinsicMedium->1],{-20,0}],
              plot,SymbolicValues->finalextrema[[1,2]], PlotType->TopView]]
```



Out[61]= {FocalPoint → {49.9903, 0, 0}, FocusType → RMSFocus, WeightedSpotSize → 0.00933056, SpotSize → 0.00933056, BackFocalLength → 40.011, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}

It is interesting to note that the calculated focal point position (49.9903) is slightly different from the final surface position (50). This is partly because the symbolic model uses an infinite number of rays in its integration calculation while **TurboPlot** is using only a finite number of rays. In addition, however, the symbolic model is a third order solution and, as such, does not perfectly model the actual system. Therefore, we will next try to improve the accuracy of our symbolic model by using a fifth order symbolic solution.

```
In[62]:= Py = (SymbolicSurfaceCoordinates/.
              SymbolicTrace[opticalsystem,{{y,0,5}, {c1>0,c2<0,y∈Reals}},
          MakeFloatingPoint->False])[[1]]
```

Out[62]=
$$\frac{1}{7776}\left(-2592\,(-3-60\,c2+10\,c1\,(7+20\,c2))\,y-72\,(2\,c1^2\,(-9+565\,c1)+3\,c1\,(-3+10\,c1\,(-43+200\,c1))\,c2+3\,(3-10\,c1)^2\,(1+80\,c1)\,c2^2+70\,(-3+10\,c1)^3\,c2^3)\,y^3-(9\,c1^4\,(-92+6285\,c1)+3\,c1^3\,(-201+25\,c1\,(-1277+5946\,c1))\,c2+c1^2\,(-3+10\,c1)\,(-693+10\,c1\,(-6489+30400\,c1))\,c2^2+15\,(3-10\,c1)^2\,c1\,(-21+2\,c1\,(-1159+5100\,c1))\,c2^3+(3-10\,c1)^4\,(27+4295\,c1)\,c2^4+765\,(-3+10\,c1)^5\,c2^5)\,y^5\right)$$

Once again, we integrate across the system aperture to calculate a symbolic merit function for the system's performance.

```
In[63]:= meritFunction = Expand[Integrate[Expand[Py^2],{y,-5,5}]]
```

Out[63]=
$$\frac{250}{3}-\frac{35000\,c1}{9}+\frac{1236250\,c1^2}{27}-\frac{968750\,c1^3}{27}+\frac{349109375\,c1^4}{567}-\frac{2329296875\,c1^5}{4536}+\frac{102431640625\,c1^6}{10206}-\frac{212001953125\,c1^7}{104976}+\frac{50938525390625\,c1^8}{769824}-\frac{2352783203125\,c1^9}{171072}+\frac{214307861328125\,c1^{10}}{456192}+\frac{10000\,c2}{3}-\frac{798125\,c1\,c2}{9}+\frac{7900000\,c1^2\,c2}{27}-\frac{2090328125\,c1^3\,c2}{1512}+\frac{25395546875\,c1^4\,c2}{4536}-\frac{2300486328125\,c1^5\,c2}{81648}+\frac{160184697265625\,c1^6\,c2}{1469664}-$$

$$\frac{2779331298828125\,c1^{7}\,c2}{13856832}+\frac{1486375732421875\,c1^{8}\,c2}{1679616}-\frac{1162784423828125\,c1^{9}\,c2}{684288}+$$
$$\frac{2534356689453125\,c1^{10}\,c2}{342144}+\frac{98125\,c2^{2}}{3}-\frac{2243750\,c1\,c2^{2}}{9}+\frac{441484375\,c1^{2}\,c2^{2}}{189}-$$
$$\frac{1473671875\,c1^{3}\,c2^{2}}{108}+\frac{12991157421875\,c1^{4}\,c2^{2}}{163296}-\frac{99485732421875\,c1^{5}\,c2^{2}}{244944}+$$
$$\frac{322325887451171875\,c1^{6}\,c2^{2}}{258660864}-\frac{6768020263671875\,c1^{7}\,c2^{2}}{1732104}+\frac{415219091430664062 5\,c1^{8}\,c2^{2}}{332563968}-$$
$$\frac{36219696655273437 5\,c1^{9}\,c2^{2}}{9237888}+\frac{24540048217773437 5\,c1^{10}\,c2^{2}}{3079296}+31250\,c2^{3}-$$
$$\frac{389421875\,c1\,c2^{3}}{168}+\frac{200234375\,c1^{2}\,c2^{3}}{9}-\frac{1510480859375\,c1^{3}\,c2^{3}}{9072}+\frac{24587552734375\,c1^{4}\,c2^{3}}{23328}-$$
$$\frac{2098391782226562 5\,c1^{5}\,c2^{3}}{4790016}+\frac{68883686767578125\,c1^{6}\,c2^{3}}{4618944}-\frac{2957855224609375\,c1^{7}\,c2^{3}}{57024}+$$
$$\frac{1246444476318359375\,c1^{8}\,c2^{3}}{6928416}-\frac{218181732177734375\,c1^{9}\,c2^{3}}{433026}+\frac{376575469970703125\,c1^{10}\,c2^{3}}{577368}+$$
$$\frac{6046875\,c2^{4}}{7}-\frac{2118203125\,c1\,c2^{4}}{168}+\frac{1580979296875\,c1^{2}\,c2^{4}}{9072}-\frac{1607212890625\,c1^{3}\,c2^{4}}{1008}+$$
$$\frac{253555792197265625\,c1^{4}\,c2^{4}}{28740096}-\frac{798616814697265625\,c1^{5}\,c2^{4}}{21555072}+\frac{210425825805664062 5\,c1^{6}\,c2^{4}}{14370048}-$$
$$\frac{3988927069091796875\,c1^{7}\,c2^{4}}{6928416}+\frac{2563099151611328125\,c1^{8}\,c2^{4}}{1299078}-\frac{940356750488281250\,c1^{9}\,c2^{4}}{216513}+$$
$$\frac{3848905181884765625\,c1^{10}\,c2^{4}}{944784}+\frac{34765625\,c2^{5}}{56}-\frac{19638671875\,c1\,c2^{5}}{216}+\frac{8404970703125\,c1^{2}\,c2^{5}}{6048}-$$
$$\frac{2823120654296875\,c1^{3}\,c2^{5}}{266112}+\frac{211972466552734375\,c1^{4}\,c2^{5}}{3592512}-\frac{6230438472900390625\,c1^{5}\,c2^{5}}{21555072}+$$
$$\frac{14390346649169921875\,c1^{6}\,c2^{5}}{10777536}-\frac{6310224212646484375\,c1^{7}\,c2^{5}}{1154736}+$$
$$\frac{84513123931884765625\,c1^{8}\,c2^{5}}{5196312}-\frac{48772490692138671875\,c1^{9}\,c2^{5}}{1732104}+$$
$$\frac{19777870178222656 25\,c1^{10}\,c2^{5}}{96228}+\frac{6326171875\,c2^{6}}{224}-\frac{1641962890625\,c1\,c2^{6}}{3024}+$$
$$\frac{6314786669921875\,c1^{2}\,c2^{6}}{1064448}-\frac{20271781005859375\,c1^{3}\,c2^{6}}{399168}+\frac{465213931884765625\,c1^{4}\,c2^{6}}{1306368}-$$
$$\frac{7652917498779296875\,c1^{5}\,c2^{6}}{3592512}+\frac{115843923309326171875\,c1^{6}\,c2^{6}}{10777536}-$$
$$\frac{6395081329345703125\,c1^{7}\,c2^{6}}{157464}+\frac{346760147857666015625\,c1^{8}\,c2^{6}}{3464208}-$$
$$\frac{120812252044677734375\,c1^{9}\,c2^{6}}{866052}+\frac{214941539764404296875\,c1^{10}\,c2^{6}}{2598156}-\frac{302734375\,c2^{7}}{32}-$$
$$\frac{66585302734375\,c1\,c2^{7}}{76032}+\frac{324571533203125\,c1^{2}\,c2^{7}}{14256}-\frac{15572891845703125\,c1^{3}\,c2^{7}}{57024}+$$
$$\frac{591092437744140625\,c1^{4}\,c2^{7}}{256608}-\frac{5760250885009765625\,c1^{5}\,c2^{7}}{384912}+\frac{13644067230224609375\,c1^{6}\,c2^{7}}{192456}-$$
$$\frac{196852710723876953125\,c1^{7}\,c2^{7}}{866052}+\frac{597395549774169921875\,c1^{8}\,c2^{7}}{1299078}-$$
$$\frac{114205551147460937500\,c1^{9}\,c2^{7}}{216513}+\frac{188550949096679687 50\,c1^{10}\,c2^{7}}{72171}+\frac{2049267578125\,c2^{8}}{5632}-$$
$$\frac{39387939453125\,c1\,c2^{8}}{4224}+\frac{2339837646484375\,c1^{2}\,c2^{8}}{16896}-\frac{56203002929687 5\,c1^{3}\,c2^{8}}{352}+$$
$$\frac{4355581665039062 5\,c1^{4}\,c2^{8}}{3168}-\frac{586810760498046875\,c1^{5}\,c2^{8}}{7128}+\frac{14212165069580078125\,c1^{6}\,c2^{8}}{42768}-$$
$$\frac{14196491241455078125\,c1^{7}\,c2^{8}}{16038}+\frac{35912551879882812500\,c1^{8}\,c2^{8}}{24057}-$$
$$\frac{31318550109863281250 0\,c1^{9}\,c2^{8}}{216513}+\frac{798340320587158203125\,c1^{10}\,c2^{8}}{1299078}-\frac{336181640625\,c2^{9}}{2816}-$$

$$\frac{43392333984375\,c1\,c2^{9}}{2816}+\frac{734930419921875\,c1^{2}\,c2^{9}}{1408}-\frac{1271575927734375\,c1^{3}\,c2^{9}}{176}+$$
$$\frac{1888427734375000\,c1^{4}\,c2^{9}}{33}-\frac{76354217529296875\,c1^{5}\,c2^{9}}{264}+\frac{34954071044921875\,c1^{6}\,c2^{9}}{36}-$$
$$\frac{1932201385498046875\,c1^{7}\,c2^{9}}{891}+\frac{2770709991455078125\,c1^{8}\,c2^{9}}{891}-\frac{6947040557861328125\,c1^{9}\,c2^{9}}{2673}+$$
$$\frac{69632530212402343750\,c1^{10}\,c2^{9}}{72171}+\frac{28575439453125\,c2^{10}}{5632}-\frac{238128662109375\,c1\,c2^{10}}{1408}+$$
$$\frac{3571929931640625\,c1^{2}\,c2^{10}}{1408}-\frac{1984405517578125\,c1^{3}\,c2^{10}}{88}+\frac{23151397705078125\,c1^{4}\,c2^{10}}{176}-$$
$$\frac{23151397705078125\,c1^{5}\,c2^{10}}{44}+\frac{192928314208984375\,c1^{6}\,c2^{10}}{132}-\frac{275611877441406250\,c1^{7}\,c2^{10}}{99}+$$
$$\frac{689029693603515625\,c1^{8}\,c2^{10}}{198}-\frac{6890296936035156250\,c1^{9}\,c2^{10}}{2673}+\frac{6890296936035156250\,c1^{10}\,c2^{10}}{8019}$$

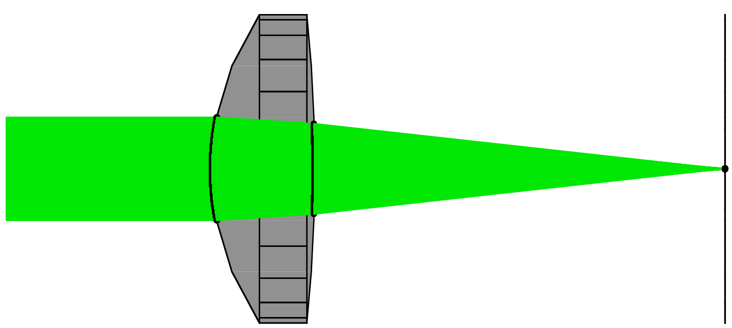
This time, since we already have the third-order global minimum for lens system, we will use its solution as a starting point for a search with **FindMinimum** on the fifth-order model.

```
In[64]:= localminimum =
         FindMinimum[meritFunction,{c1,Evaluate[c1/.finalextrema[[1,2]]]},{c2,Evaluate[c2/.fin
         alextrema[[1,2]]]}, AccuracyGoal->12]
```

```
Out[64]= {0.0008425, {c1 → 0.0376879, c2 → -0.00648358}}
```

Since the third-order solution was already determined, the fifth-order result with FindMinimum is calculated very quickly. And we can, once again, verify the resulting solution by numerically tracing the resulting system and calculating the resulting focal point and spot size with **FindFocusFast**.

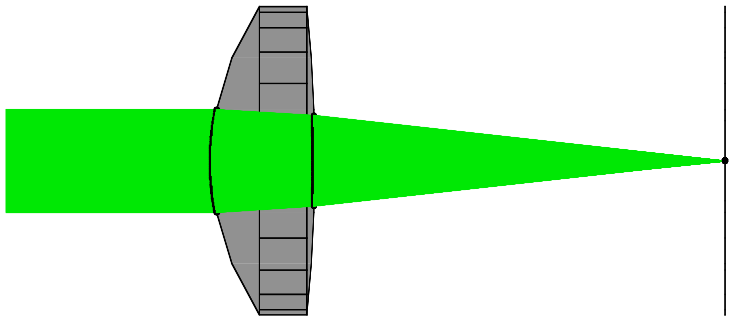
```
In[65]:= FindFocusFast[TurboPlot[
             Move[LineOfRays[10,NumberOfRays->201,IntrinsicMedium->1],{-20,0}],
             plot,SymbolicValues->localminimum[[2]], PlotType->TopView]]
```



```
Out[65]= {FocalPoint → {49.9973, 0, 0}, FocusType → RMSFocus,
          WeightedSpotSize → 0.00932762, SpotSize → 0.00932762,
          BackFocalLength → 40.0184, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}
```

When we trace the system with 101 rays, this time, the calculated focal point is much better than the third order model. However the result still does not completely match the focal surface position of the system model. However, when we increase the number of traced rays to 1001, the resulting focal position matches the final surface position precisely.

```
In[66]:= FindFocusFast[TurboPlot[
             Move[LineOfRays[10,NumberOfRays->1001,IntrinsicMedium->1],{-20,0}],
             plot,SymbolicValues->localminimum[[2]], PlotType->TopView]]
```



```
Out[66]= {FocalPoint → {50., 0, 0}, FocusType → RMSFocus,
          WeightedSpotSize → 0.00921652, SpotSize → 0.00921652,
          BackFocalLength → 40.0209, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}
```

This final calculation indicates that the fifth-order symbolic solution is a close fit to the actual system.

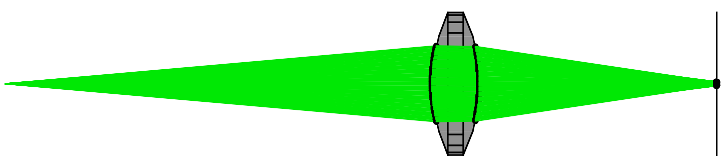
Go to list of topics

Finite-Conjugate Spherical Lens Design

In this section, we will consider the application of symbolic optimization to a finite-conjugate lens system. Finite-conjugate lens systems are considerably more difficult to accurately represent symbolically than a similar finite-conjugate system. In particular, the use of symbolic angular parameters requires a greater number of expansion terms in order order to obtain the same modelling accuracy. This can significantly increase the size of the symbolic solution as well as the amount of time required for the symbolic calculation. Let us now consider such an optical system that images a point source of light onto a surface. This is defined for the **opticalsystem** shown below.

```
In[67]:= opticalsystem = {
             Move[WedgeOfRays[10,NumberOfRays->101, IntrinsicMedium->1],-90,{θ/Degree,0}],
             SphericalLens[{1/c1,35},{1/c2,-35},30,10, ComponentMedium->3/2],
             Move[Screen[30],60]};
```

```
In[68]:= plot = TurboPlot[opticalsystem,PlotType->TopView, EmbedRays->False];
```



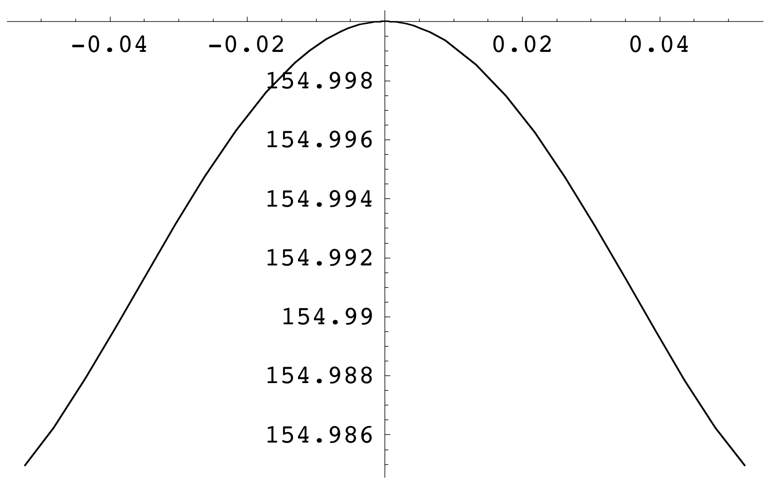
Next we use **SymbolicTrace** to determine the fifth order solution to the symbolic optical path length of the system as function of lens curvatures, **c1** and **c2**, and input ray angle, **θ**.

```
In[69]:= OL = SymbolicOpticalLength/.SymbolicTrace[opticalsystem, {{θ,0,5}, {c1>0,c2<0,θ∈
        Reals}}, ReportedParameters->SymbolicOpticalLength, MakeFloatingPoint->False]
```

Out[69]=
$$155+\frac{5}{9}\left(132+8555\,c2+105125\,c2^2+2025\,c1^2\left(51+320\,c2+500\,c2^2\right)-45\,c1\left(183+5230\,c2+14500\,c2^2\right)\right)\theta^2+\frac{25}{324}\left(378+70972\,c2+7447490\,c2^2+524363500\,c2^3+11935366875\,c2^4+820125\,c1^4\left(6183+53000\,c2+243400\,c2^2+860000\,c2^3+1350000\,c2^4\right)-72900\,c1^3\left(1584+165425\,c2+1715950\,c2^2+9567500\,c2^3+19575000\,c2^4\right)+2430\,c1^2\left(-57+115240\,c2+8920575\,c2^2+96642500\,c2^3+283837500\,c2^4\right)-36\,c1\left(1989+207815\,c2+19394875\,c2^2+791065625\,c2^3+4115643750\,c2^4\right)\right)\theta^4$$

Here is a plot of the symbolic optical path length as a function of ray angle. This uses the default initial settings of lens curvature.

```
In[70]:= oplot = Plot[OL/.{c1->1/35,c2->-1/35},{θ,-3. Degree, 3. Degree}];
```



We can now compare this symbolic result with a numeric trace of the same system to see how close the fifth-order symbolic model matches the actual system. To this end, we need to extract the numeric transfer function of the optical system via a numeric ray-trace of the default lens configuration. We use **ReadTurboRays** to extract the numeric data from the trace.

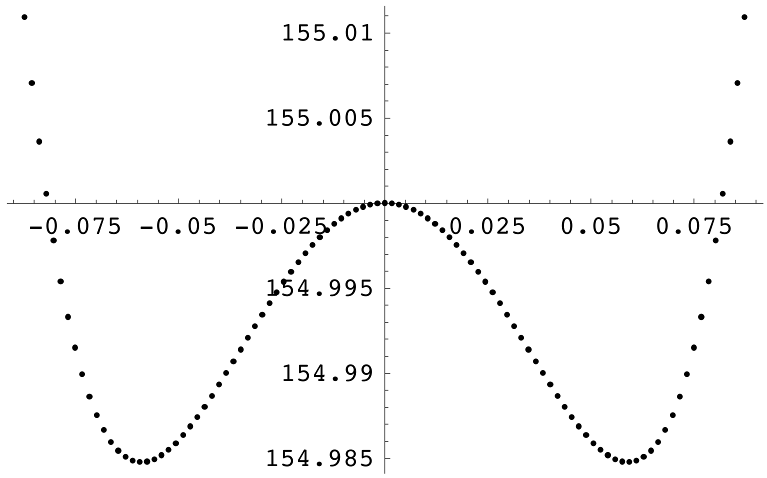
```
In[71]:= data = ReadTurboRays[plot,
        {RayTilt,OpticalLength,IntersectionNumber,RaySourceNumber}];
```

```
In[72]:= inputangles = Map[{ArcTan[#[[1,1]],#[[1,2]]],#[[4]]}&,Select[data,(#[[3]]==1)&]]

Out[72]= {{0.0872665, {{1., 101.}, {1., 0.}}},
  {0.0855211, {{2., 101.}, {2., 0.}}}, {0.0837758, {{3., 101.}, {3., 0.}}},
  {0.0820305, {{4., 101.}, {4., 0.}}}, {0.0802851, {{5., 101.}, {5., 0.}}},
  {0.0785398, {{6., 101.}, {6., 0.}}}, {0.0767945, {{7., 101.}, {7., 0.}}},
  {0.0750492, {{8., 101.}, {8., 0.}}}, {0.0733038, {{9., 101.}, {9., 0.}}},
  {0.0715585, {{10., 101.}, {10., 0.}}}, {0.0698132, {{11., 101.}, {11., 0.}}},
  {0.0680678, {{12., 101.}, {12., 0.}}}, {0.0663225, {{13., 101.}, {13., 0.}}},
  {0.0645772, {{14., 101.}, {14., 0.}}}, {0.0628319, {{15., 101.}, {15., 0.}}},
  {0.0610865, {{16., 101.}, {16., 0.}}}, {0.0593412, {{17., 101.}, {17., 0.}}},
  {0.0575959, {{18., 101.}, {18., 0.}}}, {0.0558505, {{19., 101.}, {19., 0.}}},
  {0.0541052, {{20., 101.}, {20., 0.}}}, {0.0523599, {{21., 101.}, {21., 0.}}},
  {0.0506145, {{22., 101.}, {22., 0.}}}, {0.0488692, {{23., 101.}, {23., 0.}}},
  {0.0471239, {{24., 101.}, {24., 0.}}}, {0.0453786, {{25., 101.}, {25., 0.}}},
  {0.0436332, {{26., 101.}, {26., 0.}}}, {0.0418879, {{27., 101.}, {27., 0.}}},
  {0.0401426, {{28., 101.}, {28., 0.}}}, {0.0383972, {{29., 101.}, {29., 0.}}},
  {0.0366519, {{30., 101.}, {30., 0.}}}, {0.0349066, {{31., 101.}, {31., 0.}}},
  {0.0331613, {{32., 101.}, {32., 0.}}}, {0.0314159, {{33., 101.}, {33., 0.}}},
  {0.0296706, {{34., 101.}, {34., 0.}}}, {0.0279253, {{35., 101.}, {35., 0.}}},
  {0.0261799, {{36., 101.}, {36., 0.}}}, {0.0244346, {{37., 101.}, {37., 0.}}},
  {0.0226893, {{38., 101.}, {38., 0.}}}, {0.020944, {{39., 101.}, {39., 0.}}},
  {0.0191986, {{40., 101.}, {40., 0.}}}, {0.0174533, {{41., 101.}, {41., 0.}}},
  {0.015708, {{42., 101.}, {42., 0.}}}, {0.0139626, {{43., 101.}, {43., 0.}}},
  {0.0122173, {{44., 101.}, {44., 0.}}}, {0.010472, {{45., 101.}, {45., 0.}}},
  {0.00872665, {{46., 101.}, {46., 0.}}}, {0.00698132, {{47., 101.}, {47., 0.}}},
  {0.00523599, {{48., 101.}, {48., 0.}}}, {0.00349066, {{49., 101.}, {49., 0.}}},
  {0.00174533, {{50., 101.}, {50., 0.}}}, {0., {{51., 101.}, {51., 0.}}},
  {-0.00174533, {{52., 101.}, {52., 0.}}}, {-0.00349066, {{53., 101.}, {53., 0.}}},
  {-0.00523599, {{54., 101.}, {54., 0.}}}, {-0.00698132, {{55., 101.}, {55., 0.}}},
  {-0.00872665, {{56., 101.}, {56., 0.}}}, {-0.010472, {{57., 101.}, {57., 0.}}},
  {-0.0122173, {{58., 101.}, {58., 0.}}}, {-0.0139626, {{59., 101.}, {59., 0.}}},
  {-0.015708, {{60., 101.}, {60., 0.}}}, {-0.0174533, {{61., 101.}, {61., 0.}}},
  {-0.0191986, {{62., 101.}, {62., 0.}}}, {-0.020944, {{63., 101.}, {63., 0.}}},
  {-0.0226893, {{64., 101.}, {64., 0.}}}, {-0.0244346, {{65., 101.}, {65., 0.}}},
  {-0.0261799, {{66., 101.}, {66., 0.}}}, {-0.0279253, {{67., 101.}, {67., 0.}}},
  {-0.0296706, {{68., 101.}, {68., 0.}}}, {-0.0314159, {{69., 101.}, {69., 0.}}},
  {-0.0331613, {{70., 101.}, {70., 0.}}}, {-0.0349066, {{71., 101.}, {71., 0.}}},
  {-0.0366519, {{72., 101.}, {72., 0.}}}, {-0.0383972, {{73., 101.}, {73., 0.}}},
  {-0.0401426, {{74., 101.}, {74., 0.}}}, {-0.0418879, {{75., 101.}, {75., 0.}}},
  {-0.0436332, {{76., 101.}, {76., 0.}}}, {-0.0453786, {{77., 101.}, {77., 0.}}},
  {-0.0471239, {{78., 101.}, {78., 0.}}}, {-0.0488692, {{79., 101.}, {79., 0.}}},
  {-0.0506145, {{80., 101.}, {80., 0.}}}, {-0.0523599, {{81., 101.}, {81., 0.}}},
  {-0.0541052, {{82., 101.}, {82., 0.}}}, {-0.0558505, {{83., 101.}, {83., 0.}}},
  {-0.0575959, {{84., 101.}, {84., 0.}}}, {-0.0593412, {{85., 101.}, {85., 0.}}},
  {-0.0610865, {{86., 101.}, {86., 0.}}}, {-0.0628319, {{87., 101.}, {87., 0.}}},
  {-0.0645772, {{88., 101.}, {88., 0.}}}, {-0.0663225, {{89., 101.}, {89., 0.}}},
  {-0.0680678, {{90., 101.}, {90., 0.}}}, {-0.0698132, {{91., 101.}, {91., 0.}}},
  {-0.0715585, {{92., 101.}, {92., 0.}}}, {-0.0733038, {{93., 101.}, {93., 0.}}},
  {-0.0750492, {{94., 101.}, {94., 0.}}}, {-0.0767945, {{95., 101.}, {95., 0.}}},
  {-0.0785398, {{96., 101.}, {96., 0.}}}, {-0.0802851, {{97., 101.}, {97., 0.}}},
  {-0.0820305, {{98., 101.}, {98., 0.}}}, {-0.0837758, {{99., 101.}, {99., 0.}}},
  {-0.0855211, {{100., 101.}, {100., 0.}}}, {-0.0872665, {{101., 101.}, {101., 0.}}}}
```

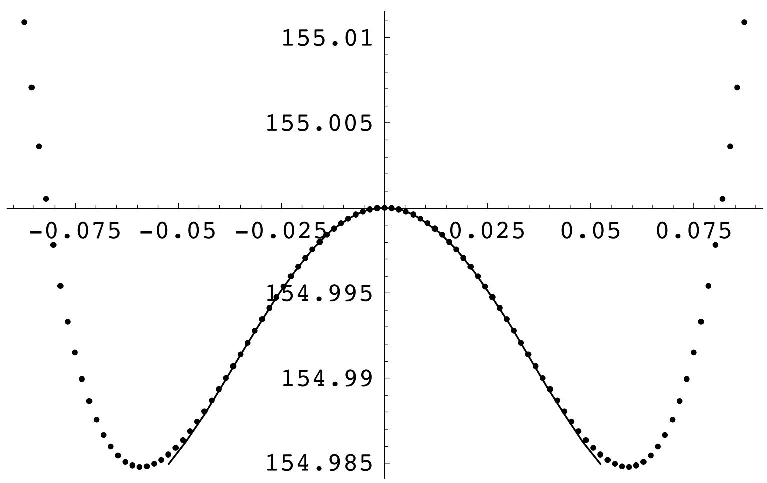
Next, we use **`ListPlot`** to plot the optical transfer function of the system.

```
In[73]:= dataplot =
ListPlot[Map[{#[[1,1]],#[[2,2]]}&,Transpose[{inputangles,Select[data,(#[[3]]==3)&]}]]
];
```



Finally, we can compare a plot of numeric ray-trace points with the result given by the symbolic equation.

```
In[74]:= Show[dataplot,oplot];
```



From this, we see that the symbolic result follows the numeric behavior quite well, although the symbolic result begins to deviate from the numeric result after about .05 radians. For this reason, we will limit the absolute angular values of our symbolic model to less than .05 radians. Based on this knowledge, we can now construct a merit function that integrates between the -.05 and .05 radians of angular space.

```
In[75]:= meritFunction=Expand[Integrate[Expand[D[OL,θ]^2],{θ,-5/100,5/100}]]
```

Out[75]=

$$
\begin{aligned}
&\frac{12419989}{6912000} - \frac{8630209\,c1}{38400} + \frac{8811546167\,c1^2}{896000} - \frac{15844701609\,c1^3}{89600} + \\
&\frac{5534665929\,c1^4}{4480} - \frac{33096701331\,c1^5}{7168} + \frac{647294186655\,c1^6}{14336} - \frac{11155832325\,c1^7}{448} + \\
&\frac{31352959666125\,c1^8}{57344} + \frac{26892241\,c2}{115200} - \frac{765076197961\,c1\,c2}{36288000} + \frac{23372228129\,c1^2\,c2}{38400} - \\
&\frac{29509050087\,c1^3\,c2}{4480} + \frac{224966885553\,c1^4\,c2}{7168} - \frac{1786598855595\,c1^5\,c2}{7168} + \\
&\frac{704189997675\,c1^6\,c2}{896} - \frac{20170970274375\,c1^7\,c2}{7168} + \frac{33594267796875\,c1^8\,c2}{3584} + \\
&\frac{61871970916463\,c2^2}{5878656000} - \frac{20605878091867\,c1\,c2^2}{32659200} + \frac{1179668318711\,c1^2\,c2^2}{103680} - \\
&\frac{693823511411\,c1^3\,c2^2}{8064} + \frac{10050522993905\,c1^4\,c2^2}{14336} - \frac{3587655036375\,c1^5\,c2^2}{1024} + \\
&\frac{193824341575875\,c1^6\,c2^2}{14336} - \frac{180088418863125\,c1^7\,c2^2}{3584} + \frac{42609040790625\,c1^8\,c2^2}{512} + \\
&\frac{57939265257631\,c2^3}{293932800} - \frac{25056690243373\,c1\,c2^3}{3265920} + \frac{246343020685\,c1^2\,c2^3}{2268} - \\
&\frac{41202259097225\,c1^3\,c2^3}{32256} + \frac{61675184754625\,c1^4\,c2^3}{7168} - \frac{303799448021625\,c1^5\,c2^3}{7168} + \\
&\frac{651472050369375\,c1^6\,c2^3}{3584} - \frac{873560187421875\,c1^7\,c2^3}{1792} + \frac{466896649921875\,c1^8\,c2^3}{896} + \\
&\frac{33835684341709\,c2^4}{19595520} - \frac{109573523754215\,c1\,c2^4}{1741824} + \frac{514226287860895\,c1^2\,c2^4}{387072} - \\
&\frac{19371732084925\,c1^3\,c2^4}{1536} + \frac{2328575367204375\,c1^4\,c2^4}{28672} - \frac{1528970573739375\,c1^5\,c2^4}{3584} + \\
&\frac{5515791492515625\,c1^6\,c2^4}{3584} - \frac{2711369369671875\,c1^7\,c2^4}{896} + \frac{4282530262734375\,c1^8\,c2^4}{1792} + \\
&\frac{643459028623135\,c2^5}{47029248} - \frac{3471452792609575\,c1\,c2^5}{5225472} + \frac{2760340174365125\,c1^2\,c2^5}{290304} - \\
&\frac{5769695677095625\,c1^3\,c2^5}{64512} + \frac{2271449403078125\,c1^4\,c2^5}{3584} - \frac{2689378265671875\,c1^5\,c2^5}{896} + \\
&\frac{3806609801484375\,c1^6\,c2^5}{448} - \frac{5769630917578125\,c1^7\,c2^5}{448} + \frac{1799623353515625\,c1^8\,c2^5}{224} + \\
&\frac{11023412952266075\,c2^6}{94058496} - \frac{15882376167869125\,c1\,c2^6}{5225472} + \frac{62291223823971875\,c1^2\,c2^6}{1161216} - \\
&\frac{18878932318259375\,c1^3\,c2^6}{32256} + \frac{13443069926328125\,c1^4\,c2^6}{3584} - \frac{3198944306953125\,c1^5\,c2^6}{224} + \\
&\frac{28628237942578125\,c1^6\,c2^6}{896} - \frac{8719032216796875\,c1^7\,c2^6}{224} + \frac{639248994140625\,c1^8\,c2^6}{32} + \\
&\frac{463590425804375\,c2^7}{1741824} - \frac{3436963501653125\,c1\,c2^7}{193536} + \frac{482332488203125\,c1^2\,c2^7}{1536} - \\
&\frac{691422433828125\,c1^3\,c2^7}{256} + \frac{1714422849609375\,c1^4\,c2^7}{128} - \frac{2591658123046875\,c1^5\,c2^7}{64} + \\
&\frac{2370310751953125\,c1^6\,c2^7}{32} - \frac{8471955322265625\,c1^7\,c2^7}{112} + \frac{1859697509765625\,c1^8\,c2^7}{56} + \\
&\frac{1563270040503125\,c2^8}{516096} - \frac{269529317328125\,c1\,c2^8}{3584} + \frac{418235147578125\,c1^2\,c2^8}{512} - \\
&\frac{648985573828125\,c1^3\,c2^8}{128} + \frac{5035232900390625\,c1^4\,c2^8}{256} - \frac{1562658486328125\,c1^5\,c2^8}{32} + \\
&\frac{2424814892578125\,c1^6\,c2^8}{32} - \frac{3762643798828125\,c1^7\,c2^8}{56} + \frac{5838585205078125\,c1^8\,c2^8}{224}
\end{aligned}
$$

```
In[76]:= Dc1 = D[meritFunction,c1]
```

Out[76]=

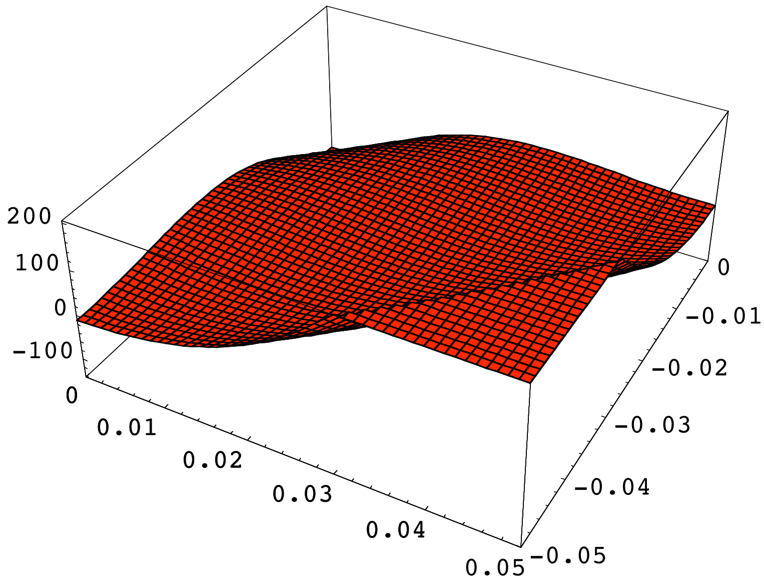
$$-\frac{8630209}{38400}+\frac{8811546167\,c1}{448000}-\frac{47534104827\,c1^{2}}{89600}+\frac{5534665929\,c1^{3}}{1120}-$$
$$\frac{165483506655\,c1^{4}}{7168}+\frac{1941882559965\,c1^{5}}{7168}-\frac{11155832325\,c1^{6}}{64}+\frac{31352959666125\,c1^{7}}{7168}-$$
$$\frac{765076197961\,c2}{36288000}+\frac{23372228129\,c1\,c2}{19200}-\frac{88527150261\,c1^{2}\,c2}{4480}+\frac{224966885553\,c1^{3}\,c2}{1792}-$$
$$\frac{8932994277975\,c1^{4}\,c2}{7168}+\frac{2112569993025\,c1^{5}\,c2}{448}-\frac{20170970274375\,c1^{6}\,c2}{1024}+$$
$$\frac{33594267796875\,c1^{7}\,c2}{448}-\frac{20605878091867\,c2^{2}}{32659200}+\frac{1179668318711\,c1\,c2^{2}}{51840}-$$
$$\frac{693823511411\,c1^{2}\,c2^{2}}{2688}+\frac{10050522993905\,c1^{3}\,c2^{2}}{3584}-\frac{17938275181875\,c1^{4}\,c2^{2}}{1024}+$$
$$\frac{581473024727625\,c1^{5}\,c2^{2}}{7168}-\frac{180088418863125\,c1^{6}\,c2^{2}}{512}+\frac{42609040790625\,c1^{7}\,c2^{2}}{64}-$$
$$\frac{25056690243373\,c2^{3}}{3265920}+\frac{246343020685\,c1\,c2^{3}}{1134}-\frac{41202259097225\,c1^{2}\,c2^{3}}{10752}+$$
$$\frac{61675184754625\,c1^{3}\,c2^{3}}{1792}-\frac{1518997240108125\,c1^{4}\,c2^{3}}{7168}+\frac{1954416151108125\,c1^{5}\,c2^{3}}{1792}-$$
$$\frac{873560187421875\,c1^{6}\,c2^{3}}{256}+\frac{466896649921875\,c1^{7}\,c2^{3}}{112}-\frac{109573523754215\,c2^{4}}{1741824}+$$
$$\frac{514226287860895\,c1\,c2^{4}}{193536}-\frac{19371732084925\,c1^{2}\,c2^{4}}{512}+\frac{2328575367204375\,c1^{3}\,c2^{4}}{7168}-$$
$$\frac{7644852868696875\,c1^{4}\,c2^{4}}{3584}+\frac{16547374477546875\,c1^{5}\,c2^{4}}{1792}-\frac{2711369369671875\,c1^{6}\,c2^{4}}{128}+$$
$$\frac{4282530262734375\,c1^{7}\,c2^{4}}{224}-\frac{3471452792609575\,c2^{5}}{5225472}+\frac{2760340174365125\,c1\,c2^{5}}{145152}-$$
$$\frac{5769695677095625\,c1^{2}\,c2^{5}}{21504}+\frac{2271449403078125\,c1^{3}\,c2^{5}}{896}-\frac{13446891328359375\,c1^{4}\,c2^{5}}{896}+$$
$$\frac{11419829404453125\,c1^{5}\,c2^{5}}{224}-\frac{5769630917578125\,c1^{6}\,c2^{5}}{64}+\frac{1799623353515625\,c1^{7}\,c2^{5}}{28}-$$
$$\frac{15882376167869125\,c2^{6}}{5225472}+\frac{62291223823971875\,c1\,c2^{6}}{580608}-\frac{18878932318259375\,c1^{2}\,c2^{6}}{10752}+$$
$$\frac{13443069926328125\,c1^{3}\,c2^{6}}{896}-\frac{15994721534765625\,c1^{4}\,c2^{6}}{224}+\frac{85884713827734375\,c1^{5}\,c2^{6}}{448}-$$
$$\frac{8719032216796875\,c1^{6}\,c2^{6}}{32}+\frac{639248994140625\,c1^{7}\,c2^{6}}{4}-\frac{3436963501653125\,c2^{7}}{193536}+$$
$$\frac{482332488203125\,c1\,c2^{7}}{768}-\frac{2074267301484375\,c1^{2}\,c2^{7}}{256}+\frac{1714422849609375\,c1^{3}\,c2^{7}}{32}-$$
$$\frac{12958290615234375\,c1^{4}\,c2^{7}}{64}+\frac{7110932255859375\,c1^{5}\,c2^{7}}{16}-\frac{8471955322265625\,c1^{6}\,c2^{7}}{16}+$$
$$\frac{1859697509765625\,c1^{7}\,c2^{7}}{7}-\frac{269529317328125\,c2^{8}}{3584}+\frac{418235147578125\,c1\,c2^{8}}{256}-$$
$$\frac{1946956721484375\,c1^{2}\,c2^{8}}{128}+\frac{5035232900390625\,c1^{3}\,c2^{8}}{64}-\frac{7813292431640625\,c1^{4}\,c2^{8}}{32}+$$
$$\frac{7274444677734375\,c1^{5}\,c2^{8}}{16}-\frac{3762643798828125\,c1^{6}\,c2^{8}}{8}+\frac{5838585205078125\,c1^{7}\,c2^{8}}{28}$$

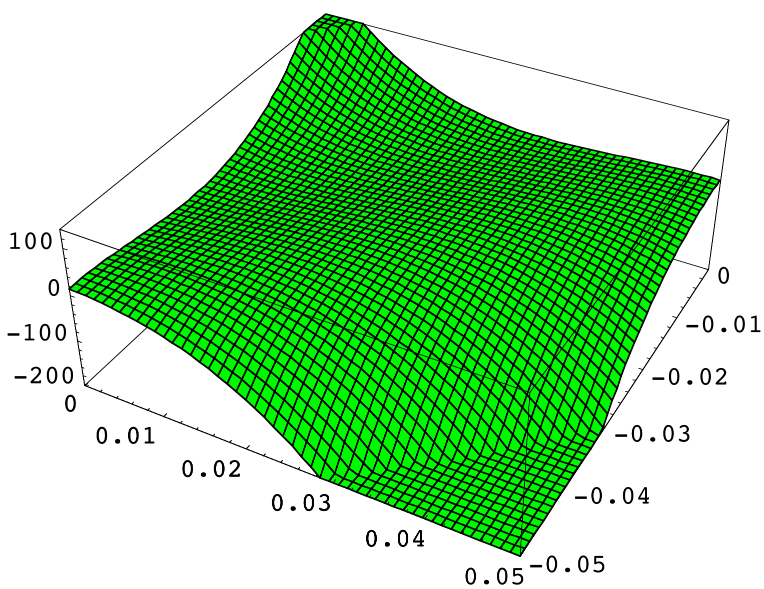
```
In[77]:= Dc2 = D[meritFunction,c2]
```

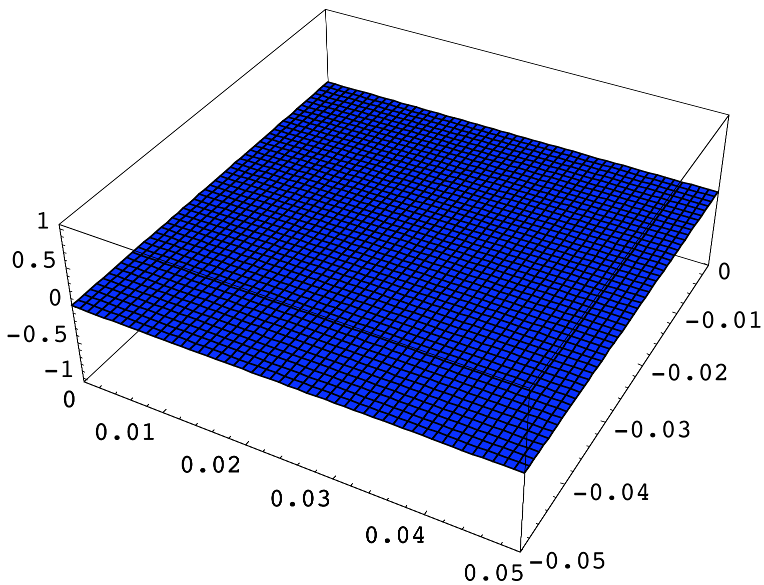
Out[77]=

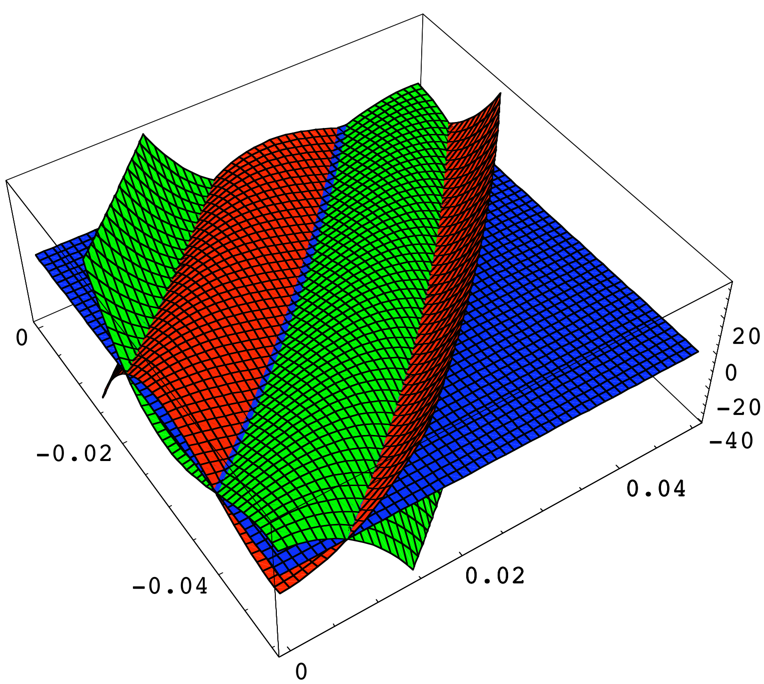
$$\frac{26892241}{115200}-\frac{765076197961\,c1}{36288000}+\frac{23372228129\,c1^2}{38400}-\frac{29509050087\,c1^3}{4480}+\frac{224966885553\,c1^4}{7168}-\frac{1786598855595\,c1^5}{7168}+\frac{704189997675\,c1^6}{896}-\frac{20170970274375\,c1^7}{7168}+\frac{33594267796875\,c1^8}{3584}+\frac{61871970916463\,c2}{2939328000}-\frac{20605878091867\,c1\,c2}{16329600}+\frac{1179668318711\,c1^2\,c2}{51840}-\frac{693823511411\,c1^3\,c2}{4032}+\frac{10050522993905\,c1^4\,c2}{7168}-\frac{3587655036375\,c1^5\,c2}{512}+\frac{193824341575875\,c1^6\,c2}{7168}-\frac{180088418863125\,c1^7\,c2}{1792}+\frac{42609040790625\,c1^8\,c2}{256}+\frac{57939265257631\,c2^2}{97977600}-\frac{25056690243373\,c1\,c2^2}{1088640}+\frac{246343020685\,c1^2\,c2^2}{756}-\frac{41202259097225\,c1^3\,c2^2}{10752}+\frac{185025554263875\,c1^4\,c2^2}{7168}-\frac{911398344064875\,c1^5\,c2^2}{7168}+\frac{1954416151108125\,c1^6\,c2^2}{3584}-\frac{2620680562265625\,c1^7\,c2^2}{1792}+\frac{1400689949765625\,c1^8\,c2^2}{896}+\frac{33835684341709\,c2^3}{4898880}-\frac{109573523754215\,c1\,c2^3}{435456}+\frac{514226287860895\,c1^2\,c2^3}{96768}-\frac{19371732084925\,c1^3\,c2^3}{384}+\frac{2328575367204375\,c1^4\,c2^3}{7168}-\frac{1528970573739375\,c1^5\,c2^3}{896}+\frac{5515791492515625\,c1^6\,c2^3}{896}-\frac{2711369369671875\,c1^7\,c2^3}{224}+\frac{4282530262734375\,c1^8\,c2^3}{448}+\frac{3217295143115675\,c2^4}{47029248}-\frac{17357263963047875\,c1\,c2^4}{5225472}+\frac{13801700871825625\,c1^2\,c2^4}{290304}-\frac{28848478385478125\,c1^3\,c2^4}{64512}+\frac{11357247015390625\,c1^4\,c2^4}{3584}-\frac{13446891328359375\,c1^5\,c2^4}{896}+\frac{19033049007421875\,c1^6\,c2^4}{448}-\frac{28848154587890625\,c1^7\,c2^4}{448}+\frac{8998116767578125\,c1^8\,c2^4}{224}+\frac{1102341295226075\,c2^5}{15676416}-\frac{15882376167869125\,c1\,c2^5}{870912}+\frac{62291223823971875\,c1^2\,c2^5}{193536}-\frac{18878932318259375\,c1^3\,c2^5}{5376}+\frac{40329209778984375\,c1^4\,c2^5}{1792}-\frac{9596832920859375\,c1^5\,c2^5}{112}+\frac{85884713827734375\,c1^6\,c2^5}{448}-\frac{26157096650390625\,c1^7\,c2^5}{112}+\frac{1917746982421875\,c1^8\,c2^5}{16}+\frac{463590425804375\,c2^6}{248832}-\frac{3436963501653125\,c1\,c2^6}{27648}+\frac{3376327417421875\,c1^2\,c2^6}{1536}-\frac{4839957036796875\,c1^3\,c2^6}{256}+\frac{12000959947265625\,c1^4\,c2^6}{128}-\frac{18141606861328125\,c1^5\,c2^6}{64}+\frac{16592175263671875\,c1^6\,c2^6}{32}-\frac{8471955322265625\,c1^7\,c2^6}{16}+\frac{1859697509765625\,c1^8\,c2^6}{8}+\frac{1563270040503125\,c2^7}{64512}-\frac{269529317328125\,c1\,c2^7}{448}+\frac{418235147578125\,c1^2\,c2^7}{64}-\frac{648985573828125\,c1^3\,c2^7}{16}+\frac{5035232900390625\,c1^4\,c2^7}{32}-\frac{1562658486328125\,c1^5\,c2^7}{4}+\frac{2424814892578125\,c1^6\,c2^7}{4}-\frac{3762643798828125\,c1^7\,c2^7}{7}+\frac{5838585205078125\,c1^8\,c2^7}{28}$$

```
In[78]:= Show[{Plot3D[Dc1,{c1,0,.05},{c2,-.05,0},ColorFunction->(Red&), PlotPoints->50],
       Plot3D[Dc2,{c1,0,.05},{c2,-.05,0},ColorFunction->(Green&), PlotPoints->50],
       Plot3D[0,{c1,0,.05},{c2,-.05,0},ColorFunction->(Blue&), PlotPoints->50]},
       ViewPoint->{-1.309, -1.983, 2.409}];
```


200
100
0
-100
0
0.01
0.02
0.03
0.04
0.05
0
-0.01
-0.02
-0.03
-0.04
-0.05


100
0
-100
-200
0
0.01
0.02
0.03
0.04
0.05
0
-0.01
-0.02
-0.03
-0.04
-0.05


1
0.5
0
-0.5
-1
0
0.01
0.02
0.03
0.04
0.05
0
-0.01
-0.02
-0.03
-0.04
-0.05


0
-0.02
-0.04
0
0.02
0.04
20
0
-20
-40

```
In[79]:= extrema = NSolve[{Dc1==0,Dc2==0},{c1,c2}]

Out[79]= {{c1 → 0.399399 + 0.0579707 ⅈ, c2 → 0.692599 + 1.45773 ⅈ},
  {c1 → 0.399399 - 0.0579707 ⅈ, c2 → 0.692599 - 1.45773 ⅈ},
  {c1 → 0.460348, c2 → -0.734278}, {c1 → 0.417163 + 0.0541652 ⅈ, c2 → 0.0858972 + 1.0192 ⅈ},
  {c1 → 0.417163 - 0.0541652 ⅈ, c2 → 0.0858972 - 1.0192 ⅈ}, {c1 → 0.546813, c2 → -0.422202},
  {c1 → 0.301225 + 0.108293 ⅈ, c2 → -0.457425 + 0.145537 ⅈ},
  {c1 → 0.301225 - 0.108293 ⅈ, c2 → -0.457425 - 0.145537 ⅈ},
  {c1 → 0.286789 + 0.156934 ⅈ, c2 → -0.349915 + 0.233135 ⅈ},
  {c1 → 0.286789 - 0.156934 ⅈ, c2 → -0.349915 - 0.233135 ⅈ},
  {c1 → 0.277105, c2 → -0.30706}, {c1 → 0.162073 + 0.224214 ⅈ, c2 → -0.150149 + 0.265798 ⅈ},
  {c1 → 0.162073 - 0.224214 ⅈ, c2 → -0.150149 - 0.265798 ⅈ},
  {c1 → 0.143053 + 0.262228 ⅈ, c2 → -0.158395 + 0.230927 ⅈ},
  {c1 → 0.143053 - 0.262228 ⅈ, c2 → -0.158395 - 0.230927 ⅈ},
  {c1 → 0.104177 + 0.229997 ⅈ, c2 → -0.116725 + 0.206965 ⅈ},
  {c1 → 0.104177 - 0.229997 ⅈ, c2 → -0.116725 - 0.206965 ⅈ},
  {c1 → 0.0640388 - 0.2553 ⅈ, c2 → -0.135983 - 0.18055 ⅈ},
  {c1 → 0.0640388 + 0.2553 ⅈ, c2 → -0.135983 + 0.18055 ⅈ},
  {c1 → -0.0358299 - 0.150642 ⅈ, c2 → 0.026303 + 0.0491706 ⅈ},
  {c1 → -0.0358299 + 0.150642 ⅈ, c2 → 0.026303 - 0.0491706 ⅈ},
  {c1 → -0.0026793 - 0.138484 ⅈ, c2 → 0.0146997 + 0.0545705 ⅈ},
  {c1 → -0.0026793 + 0.138484 ⅈ, c2 → 0.0146997 - 0.0545705 ⅈ},
  {c1 → 0.12057 + 0.0979569 ⅈ, c2 → 0.0486197 + 0.234902 ⅈ},
  {c1 → 0.12057 - 0.0979569 ⅈ, c2 → 0.0486197 - 0.234902 ⅈ},
  {c1 → 0.0817797 + 0.126848 ⅈ, c2 → -0.0182315 + 0.178538 ⅈ},
  {c1 → 0.0817797 - 0.126848 ⅈ, c2 → -0.0182315 - 0.178538 ⅈ},
  {c1 → 0.0410096 - 0.188667 ⅈ, c2 → -0.0792984 - 0.14807 ⅈ},
  {c1 → 0.0410096 + 0.188667 ⅈ, c2 → -0.0792984 + 0.14807 ⅈ},
  {c1 → -0.0928526 - 0.221853 ⅈ, c2 → -0.114414 - 0.0726756 ⅈ},
  {c1 → -0.0928526 + 0.221853 ⅈ, c2 → -0.114414 + 0.0726756 ⅈ},
  {c1 → 0.115138 + 0.096421 ⅈ, c2 → 0.0514561 + 0.167566 ⅈ},
  {c1 → 0.115138 - 0.096421 ⅈ, c2 → 0.0514561 - 0.167566 ⅈ},
  {c1 → -0.0209427 - 0.194 ⅈ, c2 → -0.0954919 - 0.0979506 ⅈ},
  {c1 → -0.0209427 + 0.194 ⅈ, c2 → -0.0954919 + 0.0979506 ⅈ},
  {c1 → 0.0480755 + 0.138782 ⅈ, c2 → -0.0413295 + 0.131875 ⅈ},
  {c1 → 0.0480755 - 0.138782 ⅈ, c2 → -0.0413295 - 0.131875 ⅈ},
  {c1 → 0.0368585 + 0.0256288 ⅈ, c2 → -0.0271366 + 0.0280986 ⅈ},
  {c1 → 0.0368585 - 0.0256288 ⅈ, c2 → -0.0271366 - 0.0280986 ⅈ},
  {c1 → 0.037051, c2 → -0.0221803},
  {c1 → -0.0381115 - 0.0337223 ⅈ, c2 → -0.0674266 - 0.0198861 ⅈ},
  {c1 → -0.0381115 + 0.0337223 ⅈ, c2 → -0.0674266 + 0.0198861 ⅈ},
  {c1 → -0.10841 - 0.0702 ⅈ, c2 → -0.110455 - 0.0273702 ⅈ},
  {c1 → -0.10841 + 0.0702 ⅈ, c2 → -0.110455 + 0.0273702 ⅈ},
  {c1 → 0.00343476 + 0.0105195 ⅈ, c2 → -0.0179873 + 0.00930596 ⅈ},
  {c1 → 0.00343476 - 0.0105195 ⅈ, c2 → -0.0179873 - 0.00930596 ⅈ},
  {c1 → 0.00353901, c2 → -0.0175553},
  {c1 → 0.100094, c2 → 0.0987598}, {c1 → 0.0926163, c2 → 0.0675519}}

In[80]:= Length[extrema]

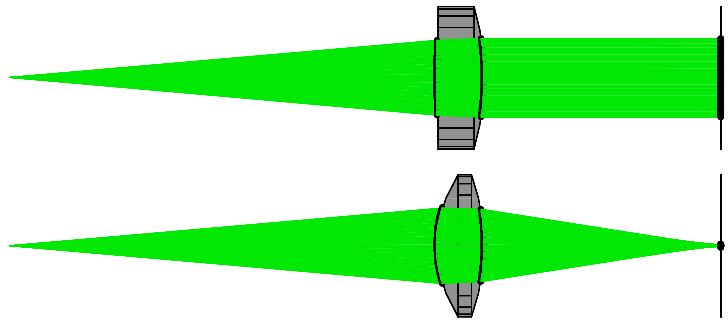
Out[80]= 49

In[81]:= D2c1 = D[meritFunction,{c1,2}];
         D2c2 = D[meritFunction,{c2,2}];
         Dc1c2 = D[meritFunction,c1,c2];
```

In[84]:=
```
finalextrema =
    Union[Select[extrema,
        ((  c1==Re[c1]&&c2==Re[c2]&&
            Abs[c1]<.1&&Abs[c2]<.1&&
            (D2c1*D2c2-Dc1c2^2)>0&&D2c1>0
            )/.#)&]]
```

Out[84]=
```
{{c1 → 0.00353901, c2 → -0.0175553}, {c1 → 0.037051, c2 → -0.0221803}}
```

In[85]:=
```
Map[{#,FindFocusFast[TurboPlot[
    plot,SymbolicValues->#, PlotType->TopView]]}&,
    finalextrema]
```



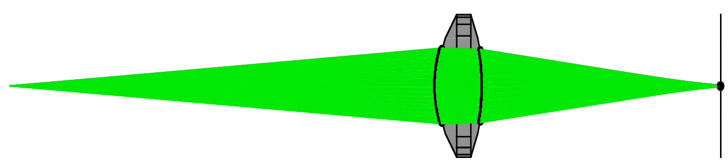
Out[85]=
```
{{{c1 → 0.00353901, c2 → -0.0175553}, {FocalPoint → {20306.1, 0, 0},
   FocusType → RMSFocus, WeightedSpotSize → 2.63622, SpotSize → 2.63622,
   BackFocalLength → 20296.3, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
 {{c1 → 0.037051, c2 → -0.0221803}, {FocalPoint → {59.6157, 0, 0},
   FocusType → RMSFocus, WeightedSpotSize → 0.123904, SpotSize → 0.123904,
   BackFocalLength → 49.8316, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}}}
```

In[86]:=
```
basicsystem = {
    Move[WedgeOfRays[10,NumberOfRays->101, IntrinsicMedium->1],-90],
    SphericalLens[{1/c1,35},{1/c2,-35},30,10, ComponentMedium->3/2],
    Move[Screen[30],60]};
```

In[87]:=
```
sol =
OptimizeSystem[basicsystem,SymbolicValues->localminimum[[2]],SequentialTrace->True]
```

Out[87]=
```
{SymbolicValues → {c1 → 0.0353375, c2 → -0.0237914},
 NumberOfCycles → 253, FinalMerit → 0.122681}
```

In[88]:=
```
FindFocusFast[TurboPlot[plot, sol, PlotType->TopView]]
```



Out[88]=
```
{FocalPoint → {60.0307, 0, 0}, FocusType → RMSFocus,
 WeightedSpotSize → 0.122651, SpotSize → 0.122651, BackFocalLength → 50.2649,
 FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}
```

Go to list of topics

Doublet Lens Design

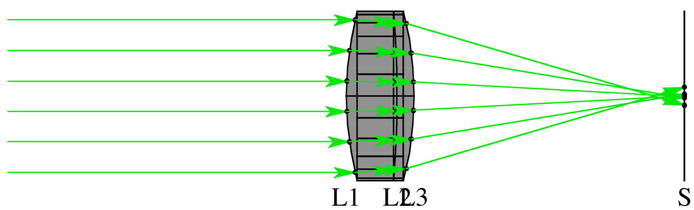
Now we define a 3 surfaced lens system, using r1 = 100 mm, r2 = -300 mm, and r3 = -100 as initial curvature starting points for the lens. Although important parameters to optimize, in this example we will fix the spacings between the different lens surfaces to be s1 = 15 mm and s2 = 5 mm.

```
In[89]:= optics =
          {ComponentRendering[
            {Move[SphericalLensSurface[{1 / c1, 100}, 50, ComponentMedium -> BK7], 100],
               Move[SphericalLensSurface[{1 / c2, -300}, 50,
              ComponentMedium -> {BK7, SF11}], 115],
               Move[SphericalLensSurface[{1 / c3, -100}, 50, ComponentMedium -> SF11], 120]}],
               Move[Screen[50], 200]};
```

```
In[90]:= opticalsystem= {Move[LineOfRays[45,NumberOfRays->6],{0,{y,0}}],optics};
```

Here is a plot of this initial system:

```
In[91]:= AnalyzeSystem[opticalsystem, PlotType -> TopView];
```




```
In[92]:= ModelRefractiveIndex[BK7][WaveLength->.532]

Out[92]= 1.51947
```

```
In[93]:= ModelRefractiveIndex[SF11][WaveLength->.532]

Out[93]= 1.7948
```

```
In[94]:= ?Rationalize

   Rationalize[x] takes Real numbers in x that are close to rationals, and
      converts them to exact Rational numbers. Rationalize[x, dx] performs the
      conversion whenever the error made is smaller in magnitude than dx. More…
```

```
In[95]:= Rationalize[ModelRefractiveIndex[SF11][WaveLength->.532],.02]
```

Out[95]= $\frac{9}{5}$

```
In[96]:= Rationalize[ModelRefractiveIndex[BK7][WaveLength->.532],.02]
```

Out[96]= $\frac{3}{2}$

```
In[97]:= Py = (SymbolicRayEnd/.
            SymbolicTrace[opticalsystem,{{y,0,3},{c1>0,c2<0,c3<0,y∈Reals}},
         NormalizeSurfaceNormal->False, ReportedParameters->SymbolicRayEnd,
         RunningCommentary->All, Simplify->True, FinalFormat->False,
            SymbolicRefractiveModels->{BK7->3/2, SF11->9/5}])[[2]];
```

In[98]:= **ByteCount[Py]**

Out[98]= 4096

In[99]:= **Py//InputForm**

Out[99]//InputForm=

```
y*(1. - 0.8333333333333331*c2 + c1*(-6.385445898048419 + 4.164421237857663*c2) +
c2*(-23.99353490705878 - 53.30100786862726*c3) + 63.961209442352725*c3 +
  c1*(-39.96767453529393 - 408.42084246788704*c3 + c2*(119.90302360588177 +
266.3614190007958*c3)) +
  (-79.9515118029409*c2*(-0.0017089009379546 + c3)*c3*(0.48764285560680765 + c3) +
66.62625983578405*c2^2*(0.007971515150067884 + c3)*(0.20283168300952217 +
c3)*(0.7735647111781162 + c3) +
    c3^2*(-0.39975755901470456 + 31.980604721176363*c3) -
18.5072943988289*c2^3*(0.5169292315766226 + c3)*(1.2975315262148452 +
0.7658726324299371*c3 + c3^2) +
    c1*(-612.6312637018306*(-0.0018540657824575426 + c3)*c3*(0.09770927331916474 + c3)
+ 1420.5942346709107*c2*(-0.0008775492072184491 + c3)*(0.042660433177280084 +
c3)*(0.5220313761158 + c3) -
      1091.3419250727047*c2^2*(-0.028920975829296026 + c3)*(0.24057370655264448 +
c3)*(0.8093056267501366 + c3) + 277.4598114591622*c2^3*(0.5169292315766226 + c3)*
       (1.297531526214845 + 0.7658726324299366*c3 + c3^2)) +
c1^2*(3911.923789821072*(0.009138592301390268 + c3)*(0.02786745725476436 +
c3)*(0.1672043655172598 + c3) +
      5915.952801241696*c2^2*(-0.05172453852830349 + c3)*(0.2630887177228902 +
c3)*(0.840467368893461 + c3) - 8362.445782588522*c2*(0.5558346769858834 + c3)*
       (0.0021322577247055046 + 0.0794664514099874*c3 + c3^2) -
1386.5514377910222*c2^3*(0.5169292315766221 + c3)*(1.2975315262148461 +
0.7658726324299368*c3 + c3^2)) +
    c1^3*(-10624.499029050985*c2^2*(-0.0305711690535631 + c3)*(0.25638873105418436 +
c3)*(0.852412801250604 + c3) - 8326.459239063663*(0.19890623747890349 + c3)*
       (0.01399394675838518 + 0.11160464690486825*c3 + c3^2) +
16290.898511211512*c2*(0.550657007706981 + c3)*(0.010673489313858552 +
0.14883676342155*c3 + c3^2) +
      2309.6737019676048*c2^3*(0.5169292315766224 + c3)*(1.2975315262148457 +
0.7658726324299371*c3 + c3^2)))*y^2)
```

In[100]:=

Timing[meritFunction=ExpandAll[Integrate[Py^2,{y,-5,5}]];][[1]]

Out[100]=

38.71 Second

In[101]:=

ByteCount[meritFunction]

Out[101]=

43708

In[102]:=

meritFunction//InputForm

Out[102]//InputForm=

```
83.33333333333333 - 7725.520072223724*c1 + 179467.42328278074*c1^2 -
77244.79245903643*c1^3 + 2.686384088965151*^6*c1^4 - 172351.7135478436*c1^5 +
1.1990021059754131*^7*c1^6 -
 4137.811373398686*c2 + 212408.9693749286*c1*c2 - 990384.8241276548*c1^2*c2 +
2.877853121571819*^6*c1^3*c2 - 1.832924021410707*^7*c1^4*c2 +
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1.0966652503587645*^7*c1^5*c2 -
 9.906804433304237*^7*c1^6*c2 + 51572.782218815875*c2^2 - 505938.4792684064*c1*c2^2 +
1.0086273901604646*^6*c1^2*c2^2 - 1.0268587890209488*^6*c1^3*c2^2 +
1.6684772775022842*^7*c1^4*c2^2 +
 2.817089520420558*^7*c1^5*c2^2 + 1.3119207727802566*^8*c1^6*c2^2 -
36205.8495172385*c2^3 + 1.548086693855232*^6*c1*c2^3 - 1.7922109105472267*^7*c1^2*c2^3
+ 9.820077896997187*^7*c1^3*c2^3 -
 3.008567653501071*^8*c1^4*c2^3 + 6.531375734981651*^8*c1^5*c2^3 -
1.2994516382756152*^9*c1^6*c2^3 + 770621.7432028732*c2^4 -
1.5362783655369086*^7*c1*c2^4 + 1.2129844142151959*^8*c1^2*c2^4 -
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4.875158776606653*^9*c1^5*c2^4 + 6.73440111955497*^9*c1^6*c2^4 -
46180.930506681754*c2^5 - 2.7131400739199156*^6*c1*c2^5 +
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6.431860511710032*^10*c1^5*c2^6 +
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5.947476723052926*^9*c1^3*c2^2*c3^2 +
```

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6.103253617905876*^11*c1^4*c2^5*c3^6 +
 1.2676488844486382*^12*c1^5*c2^5*c3^6 - 1.0954966966062238*^12*c1^6*c2^5*c3^6 +
7.645534508145615*^6*c2^6*c3^6 - 2.2924242921956587*^8*c1*c2^6*c3^6 +
2.8639861225801277*^9*c1^2*c2^6*c3^6 -
 1.90829514140837*^10*c1^3*c2^6*c3^6 + 7.152250333483447*^10*c1^4*c2^6*c3^6 -
1.4296791929871854*^11*c1^5*c2^6*c3^6 + 1.190757278919808*^11*c1^6*c2^6*c3^6
```

In[103]:=

```
{Max[#], Min[#], Min[Abs[#]], Apply[Plus,#], Apply[Plus,Abs[#]],
Apply[Plus,Abs[#]]/Length[#]}&[
    (Apply[List,meritFunction]/.{c1->.1,c2->-.1,c3->-.1})]
```

Out[103]=

{5619.26, -4436.59, 1.19076×10^{-7}, 2285.3, 91804.9, 267.653}

In[104]:=

```
meritFunctionTruncated =
    Apply[Plus,Transpose[
    Select[
        Transpose[{(Apply[List,meritFunction]/.{c1->.1,c2->-.1,c3->-.1})//N,
            Apply[List,meritFunction]}]
    ,
        (Abs[#[[1]]]>300)&
    ]][[2]]];
ByteCount[meritFunctionTruncated]
```

Out[105]=

6352

In[106]:=

```
{Max[#], Min[#], Min[Abs[#]], Apply[Plus,#], Apply[Plus,Abs[#]],
Apply[Plus,Abs[#]]/Length[#]}&[
    (Apply[List,meritFunctionTruncated]/.{c1->.1,c2->-.1,c3->-.1})]
```

Out[106]=

{5619.26, -4436.59, 356.466, 2751.13, 79710., 1306.72}

In[107]:=

```
Dc1 = D[meritFunctionTruncated,c1];
ByteCount[Dc1]
```

Out[108]=

5032

In[109]:=
```
Dc2 = D[meritFunctionTruncated,c2];
ByteCount[Dc2]
```

Out[110]=
```
3688
```

In[111]:=
```
Dc3 = D[meritFunctionTruncated,c3];
ByteCount[Dc3]
```

Out[112]=
```
5308
```

In[113]:=
```
extrema = NSolve[{Dc1==0,Dc2==0,Dc3==0},{c1,c2,c3}]
```

Out[113]=
```
{{c1 → 0.116051, c2 → 152.146, c3 → -1.19165}, {c1 → 0.111344, c2 → 3.73685, c3 → 4.08557},
 {c1 → -0.0133567 - 0.000490589 i, c2 → -1.11764 + 0.68226 i, c3 → -0.69258 + 0.0317593 i},
 {c1 → -0.0133567 + 0.000490589 i, c2 → -1.11764 - 0.68226 i, c3 → -0.69258 - 0.0317593 i},
 {c1 → 0.18553 + 0.0327454 i, c2 → -0.896976 + 0.383596 i, c3 → -0.156606 + 0.254615 i},
 {c1 → 0.18553 - 0.0327454 i, c2 → -0.896976 - 0.383596 i, c3 → -0.156606 - 0.254615 i},
 {c1 → 0.200027, c2 → 0.340004, c3 → 1.03561},
 {c1 → 0.00537628 - 0.0583189 i, c2 → -0.324857 + 0.295878 i, c3 → -0.187878 + 0.273878 i},
 {c1 → 0.00537628 + 0.0583189 i, c2 → -0.324857 - 0.295878 i, c3 → -0.187878 - 0.273878 i},
 {c1 → 0.0308946 + 0.0197294 i, c2 → 0.347781 + 0.354764 i, c3 → 0.23094 + 0.247769 i},
 {c1 → 0.0308946 - 0.0197294 i, c2 → 0.347781 - 0.354764 i, c3 → 0.23094 - 0.247769 i},
 {c1 → -0.577065 - 0.112761 i, c2 → 0.105643 - 0.0413966 i, c3 → -0.124563 + 0.0390581 i},
 {c1 → -0.577065 + 0.112761 i, c2 → 0.105643 + 0.0413966 i, c3 → -0.124563 - 0.0390581 i},
 {c1 → 0.0299428 - 0.045671 i, c2 → 0.00401216 + 0.216212 i, c3 → 0.0789409 + 0.391105 i},
 {c1 → 0.0299428 + 0.045671 i, c2 → 0.00401216 - 0.216212 i, c3 → 0.0789409 - 0.391105 i},
 {c1 → -0.42725 + 0.0683073 i, c2 → 0.13833 - 0.029227 i, c3 → 0.0119192 + 0.123766 i},
 {c1 → -0.42725 - 0.0683073 i, c2 → 0.13833 + 0.029227 i, c3 → 0.0119192 - 0.123766 i},
 {c1 → 0.156813 - 0.432484 i, c2 → 0.19193 + 0.0650504 i, c3 → -0.0433872 + 0.0422782 i},
 {c1 → 0.156813 + 0.432484 i, c2 → 0.19193 - 0.0650504 i, c3 → -0.0433872 - 0.0422782 i},
 {c1 → 0.0445461, c2 → -0.165016, c3 → -0.375492},
 {c1 → -0.0313534 + 0.0137393 i, c2 → 0.310162 + 0.0245166 i, c3 → 0.238506 + 0.0293923 i},
 {c1 → -0.0313534 - 0.0137393 i, c2 → 0.310162 - 0.0245166 i, c3 → 0.238506 - 0.0293923 i},
 {c1 → -0.0119411 + 0.0102891 i, c2 → 0.32471 + 0.0241731 i, c3 → 0.214826 + 0.0705476 i},
 {c1 → -0.0119411 - 0.0102891 i, c2 → 0.32471 - 0.0241731 i, c3 → 0.214826 - 0.0705476 i},
 {c1 → -0.370746 - 0.27228 i, c2 → 0.118837 - 0.029152 i, c3 → -0.0386355 - 0.125432 i},
 {c1 → -0.370746 + 0.27228 i, c2 → 0.118837 + 0.029152 i, c3 → -0.0386355 + 0.125432 i},
 {c1 → -0.0702117 - 0.0487969 i, c2 → -0.160229 + 0.234437 i, c3 → -0.246608 + 0.0984436 i},
 {c1 → -0.0702117 + 0.0487969 i, c2 → -0.160229 - 0.234437 i, c3 → -0.246608 - 0.0984436 i},
 {c1 → 0.0328278 - 0.288642 i, c2 → 0.125293 + 0.0939771 i, c3 → 0.0153565 + 0.0193778 i},
 {c1 → 0.0328278 + 0.288642 i, c2 → 0.125293 - 0.0939771 i, c3 → 0.0153565 - 0.0193778 i},
 {c1 → -0.0895077, c2 → -0.20884, c3 → -0.339199},
 {c1 → 0.0358917 - 0.0190159 i, c2 → 0.188572 + 0.0995932 i, c3 → 0.139061 + 0.204815 i},
 {c1 → 0.0358917 + 0.0190159 i, c2 → 0.188572 - 0.0995932 i, c3 → 0.139061 - 0.204815 i},
 {c1 → 0.0405948 - 0.0108437 i, c2 → 0.178528 + 0.143615 i, c3 → 0.0942332 + 0.204718 i},
 {c1 → 0.0405948 + 0.0108437 i, c2 → 0.178528 - 0.143615 i, c3 → 0.0942332 - 0.204718 i},
 {c1 → 0.351902 - 0.200402 i, c2 → 0.130951 + 0.0906014 i, c3 → 0.000878633 - 0.022297 i},
 {c1 → 0.351902 + 0.200402 i, c2 → 0.130951 - 0.0906014 i, c3 → 0.000878633 + 0.022297 i},
 {c1 → 0.406459 - 0.134503 i, c2 → 0.186784 + 0.0980963 i, c3 → -0.0440003 - 0.0358689 i},
 {c1 → 0.406459 + 0.134503 i, c2 → 0.186784 - 0.0980963 i, c3 → -0.0440003 + 0.0358689 i},
 {c1 → 0.12407 + 0.0200613 i, c2 → -0.209515 + 0.282329 i, c3 → 0.0400405 + 0.00708122 i},
 {c1 → 0.12407 - 0.0200613 i, c2 → -0.209515 - 0.282329 i, c3 → 0.0400405 - 0.00708122 i},
```

```
{c1 → -0.0787902 - 0.209461 ⅈ, c2 → 0.07958 + 0.0384572 ⅈ, c3 → 0.0156969 + 0.0444506 ⅈ},
{c1 → -0.0787902 + 0.209461 ⅈ, c2 → 0.07958 - 0.0384572 ⅈ, c3 → 0.0156969 - 0.0444506 ⅈ},
{c1 → 0.0521868 - 0.154544 ⅈ, c2 → 0.102196 + 0.11409 ⅈ, c3 → -0.0831145 + 0.0780083 ⅈ},
{c1 → 0.0521868 + 0.154544 ⅈ, c2 → 0.102196 - 0.11409 ⅈ, c3 → -0.0831145 - 0.0780083 ⅈ},
{c1 → 0.0116013 + 0.00554456 ⅈ, c2 → 0.215841 + 0.0806992 ⅈ, c3 → 0.111749 + 0.135767 ⅈ},
{c1 → 0.0116013 - 0.00554456 ⅈ, c2 → 0.215841 - 0.0806992 ⅈ, c3 → 0.111749 - 0.135767 ⅈ},
{c1 → -0.0425751 + 0.0410848 ⅈ, c2 → 0.1881 + 0.199533 ⅈ, c3 → 0.109323 - 0.011354 ⅈ},
{c1 → -0.0425751 - 0.0410848 ⅈ, c2 → 0.1881 - 0.199533 ⅈ, c3 → 0.109323 + 0.011354 ⅈ},
{c1 → -0.0169039 - 0.108652 ⅈ, c2 → -0.121487 + 0.0289546 ⅈ, c3 → -0.0414059 + 0.150889 ⅈ},
{c1 → -0.0169039 + 0.108652 ⅈ, c2 → -0.121487 - 0.0289546 ⅈ, c3 → -0.0414059 - 0.150889 ⅈ},
{c1 → -0.0475405 - 0.235945 ⅈ, c2 → 0.0879539 - 0.0191063 ⅈ, c3 → 0.0389638 + 0.0165313 ⅈ},
{c1 → -0.0475405 + 0.235945 ⅈ, c2 → 0.0879539 + 0.0191063 ⅈ, c3 → 0.0389638 - 0.0165313 ⅈ},
{c1 → 0.042998 - 0.0157239 ⅈ, c2 → -0.0737167 + 0.0682165 ⅈ, c3 → 0.0821024 + 0.228492 ⅈ},
{c1 → 0.042998 + 0.0157239 ⅈ, c2 → -0.0737167 - 0.0682165 ⅈ, c3 → 0.0821024 - 0.228492 ⅈ},
{c1 → 0.101703 - 0.0595435 ⅈ, c2 → -0.0777946 + 0.0064114 ⅈ, c3 → -0.103974 + 0.013293 ⅈ},
{c1 → 0.101703 + 0.0595435 ⅈ, c2 → -0.0777946 - 0.0064114 ⅈ, c3 → -0.103974 - 0.013293 ⅈ},
{c1 → 0.0999586 + 0.00565827 ⅈ, c2 → -0.208746 + 0.0689067 ⅈ, c3 → 0.00332013 - 0.0507095 ⅈ},
{c1 → 0.0999586 - 0.00565827 ⅈ, c2 → -0.208746 - 0.0689067 ⅈ, c3 → 0.00332013 + 0.0507095 ⅈ},
{c1 → -0.0364854 - 0.0917868 ⅈ, c2 → 0.0187459 + 0.0673587 ⅈ, c3 → 0.0419489 + 0.108571 ⅈ},
{c1 → -0.0364854 + 0.0917868 ⅈ, c2 → 0.0187459 - 0.0673587 ⅈ, c3 → 0.0419489 - 0.108571 ⅈ},
{c1 → 0.0974918 + 0.00885572 ⅈ, c2 → -0.0771382 + 0.129405 ⅈ, c3 → -0.106027 - 0.0697945 ⅈ},
{c1 → 0.0974918 - 0.00885572 ⅈ, c2 → -0.0771382 - 0.129405 ⅈ, c3 → -0.106027 + 0.0697945 ⅈ},
{c1 → 0.00920937 + 0.0715221 ⅈ, c2 → 0.122228 + 0.147514 ⅈ, c3 → 0.158711 + 0.0730139 ⅈ},
{c1 → 0.00920937 - 0.0715221 ⅈ, c2 → 0.122228 - 0.147514 ⅈ, c3 → 0.158711 - 0.0730139 ⅈ},
{c1 → -0.0998137 - 0.183579 ⅈ, c2 → 0.0120492 + 0.0586919 ⅈ, c3 → -0.116672 + 0.0156943 ⅈ},
{c1 → -0.0998137 + 0.183579 ⅈ, c2 → 0.0120492 - 0.0586919 ⅈ, c3 → -0.116672 - 0.0156943 ⅈ},
{c1 → -0.144009 - 0.152653 ⅈ, c2 → 0.030221 - 0.0984107 ⅈ, c3 → -0.0274538 + 0.138534 ⅈ},
{c1 → -0.144009 + 0.152653 ⅈ, c2 → 0.030221 + 0.0984107 ⅈ, c3 → -0.0274538 - 0.138534 ⅈ},
{c1 → 0.0993904 + 0.00949274 ⅈ, c2 → -0.172452 - 0.0190594 ⅈ, c3 → 0.0243215 + 0.0938225 ⅈ},
{c1 → 0.0993904 - 0.00949274 ⅈ, c2 → -0.172452 + 0.0190594 ⅈ, c3 → 0.0243215 - 0.0938225 ⅈ},
{c1 → -0.0451961 - 0.0369021 ⅈ, c2 → -0.0328739 + 0.073516 ⅈ, c3 → 0.000449387 + 0.143943 ⅈ},
{c1 → -0.0451961 + 0.0369021 ⅈ, c2 → -0.0328739 - 0.073516 ⅈ, c3 → 0.000449387 - 0.143943 ⅈ},
{c1 → 0.163826, c2 → 0.0273642, c3 → -0.0613021},
{c1 → 0.157187 - 0.150104 ⅈ, c2 → -0.0128738 + 0.0149678 ⅈ, c3 → 0.00201518 - 0.0707707 ⅈ},
{c1 → 0.157187 + 0.150104 ⅈ, c2 → -0.0128738 - 0.0149678 ⅈ, c3 → 0.00201518 + 0.0707707 ⅈ},
{c1 → 0.106863 + 0.03644 ⅈ, c2 → -0.176451 + 0.110105 ⅈ, c3 → 0.110703 + 0.0755979 ⅈ},
{c1 → 0.106863 - 0.03644 ⅈ, c2 → -0.176451 - 0.110105 ⅈ, c3 → 0.110703 - 0.0755979 ⅈ},
{c1 → 0.0689627 - 0.0976886 ⅈ, c2 → -0.0886364 + 0.106007 ⅈ, c3 → 0.152487 + 0.0092101 ⅈ},
{c1 → 0.0689627 + 0.0976886 ⅈ, c2 → -0.0886364 - 0.106007 ⅈ, c3 → 0.152487 - 0.0092101 ⅈ},
{c1 → 0.140965 - 0.00810909 ⅈ, c2 → 0.0755858 + 0.116948 ⅈ, c3 → -0.0471336 + 0.0310665 ⅈ},
{c1 → 0.140965 + 0.00810909 ⅈ, c2 → 0.0755858 - 0.116948 ⅈ, c3 → -0.0471336 - 0.0310665 ⅈ},
{c1 → 0.0704937 - 0.0376787 ⅈ, c2 → 0.145561 - 0.0635685 ⅈ, c3 → 0.129029 + 0.102843 ⅈ},
{c1 → 0.0704937 + 0.0376787 ⅈ, c2 → 0.145561 + 0.0635685 ⅈ, c3 → 0.129029 - 0.102843 ⅈ},
{c1 → -0.0303199 + 0.0495754 ⅈ, c2 → 0.0243343 + 0.23539 ⅈ, c3 → 0.053238 - 0.0402239 ⅈ},
{c1 → -0.0303199 - 0.0495754 ⅈ, c2 → 0.0243343 - 0.23539 ⅈ, c3 → 0.053238 + 0.0402239 ⅈ},
{c1 → -0.0337 + 0.0114872 ⅈ, c2 → 0.0317955 + 0.00479423 ⅈ, c3 → -0.00933275 + 0.159161 ⅈ},
{c1 → -0.0337 - 0.0114872 ⅈ, c2 → 0.0317955 - 0.00479423 ⅈ, c3 → -0.00933275 - 0.159161 ⅈ},
{c1 → 0.0931603 - 0.109623 ⅈ, c2 → 0.124862 - 0.0192527 ⅈ, c3 → -0.102349 + 0.0233032 ⅈ},
{c1 → 0.0931603 + 0.109623 ⅈ, c2 → 0.124862 + 0.0192527 ⅈ, c3 → -0.102349 - 0.0233032 ⅈ},
{c1 → 0.151183, c2 → 0.0714225, c3 → -0.0539456},
{c1 → 0.0253776, c2 → 0.131754, c3 → 0.0679543},
{c1 → 0.152096, c2 → 0.0777346, c3 → -0.0548082},
{c1 → 0.124975 + 0.00581199 ⅈ, c2 → 0.343181 + 0.0544836 ⅈ, c3 → -0.0271248 + 0.0110545 ⅈ},
{c1 → 0.124975 - 0.00581199 ⅈ, c2 → 0.343181 - 0.0544836 ⅈ, c3 → -0.0271248 - 0.0110545 ⅈ},
{c1 → 0.164528 - 0.0580539 ⅈ, c2 → 0.144661 + 0.0822514 ⅈ, c3 → -0.0816784 - 0.0724125 ⅈ},
{c1 → 0.164528 + 0.0580539 ⅈ, c2 → 0.144661 - 0.0822514 ⅈ, c3 → -0.0816784 + 0.0724125 ⅈ},
{c1 → -0.0280164 - 0.0120479 ⅈ, c2 → -0.1251 + 0.157345 ⅈ, c3 → -0.05463 - 0.0979007 ⅈ},
{c1 → -0.0280164 + 0.0120479 ⅈ, c2 → -0.1251 - 0.157345 ⅈ, c3 → -0.05463 + 0.0979007 ⅈ},
```

```
{c1 → 0.103278 - 0.0732659 ⅈ, c2 → 0.0542237 + 0.0983263 ⅈ, c3 → -0.0590357 - 0.0979677 ⅈ},
{c1 → 0.103278 + 0.0732659 ⅈ, c2 → 0.0542237 - 0.0983263 ⅈ, c3 → -0.0590357 + 0.0979677 ⅈ},
{c1 → -0.00338734 - 0.0394432 ⅈ, c2 → 0.0261043 + 0.0905731 ⅈ, c3 → -0.0122944 + 0.0113879 ⅈ},
{c1 → -0.00338734 + 0.0394432 ⅈ, c2 → 0.0261043 - 0.0905731 ⅈ, c3 → -0.0122944 - 0.0113879 ⅈ},
{c1 → -0.0858561 - 0.0637979 ⅈ, c2 → -0.0726009 - 0.0288004 ⅈ, c3 → -0.0136098 + 0.0920209 ⅈ},
{c1 → -0.0858561 + 0.0637979 ⅈ, c2 → -0.0726009 + 0.0288004 ⅈ, c3 → -0.0136098 - 0.0920209 ⅈ},
{c1 → 0.300025, c2 → 0.24399, c3 → -0.0186904},
{c1 → 0.00766179 - 0.0322848 ⅈ, c2 → -0.0799485 - 0.0700759 ⅈ, c3 → 0.0175281 + 0.144251 ⅈ},
{c1 → 0.00766179 + 0.0322848 ⅈ, c2 → -0.0799485 + 0.0700759 ⅈ, c3 → 0.0175281 - 0.144251 ⅈ},
{c1 → 0.0795499 - 0.1068 ⅈ, c2 → 0.00122989 + 0.0253571 ⅈ, c3 → -0.0206234 - 0.0806288 ⅈ},
{c1 → 0.0795499 + 0.1068 ⅈ, c2 → 0.00122989 - 0.0253571 ⅈ, c3 → -0.0206234 + 0.0806288 ⅈ},
{c1 → -0.137606 + 0.00364122 ⅈ, c2 → 0.0584118 - 0.152811 ⅈ, c3 → -0.0932163 + 0.174866 ⅈ},
{c1 → -0.137606 - 0.00364122 ⅈ, c2 → 0.0584118 + 0.152811 ⅈ, c3 → -0.0932163 - 0.174866 ⅈ},
{c1 → 0.129485 - 0.0723017 ⅈ, c2 → 0.265837 + 0.0146115 ⅈ, c3 → -0.0278767 - 0.0076578 ⅈ},
{c1 → 0.129485 + 0.0723017 ⅈ, c2 → 0.265837 - 0.0146115 ⅈ, c3 → -0.0278767 + 0.0076578 ⅈ},
{c1 → -0.0400788 - 0.0211302 ⅈ, c2 → -0.0640661 + 0.0567589 ⅈ,
 c3 → -0.0574427 + 0.00942639 ⅈ}, {c1 → -0.0400788 + 0.0211302 ⅈ,
 c2 → -0.0640661 - 0.0567589 ⅈ, c3 → -0.0574427 - 0.00942639 ⅈ},
{c1 → -0.0618507 - 0.154131 ⅈ, c2 → 0.0702936 - 0.00405524 ⅈ, c3 → -0.0907652 - 0.0887845 ⅈ},
{c1 → -0.0618507 + 0.154131 ⅈ, c2 → 0.0702936 + 0.00405524 ⅈ, c3 → -0.0907652 + 0.0887845 ⅈ},
{c1 → -0.0299763 - 0.0353739 ⅈ, c2 → 0.0500454 + 0.0269994 ⅈ, c3 → -0.0155055 - 0.00514335 ⅈ},
{c1 → -0.0299763 + 0.0353739 ⅈ, c2 → 0.0500454 - 0.0269994 ⅈ, c3 → -0.0155055 + 0.00514335 ⅈ},
{c1 → 0.128456 - 0.00392435 ⅈ, c2 → 0.263352 + 0.0160818 ⅈ, c3 → -0.0210143 + 0.0231518 ⅈ},
{c1 → 0.128456 + 0.00392435 ⅈ, c2 → 0.263352 - 0.0160818 ⅈ, c3 → -0.0210143 - 0.0231518 ⅈ},
{c1 → 0.0694214 - 0.0254624 ⅈ, c2 → -0.16109 + 0.124894 ⅈ, c3 → 0.254188 - 0.0631834 ⅈ},
{c1 → 0.0694214 + 0.0254624 ⅈ, c2 → -0.16109 - 0.124894 ⅈ, c3 → 0.254188 + 0.0631834 ⅈ},
{c1 → 0.147186 + 0.0375961 ⅈ, c2 → 0.193823 + 0.0275885 ⅈ, c3 → -0.0456651 + 0.0927173 ⅈ},
{c1 → 0.147186 - 0.0375961 ⅈ, c2 → 0.193823 - 0.0275885 ⅈ, c3 → -0.0456651 - 0.0927173 ⅈ},
{c1 → -0.0200457 - 0.0951954 ⅈ, c2 → -0.0320975 - 0.113359 ⅈ, c3 → 0.0556387 + 0.0528445 ⅈ},
{c1 → -0.0200457 + 0.0951954 ⅈ, c2 → -0.0320975 + 0.113359 ⅈ, c3 → 0.0556387 - 0.0528445 ⅈ},
{c1 → 0.0536482 + 0.0284464 ⅈ, c2 → 0.0778968 + 0.00181962 ⅈ, c3 → 0.0623728 + 0.0426674 ⅈ},
{c1 → 0.0536482 - 0.0284464 ⅈ, c2 → 0.0778968 - 0.00181962 ⅈ, c3 → 0.0623728 - 0.0426674 ⅈ},
{c1 → -0.0392616 - 0.0942197 ⅈ, c2 → -0.0304821 + 0.0746795 ⅈ, c3 → -0.0566714 - 0.115941 ⅈ},
{c1 → -0.0392616 + 0.0942197 ⅈ, c2 → -0.0304821 - 0.0746795 ⅈ, c3 → -0.0566714 + 0.115941 ⅈ},
{c1 → -0.141547 - 0.0611199 ⅈ, c2 → -0.115364 - 0.0433791 ⅈ, c3 → -0.101511 + 0.0173069 ⅈ},
{c1 → -0.141547 + 0.0611199 ⅈ, c2 → -0.115364 + 0.0433791 ⅈ, c3 → -0.101511 - 0.0173069 ⅈ},
{c1 → 0.0187382 + 0.121647 ⅈ, c2 → -0.00138481 + 0.0625933 ⅈ, c3 → -0.040134 + 0.0935903 ⅈ},
{c1 → 0.0187382 - 0.121647 ⅈ, c2 → -0.00138481 - 0.0625933 ⅈ, c3 → -0.040134 - 0.0935903 ⅈ},
{c1 → -0.0469003 + 0.100205 ⅈ, c2 → -0.0183902 + 0.0311285 ⅈ, c3 → -0.0794847 + 0.12726 ⅈ},
{c1 → -0.0469003 - 0.100205 ⅈ, c2 → -0.0183902 - 0.0311285 ⅈ, c3 → -0.0794847 - 0.12726 ⅈ},
{c1 → 0.00427675 - 0.0355482 ⅈ, c2 → 0.0489328 + 0.0131795 ⅈ, c3 → 0.00154441 - 0.0183255 ⅈ},
{c1 → 0.00427675 + 0.0355482 ⅈ, c2 → 0.0489328 - 0.0131795 ⅈ, c3 → 0.00154441 + 0.0183255 ⅈ},
{c1 → 0.0283933 + 0.0000572592 ⅈ, c2 → -0.00894282 + 0.0103764 ⅈ,
 c3 → 0.00241026 + 0.00418104 ⅈ}, {c1 → 0.0283933 - 0.0000572592 ⅈ,
 c2 → -0.00894282 - 0.0103764 ⅈ, c3 → 0.00241026 - 0.00418104 ⅈ},
{c1 → 0.0848758 - 0.0367929 ⅈ, c2 → 0.0317765 + 0.0477429 ⅈ, c3 → 0.0682901 - 0.0535792 ⅈ},
{c1 → 0.0848758 + 0.0367929 ⅈ, c2 → 0.0317765 - 0.0477429 ⅈ, c3 → 0.0682901 + 0.0535792 ⅈ},
{c1 → 0.105123 + 0.0778912 ⅈ, c2 → 0.121925 + 0.0290584 ⅈ, c3 → -0.0183137 + 0.0798155 ⅈ},
{c1 → 0.105123 - 0.0778912 ⅈ, c2 → 0.121925 - 0.0290584 ⅈ, c3 → -0.0183137 - 0.0798155 ⅈ},
{c1 → 0.0761908 + 0.016904 ⅈ, c2 → -0.000368986 - 0.0117083 ⅈ, c3 → 0.0653701 + 0.0355927 ⅈ},
{c1 → 0.0761908 - 0.016904 ⅈ, c2 → -0.000368986 + 0.0117083 ⅈ, c3 → 0.0653701 - 0.0355927 ⅈ},
{c1 → 0.0762652 - 0.108661 ⅈ, c2 → -0.0219389 - 0.046807 ⅈ, c3 → 0.107714 - 0.0584889 ⅈ},
{c1 → 0.0762652 + 0.108661 ⅈ, c2 → -0.0219389 + 0.046807 ⅈ, c3 → 0.107714 + 0.0584889 ⅈ},
{c1 → 0.142109, c2 → 0.186859, c3 → -0.0290917},
{c1 → 0.0052115, c2 → 0.000440293, c3 → -0.0120466},
{c1 → 0.0135961, c2 → 0.00572547, c3 → -0.00404364},
{c1 → -0.00590534, c2 → 0.0003676, c3 → -0.0189767}}
```

In[114]:=

```
Length[extrema]
```

Out[114]=

```
154
```

In[115]:=

```
finalextrema =
    Union[Select[extrema,
        ((  c1==Re[c1]&&c2==Re[c2]&&c3==Re[c3]&&
            Abs[Re[c1]]<.1&&Abs[Re[c2]]<.1&&Abs[Re[c3]]<.1
        )/.#)&]]
```

Out[115]=

```
{{c1 → -0.00590534, c2 → 0.0003676, c3 → -0.0189767},
 {c1 → 0.0052115, c2 → 0.000440293, c3 → -0.0120466},
 {c1 → 0.0135961, c2 → 0.00572547, c3 → -0.00404364}}
```

In[116]:=

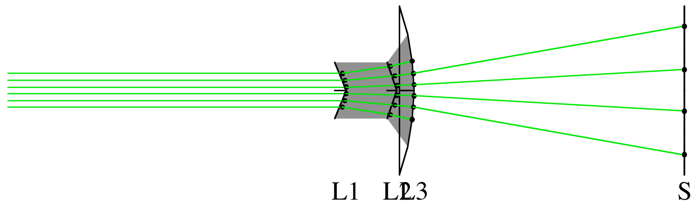
```
finalextrema =
    Map[Apply[Rule,Transpose[{{c1,c2,c3},#}],2]&,Union[Re[{c1,c2,c3}/.Select[extrema,
        ((  Abs[Re[c1]]<.1&&Abs[Re[c2]]<.1&&Abs[Re[c3]]<.1
        )/.#)&]]]]
```

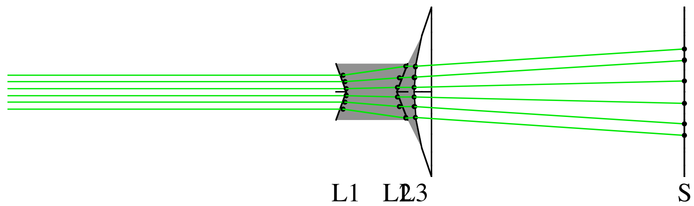
Out[116]=

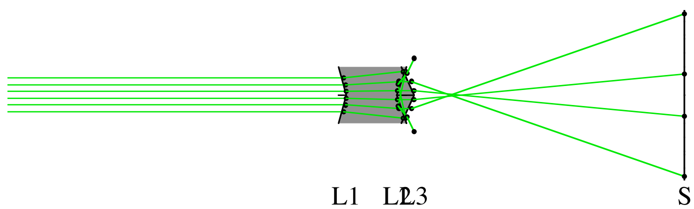
```
{{c1 → -0.0858561, c2 → -0.0726009, c3 → -0.0136098},
 {c1 → -0.0787902, c2 → 0.07958, c3 → 0.0156969},
 {c1 → -0.0618507, c2 → 0.0702936, c3 → -0.0907652},
 {c1 → -0.0475405, c2 → 0.0879539, c3 → 0.0389638},
 {c1 → -0.0469003, c2 → -0.0183902, c3 → -0.0794847},
 {c1 → -0.0451961, c2 → -0.0328739, c3 → 0.000449387},
 {c1 → -0.0400788, c2 → -0.0640661, c3 → -0.0574427},
 {c1 → -0.0392616, c2 → -0.0304821, c3 → -0.0566714},
 {c1 → -0.0364854, c2 → 0.0187459, c3 → 0.0419489},
 {c1 → -0.0337, c2 → 0.0317955, c3 → -0.00933275},
 {c1 → -0.0303199, c2 → 0.0243343, c3 → 0.053238},
 {c1 → -0.0299763, c2 → 0.0500454, c3 → -0.0155055},
 {c1 → -0.0200457, c2 → -0.0320975, c3 → 0.0556387},
 {c1 → -0.00590534, c2 → 0.0003676, c3 → -0.0189767},
 {c1 → -0.00338734, c2 → 0.0261043, c3 → -0.0122944},
 {c1 → 0.00427675, c2 → 0.0489328, c3 → 0.00154441},
 {c1 → 0.0052115, c2 → 0.000440293, c3 → -0.0120466},
 {c1 → 0.00766179, c2 → -0.0799485, c3 → 0.0175281},
 {c1 → 0.0135961, c2 → 0.00572547, c3 → -0.00404364},
 {c1 → 0.0187382, c2 → -0.00138481, c3 → -0.040134},
 {c1 → 0.0283933, c2 → -0.00894282, c3 → 0.00241026},
 {c1 → 0.0299428, c2 → 0.00401216, c3 → 0.0789409},
 {c1 → 0.042998, c2 → -0.0737167, c3 → 0.0821024},
 {c1 → 0.0536482, c2 → 0.0778968, c3 → 0.0623728},
 {c1 → 0.0761908, c2 → -0.000368986, c3 → 0.0653701},
 {c1 → 0.0795499, c2 → 0.00122989, c3 → -0.0206234},
 {c1 → 0.0848758, c2 → 0.0317765, c3 → 0.0682901}}
```

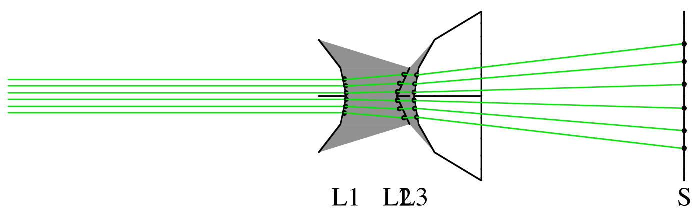
In[117]:=

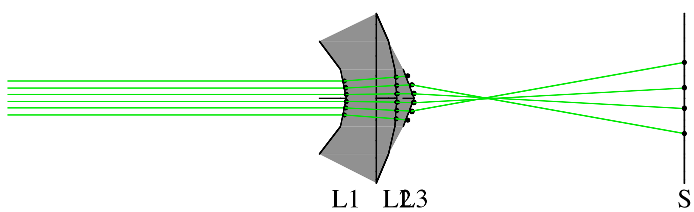
```
Map[{meritFunction/.#,FindFocusFast[AnalyzeSystem[{
    LineOfRays[10,NumberOfRays->6],
    ComponentRendering[{Move[
        SphericalLensSurface[(1/c1)/.#,50,ComponentMedium->3/2],100],
        Move[SphericalLensSurface[(1/c2)/.#,50,ComponentMedium->{3/2,9/5}],115],
        Move[SphericalLensSurface[(1/c3)/.#,50,ComponentMedium->9/5],120]}],
        Move[Screen[50],200]}, PlotType->TopView, ShowArrows->False]]}&,
        finalextrema]
```

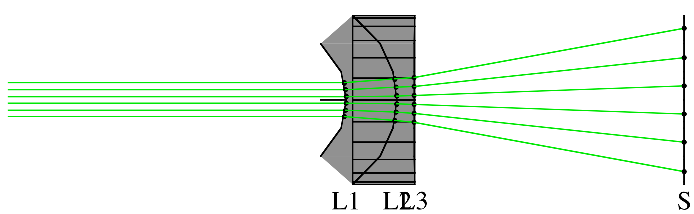


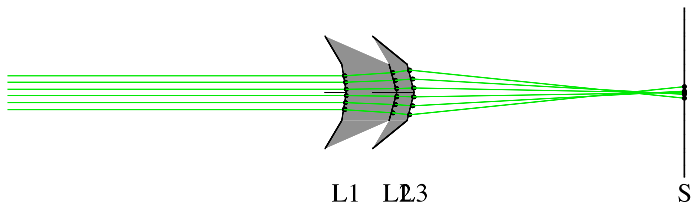


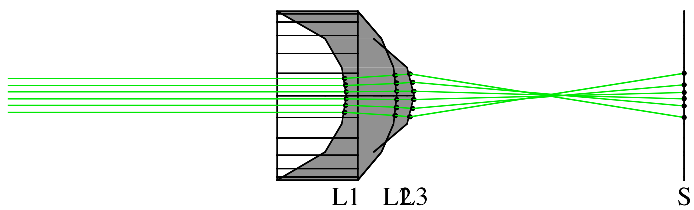


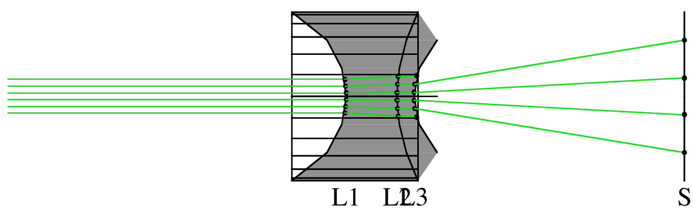


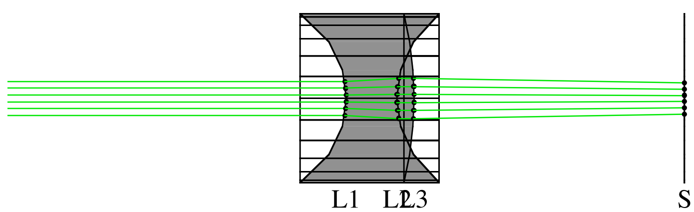


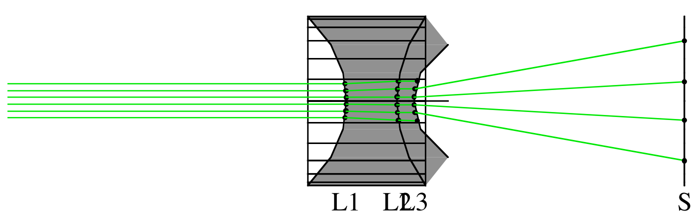


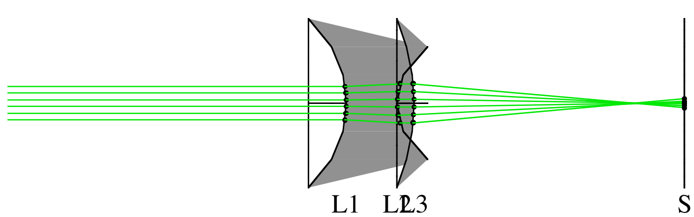

L1
L2
L3
S

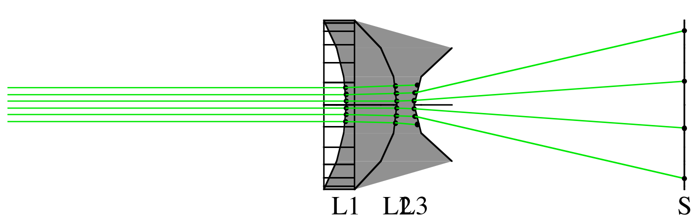

L1
L2
L3
S

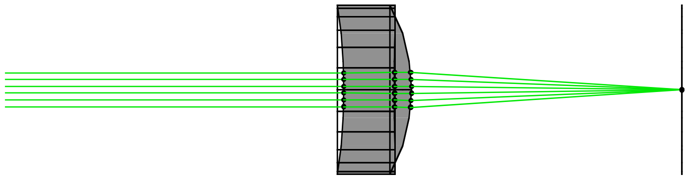

L1
L2
L3
S

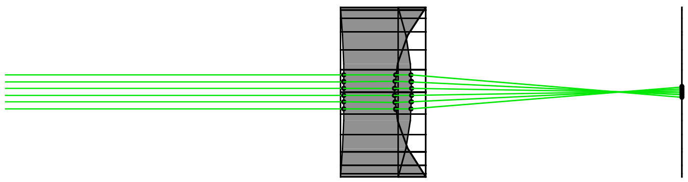

L1
L2
L3
S

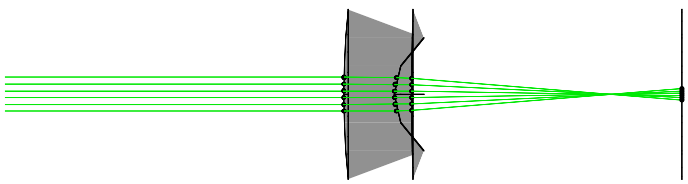

L1
L2
L3
S

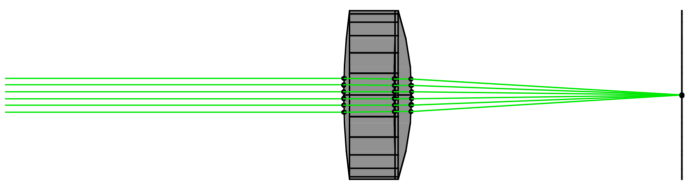

L1
L2
L3
S

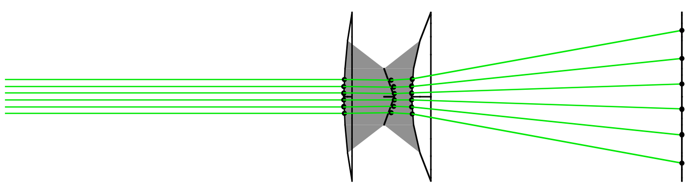

L1
L2
L3
S

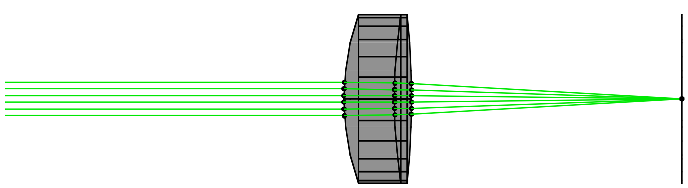

L1
L2
L3
S

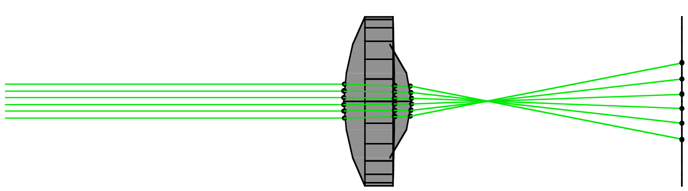

L1
L2
L3
S

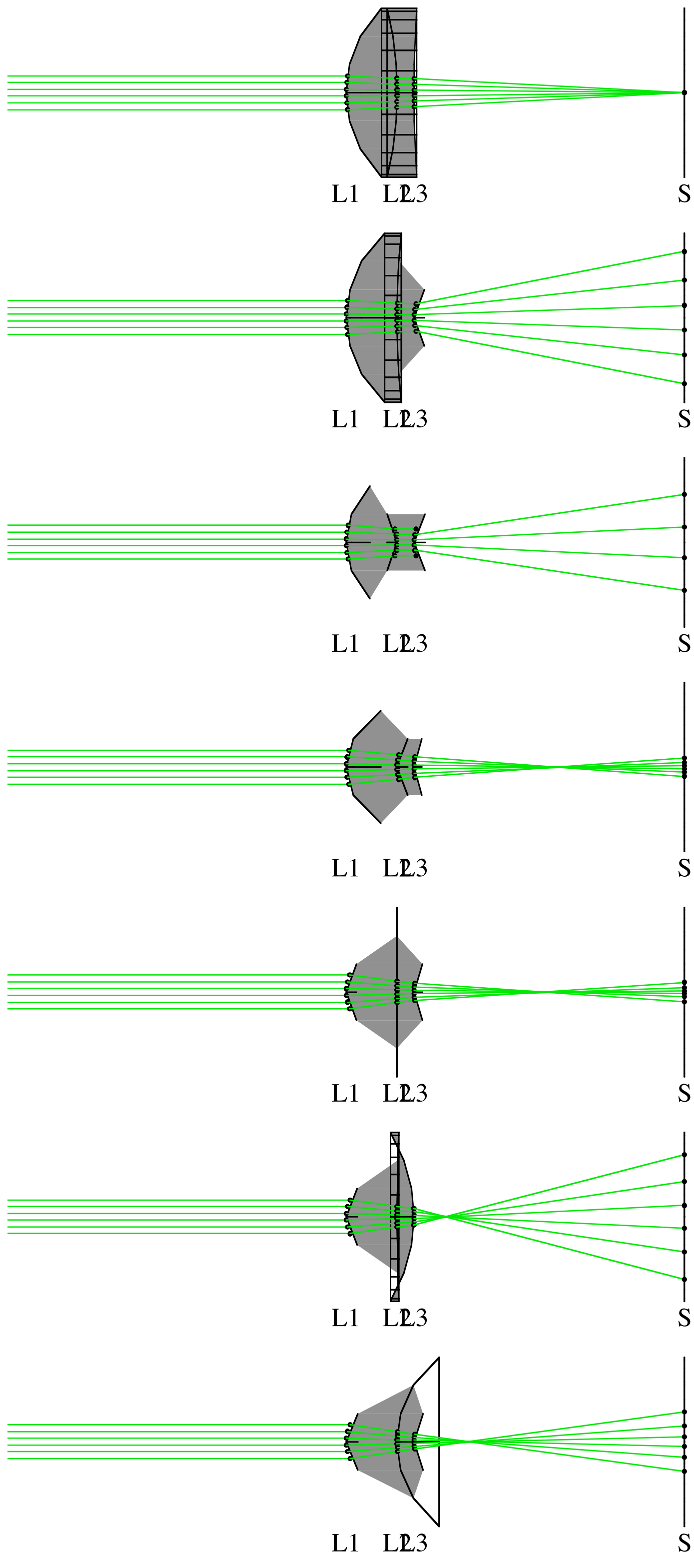

L1
L2
L3
S


L1
L2
L3
S


L1
L2
L3
S


L1
L2
L3
S


L1
L2
L3
S



Out[117]=

```
{{3514.46, {FocalPoint → {91.2895, 0, 0},
   FocusType → RMSFocus, WeightedSpotSize → 0.0134862, SpotSize → 0.0134862,
   BackFocalLength → 28.7105, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
 {775.974, {FocalPoint → {30.1484, 0, 0}, FocusType → RMSFocus,
```

```
  WeightedSpotSize → 1.34186, SpotSize → 1.34186, BackFocalLength → 89.8516,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{4192.01, {FocalPoint → {115.827, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 3.71033, SpotSize → 3.71033, BackFocalLength → 15.8272,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{874.176, {FocalPoint → {70.2517, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.362203, SpotSize → 0.362203, BackFocalLength → 49.7483,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{775.748, {FocalPoint → {141.54, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.10893, SpotSize → 0.10893, BackFocalLength → 21.5399,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{1464.65, {FocalPoint → {82.908, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0248078, SpotSize → 0.0248078, BackFocalLength → 37.092,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{0.32686, {FocalPoint → {185.227, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.238503, SpotSize → 0.238503, BackFocalLength → 65.2272,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{66.253, {FocalPoint → {160.115, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.241815, SpotSize → 0.241815, BackFocalLength → 40.1152,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{2547.3, {FocalPoint → {97.7573, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0334843, SpotSize → 0.0334843, BackFocalLength → 22.2427,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{71.8734, {FocalPoint → {468.609, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.117239, SpotSize → 0.117239, BackFocalLength → 348.609,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{2845.9, {FocalPoint → {100.495, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0441963, SpotSize → 0.0441963, BackFocalLength → 19.5054,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{3.98895, {FocalPoint → {185.524, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0870964, SpotSize → 0.0870964, BackFocalLength → 65.5237,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{4381.17, {FocalPoint → {105.13, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0563629, SpotSize → 0.0563629, BackFocalLength → 14.8695,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{0.000388439, {FocalPoint → {199.477, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0155916, SpotSize → 0.0155916, BackFocalLength → 79.4767,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{7.28524, {FocalPoint → {181.47, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0133574, SpotSize → 0.0133574, BackFocalLength → 61.4698,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{8.06859, {FocalPoint → {179.836, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0190548, SpotSize → 0.0190548, BackFocalLength → 59.8361,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{0.000138021, {FocalPoint → {199.838, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.00670352, SpotSize → 0.00670352,
  BackFocalLength → 79.8377, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{1214.38, {FocalPoint → {91.3793, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.0779076, SpotSize → 0.0779076, BackFocalLength → 28.6207,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{0.0000980211, {FocalPoint → {200.014, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.00273538, SpotSize → 0.00273538,
  BackFocalLength → 80.0142, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{387.598, {FocalPoint → {142.422, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.061056, SpotSize → 0.061056, BackFocalLength → 22.4224,
  FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
{0.000982952, {FocalPoint → {199.776, 0, 0}, FocusType → RMSFocus,
  WeightedSpotSize → 0.00510109, SpotSize → 0.00510109,
```

```
    BackFocalLength → 79.7757, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
  {1073.33, {FocalPoint → {99.5885, 0, 0}, FocusType → RMSFocus,
    WeightedSpotSize → 0.127118, SpotSize → 0.127118, BackFocalLength → 20.4115,
    FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
  {1781.94, {FocalPoint → {104.415, 0, 0}, FocusType → RMSFocus,
    WeightedSpotSize → 0.0305655, SpotSize → 0.0305655, BackFocalLength → 15.5853,
    FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
  {24.7712, {FocalPoint → {163.107, 0, 0}, FocusType → RMSFocus,
    WeightedSpotSize → 0.0374562, SpotSize → 0.0374562, BackFocalLength → 43.107,
    FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
  {27.3441, {FocalPoint → {158.923, 0, 0}, FocusType → RMSFocus,
    WeightedSpotSize → 0.104771, SpotSize → 0.104771, BackFocalLength → 38.9228,
    FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
  {1037.65, {FocalPoint → {129.452, 0, 0}, FocusType → RMSFocus,
    WeightedSpotSize → 0.049931, SpotSize → 0.049931, BackFocalLength → 9.45214,
    FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}},
  {240.481, {FocalPoint → {136.245, 0, 0}, FocusType → RMSFocus,
    WeightedSpotSize → 0.0765416, SpotSize → 0.0765416, BackFocalLength → 16.2448,
    FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}}}
```

In[118]:=

```
Timing[localminimum =
FindMinimum[meritFunction,{c1,.01},{c2,-.003},{c3,-.01},MaxIterations->200]]
```

Out[118]=

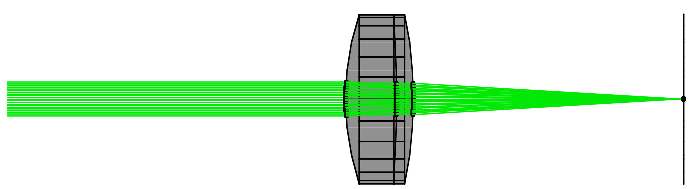
```
{0.35 Second, {0.0000540331, {c1 → 0.0120806, c2 → -0.00270979, c3 → -0.00847703}}}
```

In[119]:=

```
FindFocusFast[AnalyzeSystem[{
    LineOfRays[10,NumberOfRays->21],
    ComponentRendering[{Move[
        SphericalLensSurface[1/c1/.localminimum[[2]],50,ComponentMedium->BK7],100],

Move[SphericalLensSurface[1/c2/.localminimum[[2]],50,ComponentMedium->{BK7,SF11}],115],

Move[SphericalLensSurface[1/c3/.localminimum[[2]],50,ComponentMedium->SF11],120]}],
    Move[Screen[50],200]}, PlotType->TopView, ShowArrows->False]]
```



Out[119]=

```
{FocalPoint → {198.067, 0, 0}, FocusType → RMSFocus,
 WeightedSpotSize → 0.0035545, SpotSize → 0.0035545,
 BackFocalLength → 78.0668, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}
```

In[120]:=
```
FindFocusFast[{
    LineOfRays[10,NumberOfRays->101],
    ComponentRendering[{Move[
        SphericalLensSurface[1/c1/.localminimum[[2]],50,ComponentMedium->BK7],100],

Move[SphericalLensSurface[1/c2/.localminimum[[2]],50,ComponentMedium->{BK7,SF11}],115],

Move[SphericalLensSurface[1/c3/.localminimum[[2]],50,ComponentMedium->SF11],120]}],
    Move[Screen[50],200]}]
```

Out[120]=
```
{FocalPoint → {198.083, 0, 0}, FocusType → RMSFocus,
 WeightedSpotSize → 0.00321119, SpotSize → 0.00321119,
 BackFocalLength → 78.083, FocalPlaneTilt → {1., 0, 0}, TurboRays → TurboRays}
```

Go to list of topics

2.3 Geometric Intensity Calculations

Set-up System

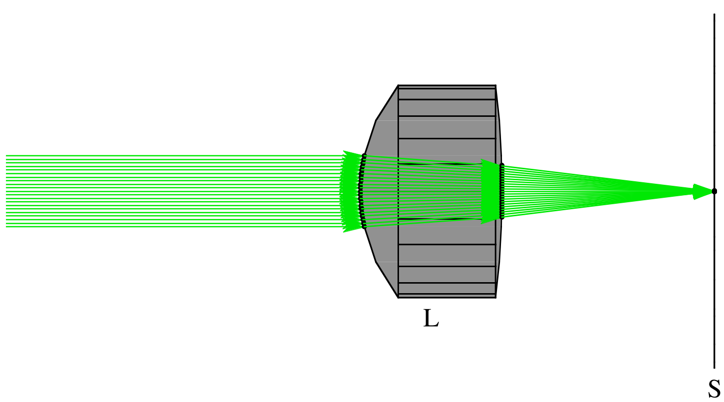
In[121]:=
```
localminimum = {c1→0.041742,c2→-0.00807904}
```

Out[121]=
```
{c1 → 0.041742, c2 → -0.00807904}
```

In[122]:=
```
optics = {
    SphericalLens[1/c1/.localminimum,1/c2/.localminimum,30,20,ComponentMedium->3/2],
    Move[Screen[50],{d+50,50}]};
```

In[123]:=
```
opticalsystem = {Move[LineOfRays[10,NumberOfRays->21],{-50,{y,0}}],optics};
```

In[124]:=
```
tracedsystem = AnalyzeSystem[opticalsystem,PlotType->TopView];
```



In[125]:=
```
data = ReadRays[tracedsystem, {SurfaceCoordinates[[1]], IntersectionNumber}];
```

In[126]:=

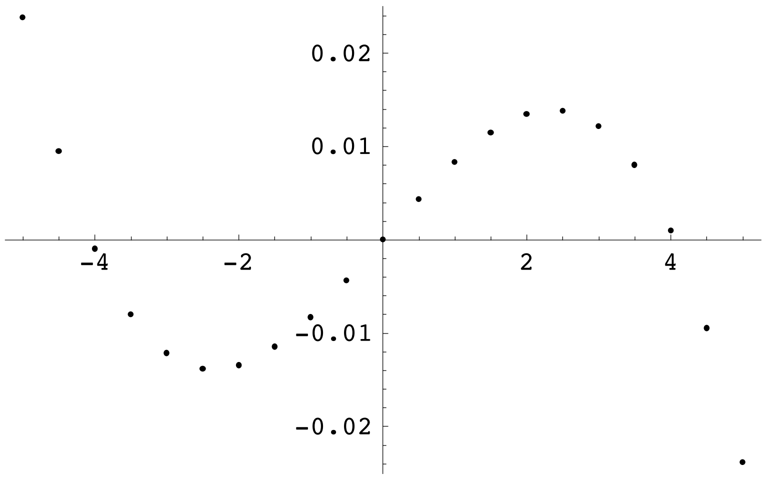
```
indata = Select[data,(#[[2]]==1)&]
```

Out[126]=

```
{{-5., 1}, {-4.5, 1}, {-4., 1}, {-3.5, 1}, {-3., 1}, {-2.5, 1},
 {-2., 1}, {-1.5, 1}, {-1., 1}, {-0.5, 1}, {0, 1}, {0.5, 1}, {1., 1},
 {1.5, 1}, {2., 1}, {2.5, 1}, {3., 1}, {3.5, 1}, {4., 1}, {4.5, 1}, {5., 1}}
```

In[127]:=

```
dataplot =
ListPlot[Map[{#[[1,1]],#[[2,1]]}&,Transpose[{indata,Select[data,(#[[2]]==3)&]}]]];
```



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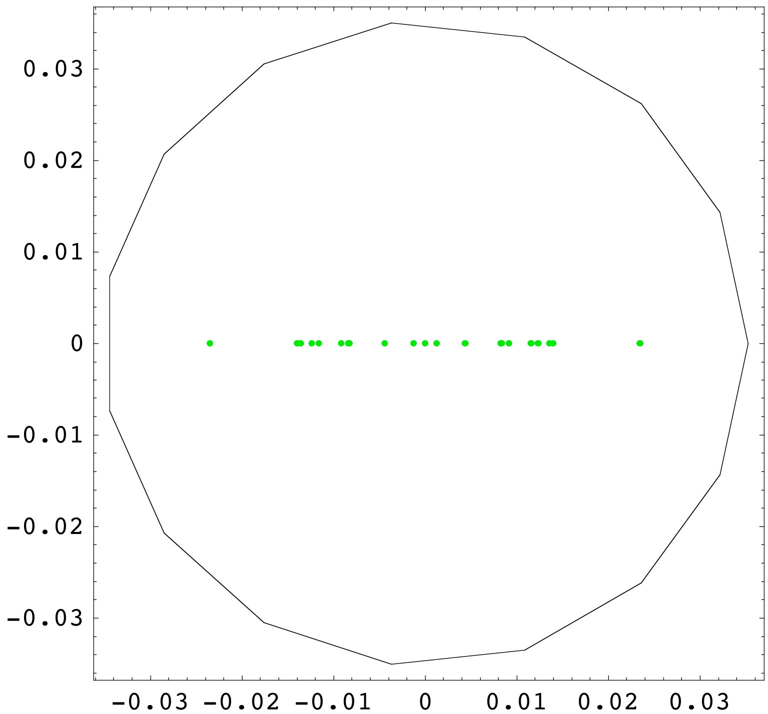
Preliminary Characterization

In[128]:=

```
FindFocus[opticalsystem]
```

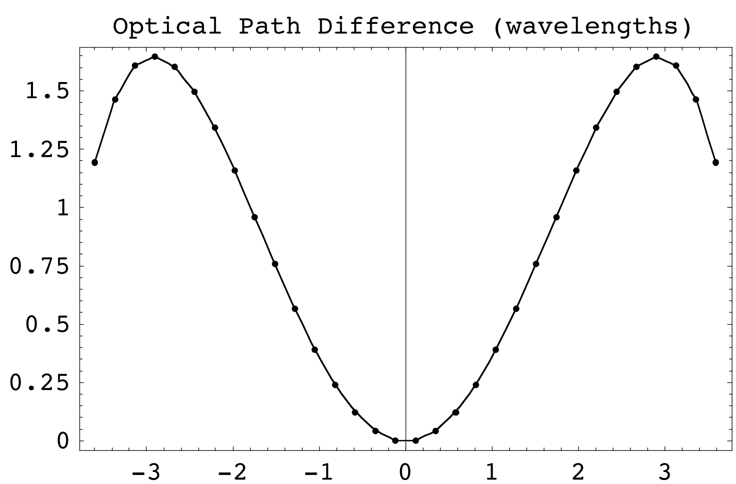


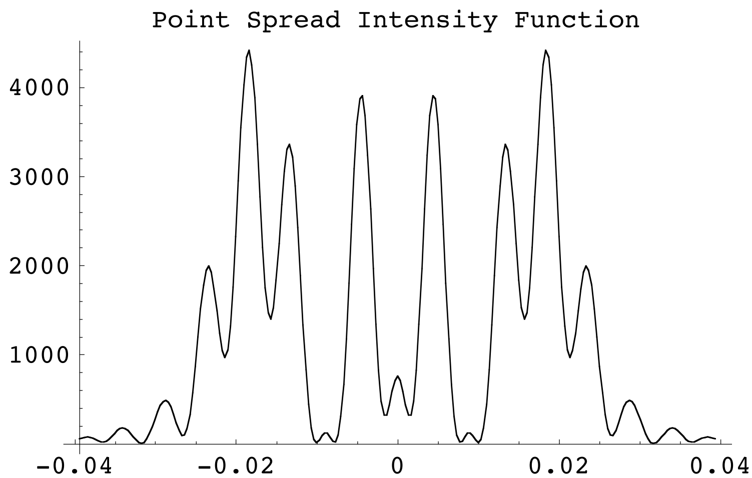
Out[128]=

```
{Screen → Move[Screen[0.0704605], 49.9971],
 TurboSystem → -traced system-, FocalPoint → {49.9971, 0, 0}, FocusType → RMSFocus,
 WeightedSpotSize → 0.0117992, SpotSize → 0.0117992, BackFocalLength → 29.9971,
 FocalPlaneTilt → {1., 0, 0}, TurboRays → -ray intercepts of 1 surfaces-}
```

In[129]:=

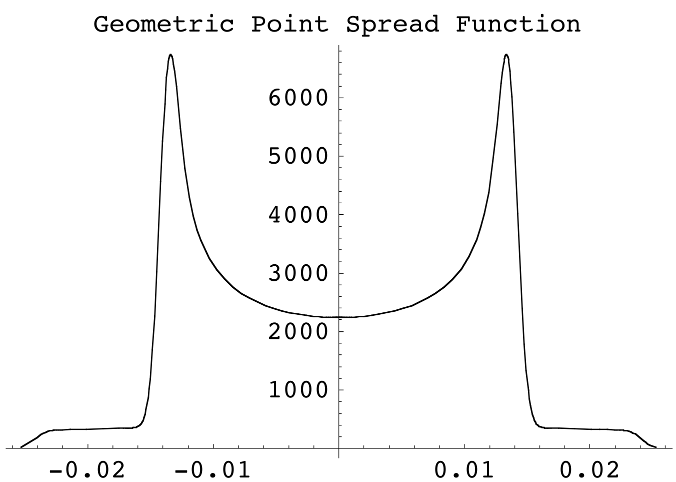
```
PSF[opticalsystem,NumberOfPoints->512,PaddingFactor->10];
```





In[130]:=

```
GPSF[opticalsystem,NumberOfRays->512];
```



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Inverse Position

In[131]:=

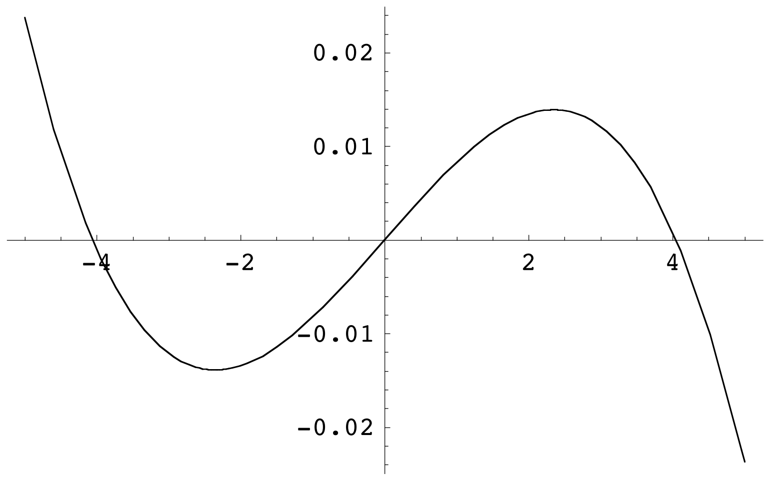
```
symbolicy = (SymbolicSurfaceCoordinates/.SymbolicTrace[opticalsystem,
    {{y,0,5},{y∈Reals,d∈Reals}}, ReportedParameters->SymbolicSurfaceCoordinates])[[1]]
```

Out[131]=

$$0.00883642\, y - 0.0237678\, d\, y - 0.00052525\, y^3 - 0.0000202087\, d\, y^3 - 7.26106 \times 10^{-7}\, y^5 - 2.73137 \times 10^{-8}\, d\, y^5$$

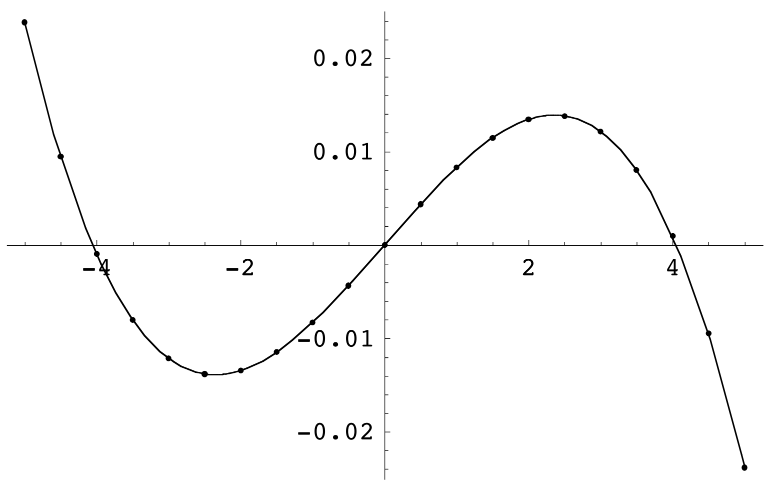
In[132]:=

```
oplot = Plot[symbolicy/.{d->0},{y,-5, 5}];
```



In[133]:=

```
Show[dataplot,oplot];
```



In[134]:=

```
Dy = D[symbolicy,y]
```

Out[134]=

$$0.00883642 - 0.0237678\, d - 0.00157575\, y^2 - 0.000060626\, d\, y^2 - 3.63053 \times 10^{-6}\, y^4 - 1.36569 \times 10^{-7}\, d\, y^4$$

In[135]:=

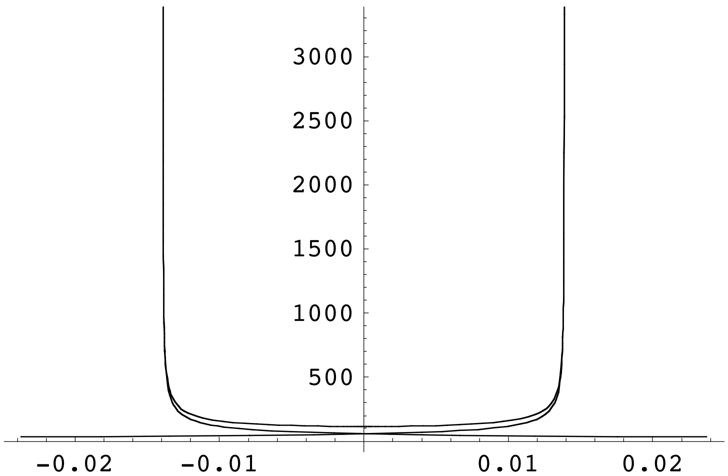
```
Cderivative = Compile[{y,d},Evaluate[Dy]];
```

In[136]:=

```
ytransfer = Compile[{y,d},Evaluate[symbolicy]];
```

In[137]:=

```
ParametricPlot[Evaluate[{symbolicy,1/Abs[Dy]}/.d->0],{y,-5,5}];
```



In[138]:=

```
inversey = Solve[ys==symbolicy,y]
```

Out[138]=

$\{\{y \to \text{Root}[ys - 0.00883642\,\#1 + 0.0237678\,d\,\#1 + 0.00052525\,\#1^3 + 0.0000202087\,d\,\#1^3 + 7.26106\times 10^{-7}\,\#1^5 + 2.73137\times 10^{-8}\,d\,\#1^5\ \&,\ 1]\},$

$\{y \to \text{Root}[ys - 0.00883642\,\#1 + 0.0237678\,d\,\#1 + 0.00052525\,\#1^3 + 0.0000202087\,d\,\#1^3 + 7.26106\times 10^{-7}\,\#1^5 + 2.73137\times 10^{-8}\,d\,\#1^5\ \&,\ 2]\},$

$\{y \to \text{Root}[ys - 0.00883642\,\#1 + 0.0237678\,d\,\#1 + 0.00052525\,\#1^3 + 0.0000202087\,d\,\#1^3 + 7.26106\times 10^{-7}\,\#1^5 + 2.73137\times 10^{-8}\,d\,\#1^5\ \&,\ 3]\},$

$\{y \to \text{Root}[ys - 0.00883642\,\#1 + 0.0237678\,d\,\#1 + 0.00052525\,\#1^3 + 0.0000202087\,d\,\#1^3 + 7.26106\times 10^{-7}\,\#1^5 + 2.73137\times 10^{-8}\,d\,\#1^5\ \&,\ 4]\},$

$\{y \to \text{Root}[ys - 0.00883642\,\#1 + 0.0237678\,d\,\#1 + 0.00052525\,\#1^3 + 0.0000202087\,d\,\#1^3 + 7.26106\times 10^{-7}\,\#1^5 + 2.73137\times 10^{-8}\,d\,\#1^5\ \&,\ 5]\}\}$

In[139]:=

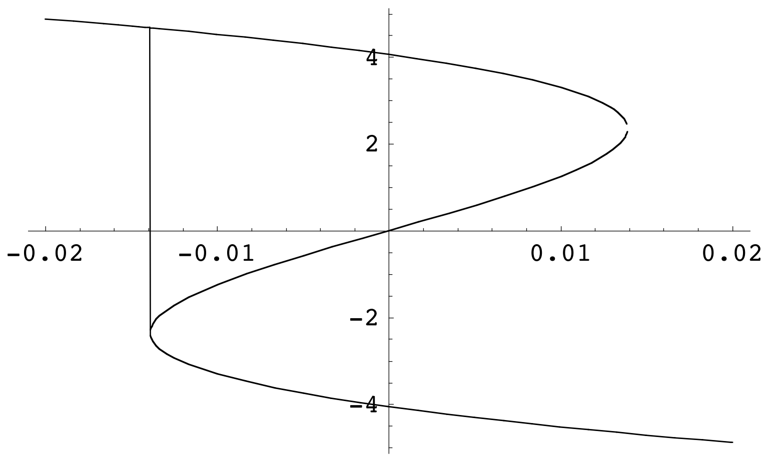
```
Plot[Evaluate[y/.inversey/.d->0],{ys,-.02,.02}];
```

Plot::plnr : Root[ys - 0.00883642 #1 + 0.00052525 #1^3 + 7.26106 × 10^{-7} #1^5 &, 2] is not a machine-size real number at ys = -0.02.

Plot::plnr : Root[ys - 0.00883642 #1 + 0.00052525 #1^3 + 7.26106 × 10^{-7} #1^5 &, 2] is not a machine-size real number at ys = -0.0183773.

Plot::plnr : Root[ys - 0.00883642 #1 + 0.00052525 #1^3 + 7.26106 × 10^{-7} #1^5 &, 2] is not a machine-size real number at ys = -0.0166076.

General::stop : Further output of Plot::plnr will be suppressed during this calculation.



Go to list of topics

Geometric Intensity Analysis

In[140]:=

```
(*Fast Version including aperture effects: presently=10*)
```

In[141]:=

```
Clear[GeometricIntensity];
GeometricIntensity =
    Function[{ys,d},
        Apply[Plus,
        Map[If[Abs[#]<=5.,1/(Abs[dummy2]/.y->#+.0001),0]&,
            Select[
                Chop[dummy1]
            ,
                (Head[#]=!=Symbol&&Im[#]==0)&
            ]
        ]]
    ]/.{dummy1->(y/.inversey),dummy2->Dy};
```

In[143]:=

```
GeometricIntensity//InputForm
```

Out[143]//InputForm=

```
Function[{ys, d},
 Plus @@ (If[Abs[#1] <= 5., 1/(Abs[0.0088364155326911 - 0.02376778501234205*d -
0.0015757513747870293*y^2 - 0.00006062604240982259*d*y^2 - 3.6305315986759584*^-6*y^4
-
         1.3656859916723253*^-7*d*y^4] /. y -> #1 + 0.0001), 0] & ) /@
   Select[Chop[{Root[ys - 0.0088364155326911*#1 + 0.02376778501234205*d*#1 +
0.0005252504582623431*#1^3 + 0.000020208680803274196*d*#1^3 +
7.261063197351916*^-7*#1^5 +
         2.7313719833446508*^-8*d*#1^5 & , 1], Root[ys - 0.0088364155326911*#1 +
0.02376778501234205*d*#1 + 0.0005252504582623431*#1^3 + 0.000020208680803274196*d*#1^3
+
         7.261063197351916*^-7*#1^5 + 2.7313719833446508*^-8*d*#1^5 & , 2], Root[ys -
0.0088364155326911*#1 + 0.02376778501234205*d*#1 + 0.0005252504582623431*#1^3 +
         0.000020208680803274196*d*#1^3 + 7.261063197351916*^-7*#1^5 +
2.7313719833446508*^-8*d*#1^5 & , 3],
      Root[ys - 0.0088364155326911*#1 + 0.02376778501234205*d*#1 +
0.0005252504582623431*#1^3 + 0.000020208680803274196*d*#1^3 +
7.261063197351916*^-7*#1^5 + 2.7313719833446508*^-8*d*#1^5 & , 4],
      Root[ys - 0.0088364155326911*#1 + 0.02376778501234205*d*#1 +
0.0005252504582623431*#1^3 + 0.000020208680803274196*d*#1^3 +
7.261063197351916*^-7*#1^5 + 2.7313719833446508*^-8*d*#1^5 & ,
       5]}], Head[#1] =!= Symbol && Im[#1] == 0 & ]]
```

In[144]:=

```
Options[NIntegrate]
```

Out[144]=

```
{AccuracyGoal → ∞, Compiled → True, GaussPoints → Automatic,
 MaxPoints → Automatic, MaxRecursion → 6, Method → Automatic, MinRecursion → 0,
 PrecisionGoal → Automatic, SingularityDepth → 4, WorkingPrecision → 16}
```

In[145]:=

```
Off[NIntegrate::slwcon];
NIntegrate[GeometricIntensity[ys,0],{ys,-.04,.04},MaxRecursion->12]
```

Out[146]=

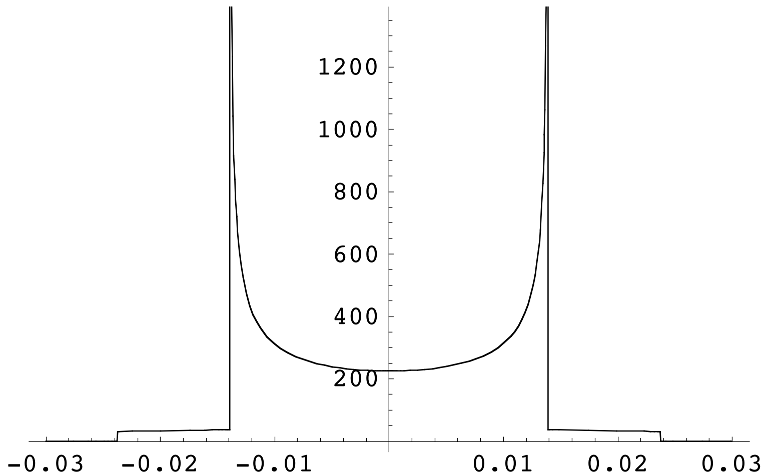
```
9.95693
```

In[147]:=

```
On[NIntegrate::slwcon];
```

In[148]:=

```
Plot[GeometricIntensity[ys,0],{ys,-.03,.03}];
```



In[149]:=

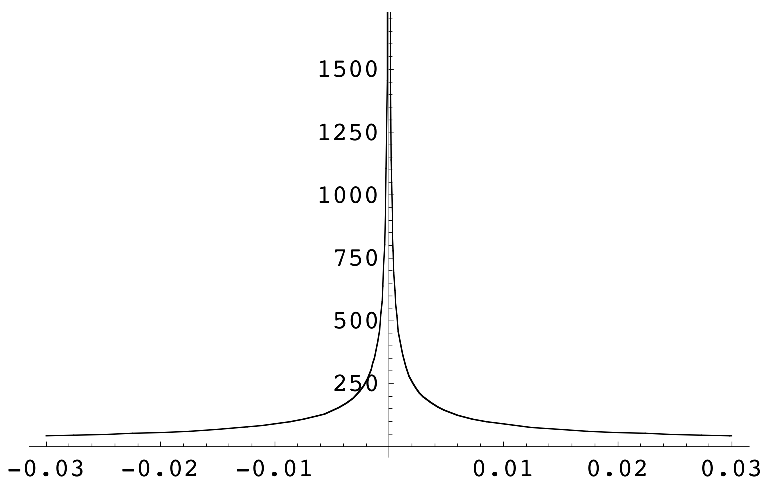
```
paraxialfocus = d/.Solve[Evaluate[(Dy/.y->0)==0],d][[1]]
```

Out[149]=

```
0.371781
```

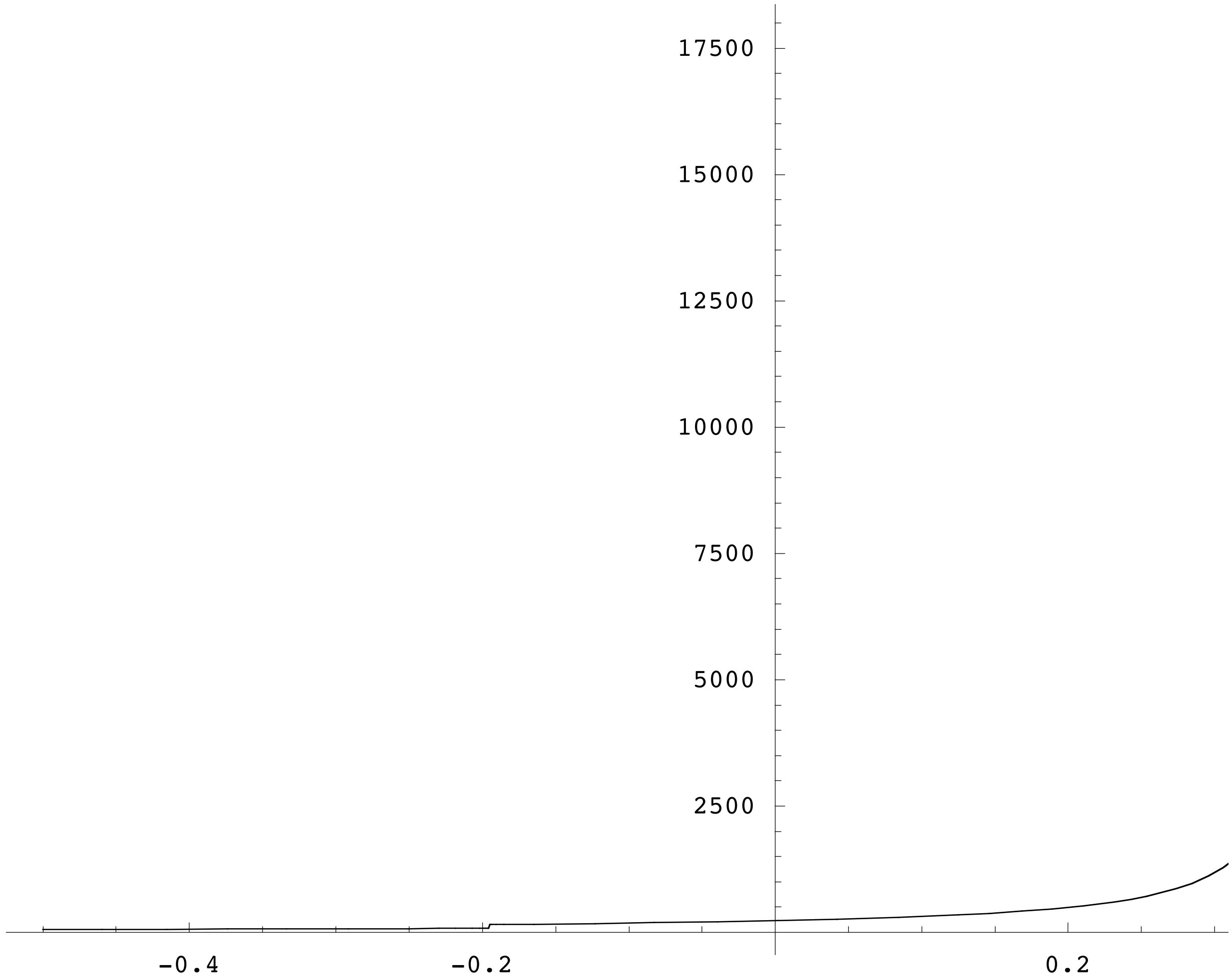
In[150]:=

```
Plot[GeometricIntensity[ys,paraxialfocus],{ys,-.03,.03}];
```



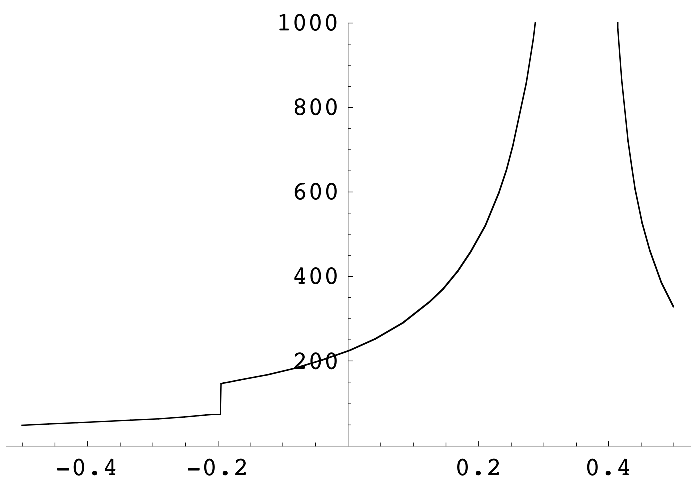
In[151]:=

```
Plot[GeometricIntensity[0,d],{d,-.5,.5}];
```



In[152]:=

```
Plot[GeometricIntensity[0,d],{d,-.5,.5},PlotRange->{0,1000}];
```

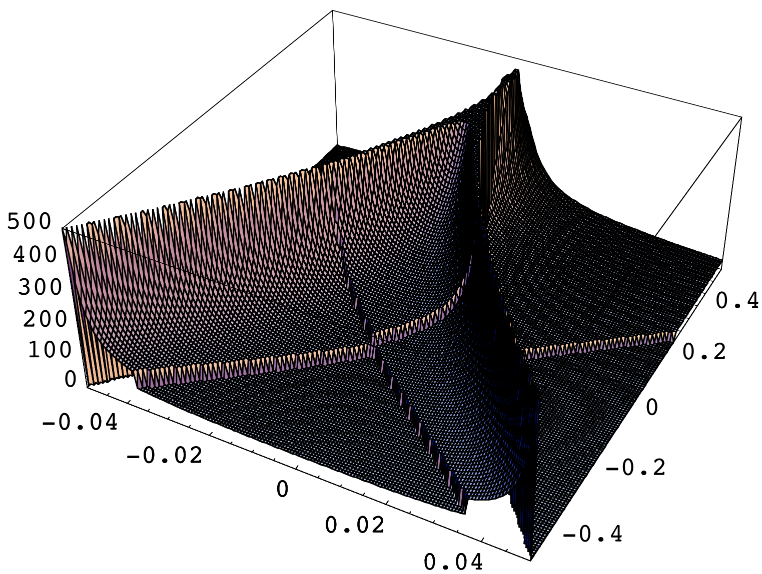


In[113]:=

Note that the following plots are very time consuming to construct. Some systems may require 10 - 15 minutes for each plot. The results are well worth the wait however.

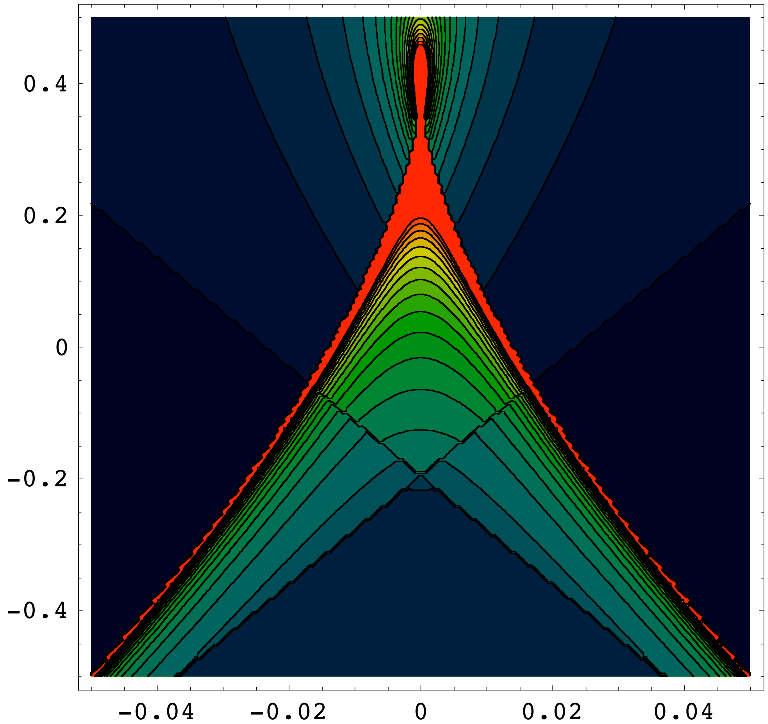
In[153]:=

```
Plot3D[GeometricIntensity[ys,d], {ys,-.05,.05}, {d,-.5,
.5},PlotPoints->150,PlotRange->{0,500}];
```



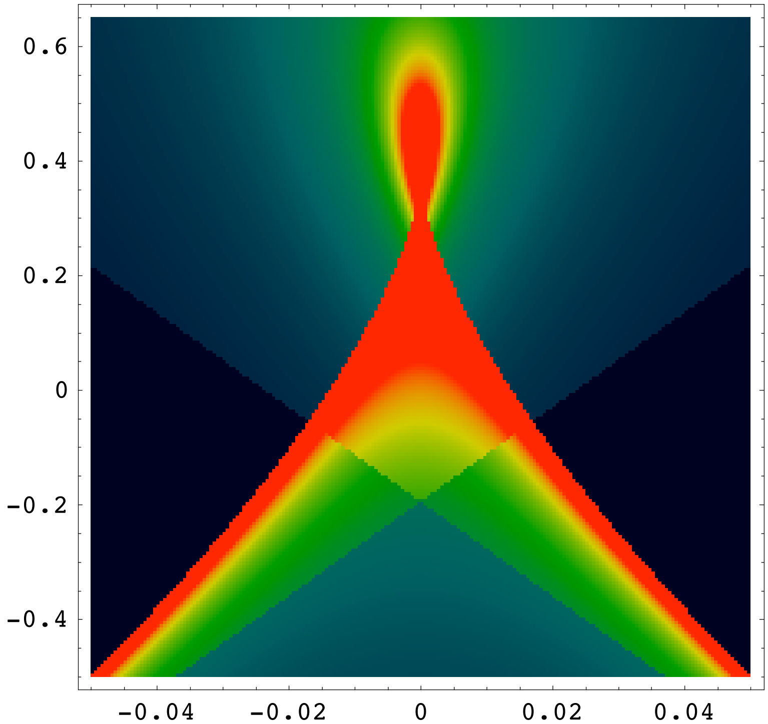
In[154]:=

```
ContourPlot[GeometricIntensity[ys,d], {ys,-.05,.05}, {d,-.5,
.5},PlotPoints->200,PlotRange->{0,500}, Contours->20,
ColorFunction->Function[Hue[.65-#*(.65),1,#*.9+.1]]];
```



In[155]:=

```
DensityPlot[GeometricIntensity[ys,d], {ys,-.05,.05}, {d,-.5, .65}, PlotPoints->200,
PlotRange->{0,260}, ColorFunction->Function[Hue[.65-#*(.65),1,#*.9+.1]], Mesh->False];
```



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3. ABCD Matrix Calculations: FindABCDMatrix

3.1 Overview

In[156]:=

```
?FindABCDMatrix
```

In[157]:=

```
Options[FindABCDMatrix]
```

In[158]:=

```
opticalsystemreflected = {
GaussianBeam[40.,.01,SymbolicIntensity->100,SymbolicWaveLength->λ,NumberOfRays->6],
Move[PlanoConvexLens[{f,100},{a,50},{t,10},DesignWaveLength->.532],{d1,50}],
Move[Mirror[{b,20}],175,-45],
Move[Screen[50],{175,50},90]};
```

In[159]:=

```
AnalyzeSystem[opticalsystemreflected,PlotType->TopView];
```

In[160]:=

```
FindABCDMatrix[opticalsystemreflected,{f>0},MatrixForm->True]
```

```
In[161]:=
     FindABCDMatrix[opticalsystemreflected,{f>0},MatrixForm->True,DecomposeABCD->True]

In[162]:=
     FindABCDMatrix[opticalsystemreflected,MatrixForm->True,DecomposeABCD->True,NumericalRes
     ults->True]
```

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3.2 SphericalMirror Examples

```
In[12]:= opticalsystem = {
             Move[LineOfRays[40,IntrinsicMedium->Vacuum],{d1,-70}],
             SphericalMirror[{R,-100},50],
             Move[Screen[25],{d2,-50}]};

In[13]:= AnalyzeSystem[opticalsystem,PlotType->TopView];

In[17]:= result = FindABCDMatrix[opticalsystem,{R<0,d1<0,d2<0},ABCDConstruction->Horizontal]

In[18]:= Simplify[result/.{d1->0,d2->0},{R<0}]
```

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3.3 PlanoConvexLens Examples

```
In[19]:= opticalsystem = {
         Move[LineOfRays[40,SymbolicIntensity->100,SymbolicWaveLength->λ], {-d1,-50}],
         PlanoConvexLens[{f,100},{a,50},{t,10},DesignWaveLength->.532],
         Move[Screen[50],{d2+t,100}]};

In[20]:= AnalyzeSystem[opticalsystem,PlotType->TopView];

In[21]:= FindABCDMatrix[opticalsystem,{f>0},MatrixForm->True, DecomposeABCD->True,
         ABCDConstruction->Horizontal]

In[23]:= FindABCDMatrix[opticalsystem,{f>0,a>0,d1>0,t>0,d2>0}, MatrixForm->True,
         DecomposeABCD->True,
             ComplexABCD->True, ABCDConstruction->Horizontal]
```

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```
In[24]:= offaxisopticalsystem = {
         Move[LineOfRays[40,SymbolicIntensity->100,SymbolicWaveLength->λ], {-d1,-50}],
         PlanoConvexLens[{f,100},{a,50},{t,20},DesignWaveLength->.532,OffAxis->{10,0}],
         Move[Screen[50],{d2+t,100}]};

In[25]:= AnalyzeSystem[offaxisopticalsystem,PlotType->TopView];

In[26]:= FindABCDMatrix[offaxisopticalsystem,{f>0,a>0,d1>0,t>0,d2>0}, MatrixForm->True,
         DecomposeABCD->True, ComplexABCD->True]
```

In[27]:=
```
resultO = FindABCDMatrix[offaxisopticalsystem,{f>0,a>0,d1>0,t>0,d2>0},
MatrixForm->False, DecomposeABCD->True, ComplexABCD->True,
ABCDConstruction->Horizontal]
```

In[28]:=
```
FindABCDMatrix[offaxisopticalsystem,{f>0,a>0,d1>0,t>0,d2>0}, MatrixForm->True,
ABCDConstruction->Horizontal]
```

Go to list of topics

In[29]:=
```
gaussiansystem3D = {
GaussianBeam[{1,1},{.1,.1},SymbolicIntensity->100,SymbolicWaveLength->λ],
Move[PlanoConvexCylindricalLens[{f,100},{{a,50},{b,40}},{t,10},DesignWaveLength->.532
],{d1,50}],
Move[Screen[{{h,30},{v,50}}],{d2+t,100}, TwistAngle->30]};
```

In[30]:=
```
gaussiansystem = {
GaussianBeam[1,.1,SymbolicIntensity->100,SymbolicWaveLength->λ],
Move[PlanoConvexCylindricalLens[{f,100},{{a,50},{b,40}},{t,10},DesignWaveLength->.532
],{d1,50}],
Move[Screen[{{h,30},{v,50}}],{d2+t,100}, TwistAngle->30]};
```

In[31]:=
```
FindABCDMatrix[gaussiansystem3D,{f>0},MatrixForm->True]
```

In[32]:=
```
FindABCDMatrix[gaussiansystem,{f>0},MatrixForm->True]
```

In[47]:=
```
cylindricalsystem = {
Move[LineOfRays[40,SymbolicIntensity->100,SymbolicWaveLength->λ,
IntrinsicMedium->Vacuum], {-d1,-50}],
PlanoConvexCylindricalLens[{f,100},{{a,50},{b,40}},{t,10},DesignWaveLength->1/2,
ComponentMedium->3/2],
Move[Screen[{{h,30},{v,50}}],{d2+t,100}, TwistAngle->30]};
```

In[34]:=
```
AnalyzeSystem[cylindricalsystem];
```

In[48]:=
```
FindABCDMatrix[cylindricalsystem, {f>0}, MatrixForm->True]
```

In[49]:=
```
FindABCDMatrix[cylindricalsystem, {f>0}, MatrixForm->True,
ABCDConstruction->Horizontal]
```

In[50]:=
```
FindABCDMatrix[cylindricalsystem, {f>0}, MatrixForm->True,
ABCDConstruction->Vertical]
```

In[38]:=
```
FindABCDMatrix[cylindricalsystem, {f>0}, MatrixForm->True, DecomposeABCD->True]
```

In[39]:=
```
FindABCDMatrix[cylindricalsystem, {f>0}, MatrixForm->True, DecomposeABCD->True,
ComplexABCD->True]
```

In[40]:=
```
FindABCDMatrix[cylindricalsystem, MatrixForm->True, NumericalResults->True]
```

In[41]:=
```
FindABCDMatrix[cylindricalsystem, MatrixForm->True, NumericalResults->True,
ComplexABCD->True]
```

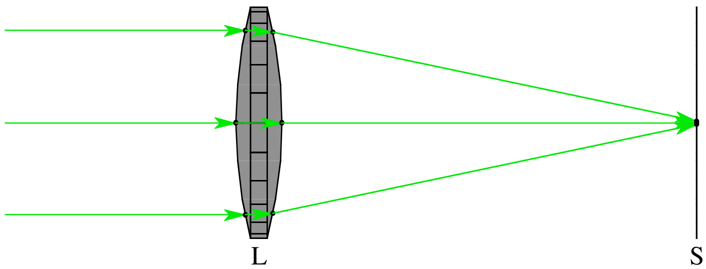
In[42]:=
```
FindABCDMatrix[cylindricalsystem, MatrixForm->True, NumericalResults->True,
ComplexABCD->True, DecomposeABCD->True]
```

Go to list of topics

3.4 SphericalLens Examples

```
In[35]:= opticalsystem = {Move[LineOfRays[40,
         IntrinsicMedium->Vacuum],{{-d1,-50},{y,0}},{s/Degree,0}],
         SphericalLens[{R1,100},{R2,-100},50,{t,10},DesignWaveLength->1/2],
         Move[Screen[50],{d2,100}]};
```

```
In[93]:= AnalyzeSystem[opticalsystem,PlotType->TopView];
```




```
In[36]:= FindABCDMatrix[opticalsystem,y,s,{R1>0,R2<0,d1>0,t>0,d2>0,n2>0,n1>0},
                   SymbolicRefractiveModels->{Vacuum->n1,BK7->n2}, MatrixForm->False]
```

Out[36]= {ABCDCenterPoint → {d2, 0, 0},

ABCDMatrix → $\left\{\left\{\frac{1}{n1\,n2\,R1\,R2}(n1\,n2\,(R1-2\,t)\,(R2+t)+n1^2\,t\,(R2+t)+n2^2\,t\,(-R1+R2+t)-d2\,(n1-n2)\,(n2\,(R1-R2-t)+n1\,t)),\ \frac{1}{n1\,n2\,R1\,R2}(d1\,(n1\,n2\,(R1-2\,t)\,(R2+t)+n1^2\,t\,(R2+t)+n2^2\,t\,(-R1+R2+t)-d2\,(n1-n2)\,(n2\,(R1-R2-t)+n1\,t))+n1\,R1\,((n1-n2)\,t\,(R2+t)+d2\,(-n1\,t+n2\,(R2+t))))\right\},\left\{-\frac{(n1-n2)\,(n2\,(R1-R2-t)+n1\,t)}{n1\,n2\,R1\,R2},\ \frac{-d1\,(n1-n2)\,(n2\,(R1-R2-t)+n1\,t)+n1\,R1\,(-n1\,t+n2\,(R2+t))}{n1\,n2\,R1\,R2}\right\}\right\}$,

ABCDRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}},

SymbolicIntensity →
 100,

SymbolicOpticalLength → d1 n1 + n1 (d2 − t) + n2 t,

SymbolicWaveLength → $\frac{133}{250}$}

```
In[37]:= result = FindABCDMatrix[opticalsystem,y,s,{R1>0,R2<0,d1>0,t>0,d2>0,n2>0,n1>0},
                   SymbolicRefractiveModels->{Vacuum->n1,BK7->n2}, MatrixForm->True]
```

Out[37]//MatrixForm=

$$\begin{pmatrix} \frac{n1\,n2\,(R1-2\,t)\,(R2+t)+n1^2\,t\,(R2+t)+n2^2\,t\,(-R1+R2+t)-d2\,(n1-n2)\,(n2\,(R1-R2-t)+n1\,t)}{n1\,n2\,R1\,R2} & \frac{d1\,(n1\,n2\,(R1-2\,t)\,(R2+t)+n1^2\,t\,(R2+t)+n2^2\,t\,(-R1+R2+t}{} \\ -\frac{(n1-n2)\,(n2\,(R1-R2-t)+n1\,t)}{n1\,n2\,R1\,R2} & \frac{-d1\,(n1-n2)\,(n2}{} \end{pmatrix}$$

```
In[38]:= result/.{d1->0,t->0,d2->0}
```

Out[38]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ -\frac{(n1-n2)\,(R1-R2)}{n1\,R1\,R2} & 1 \end{pmatrix}$$

```
In[39]:= result/.{d1->0,d2->0}
```

Out[39]//MatrixForm=

$$\begin{pmatrix} \frac{n1\ n2\ (R1-2\ t)\ (R2+t)+n1^2\ t\ (R2+t)+n2^2\ t\ (-R1+R2+t)}{n1\ n2\ R1\ R2} & \frac{(n1-n2)\ t\ (R2+t)}{n2\ R2} \\ -\frac{(n1-n2)\ (n2\ (R1-R2-t)+n1\ t)}{n1\ n2\ R1\ R2} & \frac{-n1\ t+n2\ (R2+t)}{n2\ R2} \end{pmatrix}$$

```
In[40]:= result/.{d1->d,t->0,d2->0}
```

Out[40]//MatrixForm=

$$\begin{pmatrix} 1 & d \\ -\frac{(n1-n2)\ (R1-R2)}{n1\ R1\ R2} & \frac{-d\ (n1-n2)\ n2\ (R1-R2)+n1\ n2\ R1\ R2}{n1\ n2\ R1\ R2} \end{pmatrix}$$

Go to list of topics

3.5 ThinLens, and Custom ABCD Input

```
In[99]:= thinsystem = {Move[LineOfRays[40],{-d1,-50}],
         ThinLens[{f,100},50],
         Move[Screen[50],{d2,100}]};

In[100]:=
    AnalyzeSystem[thinsystem,PlotType->TopView];

In[101]:=
    result = FindABCDMatrix[thinsystem,{f>0,d1>0,d2>0}, ABCDConstruction->Horizontal]

In[102]:=
    result/.{d1->0,d2->0}

In[103]:=
    result/.{d1->d,d2->0}

In[104]:=
    thicksystem = {Move[LineOfRays[40],{-d1,-50}],
    ThickLens[{f,100},50,{t,10}],
    Move[Screen[50],{d2,100}]};

In[105]:=
    AnalyzeSystem[thicksystem,PlotType->TopView];

In[106]:=
    TurboPlot[thicksystem,PlotType->TopView];

In[107]:=
    result = FindABCDMatrix[thicksystem,{f>0,d1>0,t>0,d2>0}, ABCDConstruction->Horizontal]

In[108]:=
    result/.{d1->0,t->0,d2->0}

In[109]:=
    result/.{d1->0,d2->0}

In[110]:=
    result/.{d1->d,t->0,d2->0}
```

You can enter your own ABCD matrix expressions using *Rayica*'s built-in **ABCDOptic** function.

In[111]:=
```
abcdsystem = {Move[LineOfRays[40],{-d1,-50}],
ABCDOptic[{{1,0},{{-1/f,-1/100},1}},{a,50}],
Move[Screen[50],{d2,100}]};
```

In[112]:=
```
AnalyzeSystem[abcdsystem,PlotType->TopView];
```

In[113]:=
```
result = FindABCDMatrix[abcdsystem,{f>0}, ABCDConstruction->Horizontal]
```

In[114]:=
```
result/.{d1->0,d2->0}
```

In[115]:=
```
FindABCDMatrix[abcdsystem,{f>0}, ABCDConstruction->Horizontal, ComplexABCD->True,
DecomposeABCD->True]
```

ABCDOptic now also works with astigmatic systems.

In[116]:=
```
abcdsystem = {Move[GridOfRays[40],{-d1,-50}],
ABCDOptic[{{{1,0},{{-1/f1,-1/100},1}},{{1,0},{{-1/f2,-1/200},1}}},{a,50}],
Move[Screen[50],{d2,100}]};
```

In[117]:=
```
AnalyzeSystem[abcdsystem];
```

In[118]:=
```
result = FindABCDMatrix[abcdsystem,{f>0}]
```

In[119]:=
```
result/.{d1->0,d2->0}
```

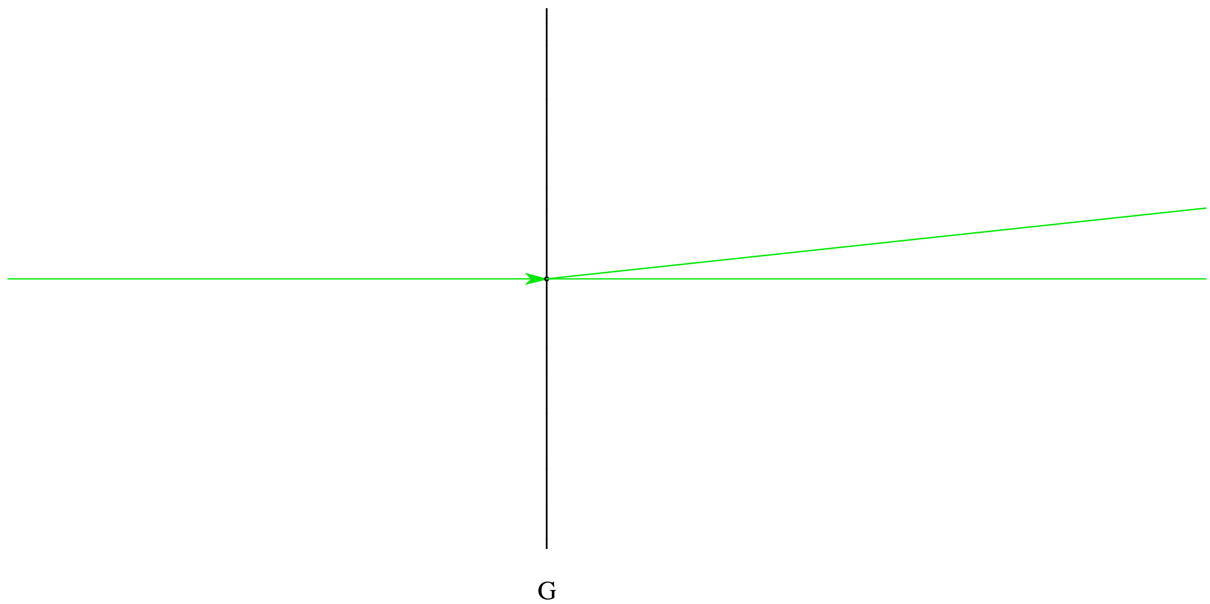
Go to list of topics

3.6 Grating Examples

In[120]:=
```
?Grating
```

In[13]:=
```
gratingsystem = {Move[SingleRay[SymbolicWaveLength->λ,
 IntrinsicMedium->Vacuum],{-d1,-50}],
 Grating[{g,200},50,GratingMedium->IntrinsicMedium,GratingThickness->0,ComponentMedium
 ->IntrinsicMedium],
 Move[Screen[50],{d2,100}]};
```

In[14]:= **sol = AnalyzeSystem[gratingsystem,PlotType->TopView];**



In[124]:=
ReadRays[sol,RayEnd]

Out[124]=
{{0., 0., 0.}, {100., 0., 0.}, {100., 10.7007, 0.}}

In[11]:= **result = FindABCDMatrix[gratingsystem, {λ>0,g>0,d1>0,t>0,d2>0},MatrixForm->True, SymbolicRefractiveModels->{Vacuum->1}, ABCDConstruction->Horizontal, FilterTrace->False]**

Out[11]= $\left\{\left\{\begin{pmatrix} 1 & d1+d2 \\ 0 & 1 \end{pmatrix}\right\}, \left\{\begin{pmatrix} -\frac{\sqrt{1000000-g^2\lambda^2}}{1000} & -\frac{d1\sqrt{1000000-g^2\lambda^2}}{1000}+\frac{1000000\ d2}{-1000000+g^2\lambda^2} \\ 0 & \frac{1000}{\sqrt{1000000-g^2\lambda^2}} \end{pmatrix}\right\}\right\}$

In[10]:= **result/.{d1->0,d2->0}**

Out[10]//MatrixForm=
$$\begin{pmatrix} -\frac{\sqrt{1000000-g^2\lambda^2}}{1000} & 0 \\ 0 & \frac{1000}{\sqrt{1000000-g^2\lambda^2}} \end{pmatrix}$$

In[12]:= **result = FindABCDMatrix[gratingsystem,{g>0,λ>0,d1>0,t>0,d2>0},MatrixForm->True, SymbolicRefractiveModels->{Vacuum->1}, ABCDConstruction->Horizontal, FilterTrace->True]**

Out[12]//MatrixForm=
$$\begin{pmatrix} 1 & d1+d2 \\ 0 & 1 \end{pmatrix}$$

```
In[13]:= FindABCDMatrix[gratingsystem, NumericalResults->True,MatrixForm->False,
         FilterTrace->False]

Out[13]= {{{ABCDCenterPoint → {{100, 0, 0}, {100, 0, 0}},
            ABCDMatrix → {{{1, 150.}, {0, 1}}, {{1, 150.}, {0, 1}}}, ABCDRotationMatrix →
             {{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}},
            SymbolicIntensity → {20, 20}, SymbolicOpticalLength → {150., 150.},
            SymbolicWaveLength → {0.532, 0.532}}},
          {{ABCDCenterPoint → {{100, 10.7007, 0}, {100, 10.7007, 0}},
            ABCDMatrix → {{{0.994323, 150.861}, {0, 1.00571}}, {{1, 150.571}, {0, 1}}},
            ABCDRotationMatrix → {{{0.994323, 0.1064, 0}, {-0.1064, 0.994323, 0}, {0, 0, 1}},
              {{0.994323, 0.1064, 0}, {-0.1064, 0.994323, 0}, {0, 0, 1}}},
            SymbolicIntensity → {66.6075, 66.6075}, SymbolicOpticalLength → {150.571, 150.571},
            SymbolicWaveLength → {0.532, 0.532}}}}
```

Go to list of topics

4. Gaussian Beam Calculations: GaussianTrace

4.1 Overview

```
In[228]:=
      ?GaussianTrace

In[229]:=
      Options[GaussianTrace]

In[230]:=
      ?GaussianPlot

In[231]:=
      Options[GaussianPlot]

In[232]:=
      ?GaussianBeam

In[233]:=
      Options[GaussianBeam]

In[17]:= gratingsystem =
         {Move[GaussianBeam[5.,.01,SymbolicWaveLength->λ,NumberOfRays->3],{-d1,-50}],
         Grating[{g,400},50,GratingMedium->IntrinsicMedium,GratingThickness->0,ComponentMedium
         ->IntrinsicMedium],
         Move[Screen[50],{d2,100}]};

In[18]:= sol = AnalyzeSystem[gratingsystem,PlotType->TopView, ShowGaussian->True];
```

Go to list of topics

4.2 Reflected Lens Example

Set-up

```
In[24]:= opticalsystemreflected = {
        GaussianBeam[20.,.01,SymbolicWaveLength->λ,NumberOfRays->6],
        Move[PlanoConvexLens[{f,100},{a,50},{t,10},DesignWaveLength->.532],{d1,50}],
        Move[Mirror[{b,20}],175,-45],
        Move[Screen[50],{175,50},90]};
```

```
In[21]:= AnalyzeSystem[opticalsystemreflected, PlotType->TopView];
```

```
In[25]:= FindABCDMatrix[opticalsystemreflected,MatrixForm->True,DecomposeABCD->True,NumericalR
        esults->True]
```

```
In[27]:= FindABCDMatrix[opticalsystemreflected,{f>0},MatrixForm->True,DecomposeABCD->True,Nume
        ricalResults->False,SymbolicRefractiveModels->{Air->1,BK7->3/2}]
```

Go to list of topics

GaussianTrace

```
In[238]:=
      GaussianTrace[opticalsystemreflected]
```

Go to list of topics

GaussianPlot

```
In[29]:= GaussianPlot[opticalsystemreflected];
```

```
In[30]:= GaussianPlot[opticalsystemreflected, RenderedParameters->BeamCurvature];
```

```
In[31]:= GaussianPlot[opticalsystemreflected, RenderedParameters->ComplexBeamParameter];
```

```
In[32]:= finalresultreflected =
            GaussianPlot[opticalsystemreflected, Plot3D->True, PlotType->TopView]
```

Go to list of topics

ShowGaussian

```
In[33]:= ShowSystem[opticalsystemreflected,PlotType->TopView,ShowGaussian->True];
```

```
In[34]:= AnalyzeSystem[opticalsystemreflected,PlotType->TopView,ShowGaussian->True];
```

Go to list of topics

4.3 Reflected ThickLens Example

Set-up

```
In[35]:= thinopticalsystemreflected2 = {
         GaussianBeam[{20.,15.},{.01,.05},SymbolicWaveLength->λ,NumberOfRays->6],
         Move[ThinLens[{{f1,100},{f2,150}},50],{d1,50}],
         Move[Mirror[{b,20}],175,-45],
         Move[Screen[50],{175,50},90]};
```

```
In[36]:= AnalyzeSystem[thinopticalsystemreflected2];
```

```
In[37]:= FindABCDMatrix[thinopticalsystemreflected2,MatrixForm->True,DecomposeABCD->True,Numer
         icalResults->True,SymbolicRefractiveModels->{Air->1}]
```

```
In[38]:= FindABCDMatrix[thinopticalsystemreflected2,MatrixForm->True,DecomposeABCD->True,Numer
         icalResults->False,SymbolicRefractiveModels->{Air->1}]
```

Go to list of topics

GaussianTrace

```
In[39]:= GaussianTrace[thinopticalsystemreflected2]
```

Go to list of topics

GaussianPlot

```
In[40]:= GaussianPlot[thinopticalsystemreflected2]
```

Go to list of topics

ShowGaussian

```
In[41]:= ShowSystem[thinopticalsystemreflected2,ShowGaussian->True,CreateStereoView->Crossed];
```

Go to list of topics

4.4 Off-Axis Beam Example

Set-up

```
In[61]:= opticalsystemoffaxis = {
         Move[GaussianBeam[{5.,5.},{.01,.01},SymbolicWaveLength->λ],{0,10}],
         Move[PlanoConvexLens[{f,100},{a,50},{t,10},DesignWaveLength->.532],{d1,50}],
         Move[Mirror[20],175,-45],
         Move[Screen[50],{175,50},90]};
```

```
In[62]:= AnalyzeSystem[opticalsystemoffaxis];
```

```
In[63]:= FindABCDMatrix[opticalsystemoffaxis,MatrixForm->True,DecomposeABCD->True,NumericalRes
         ults->True]
```

```
In[64]:= FindABCDMatrix[opticalsystemoffaxis,NumericalResults->True,MatrixForm->True]
```

Go to list of topics

GaussianTrace

```
In[65]:= GaussianTrace[opticalsystemoffaxis]
```

Go to list of topics

GaussianPlot

```
In[66]:= GaussianPlot[opticalsystemoffaxis];
```

```
In[68]:= GaussianPlot[opticalsystemoffaxis,RenderedParameters->ComplexBeamParameter];
```

Go to list of topics

ShowGaussian

```
In[69]:= ShowSystem[opticalsystemoffaxis, PlotType->TopView, ShowGaussian->True];
```

```
In[70]:= ShowSystem[opticalsystemoffaxis, PlotType->SideView, ShowGaussian->True];
```

```
In[71]:= ShowSystem[opticalsystemoffaxis, ShowGaussian->True];
```

Go to list of topics

4.5 CylindricalLens Example

Set-up

```
In[72]:= cylindricalsystem = {
         GaussianBeam[{5,10},{.01,.005},SymbolicWaveLength->λ],
         Move[PlanoConvexCylindricalLens[{f,100},{{a,50},{b,40}},{t,10},DesignWaveLength->.532
         ],{d1,50}],
         Move[Screen[{{h,30},{v,50}}],{d2+t,100}, TwistAngle->30]};
```

```
In[73]:= AnalyzeSystem[cylindricalsystem];
```

```
In[74]:= FindABCDMatrix[cylindricalsystem,{f>0},MatrixForm->True,DecomposeABCD->True]
```

```
In[75]:= FindABCDMatrix[cylindricalsystem,NumericalResults->True,MatrixForm->False]
```

Go to list of topics

GaussianTrace

```
In[76]:= GaussianTrace[cylindricalsystem]
```

Go to list of topics

GaussianPlot

```
In[77]:= GaussianPlot[cylindricalsystem];
```

```
In[78]:= GaussianPlot[cylindricalsystem,RenderedParameters->ComplexBeamParameter];
```

Go to list of topics

ShowGaussian

```
In[79]:= ShowSystem[cylindricalsystem, PlotType->TopView, ShowGaussian->True];
```

```
In[80]:= ShowSystem[cylindricalsystem, PlotType->SideView, ShowGaussian->True];
```

```
In[81]:= ShowSystem[cylindricalsystem, ShowGaussian->True];
```

Go to list of topics

4.6 Prism-Lens Example

Set-up

```
In[82]:= prism = Prism[{60,50,60},50];
         prismgaussiansystem = {
         Move[GaussianBeam[{5,5},{.01,.01}],0,20],
         Move[prism,{30,-20}],
         Move[BiConvexLens[45,30,10],60],
         Move[Screen[50],120]};
```

```
In[84]:= AnalyzeSystem[prismgaussiansystem];
```

```
In[85]:= FindABCDMatrix[prismgaussiansystem,MatrixForm->True,DecomposeABCD->True,NumbericalRes
         ults->True]
```

Go to list of topics

GaussianTrace

```
In[86]:= GaussianTrace[prismgaussiansystem]
```

Go to list of topics

GaussianPlot

```
In[87]:= GaussianPlot[prismgaussiansystem];
```

```
In[88]:= GaussianPlot[prismgaussiansystem,RenderedParameters->ComplexBeamParameter];
```

Go to list of topics

ShowGaussian

```
In[89]:= ShowSystem[prismgaussiansystem, PlotType->TopView, ShowGaussian->True];
```

```
In[90]:= ShowSystem[prismgaussiansystem, PlotType->SideView, ShowGaussian->True];
```

In[91]:= **ShowSystem[prismgaussiansystem, ShowGaussian->True];**

Go to list of topics

5. Imaging Calculations

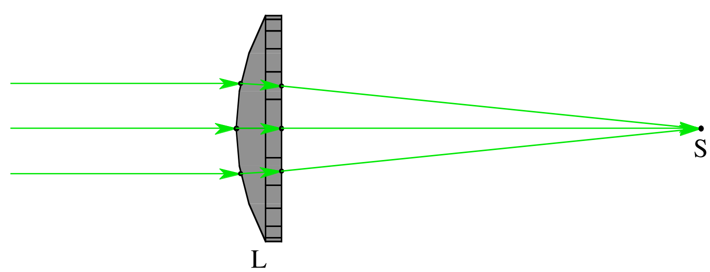
5.1 Overview

In[106]:=
Wavica has several built-in functions userful for characterizing imaging systems. These functions include: **FindLensParameters**, **PupilFunction**, **PointSpreadFunction**, **GeometricPointSpreadFunction**, and **OpticalTransferFunction**. In addition to these names, many of these functions can also take an abbreviated alias that include: **PF**, **PSF**, **GPSF**, **OTF** and **MTF**. The following examples shall serve to introduce the basic use of these highlighted functions. Next, evaluate the examples to see the resulting answers.

In[122]:=
```
optics = {PlanoConvexLens[100,50,10],
    Move[Screen[1],{d+103,103}]};
```

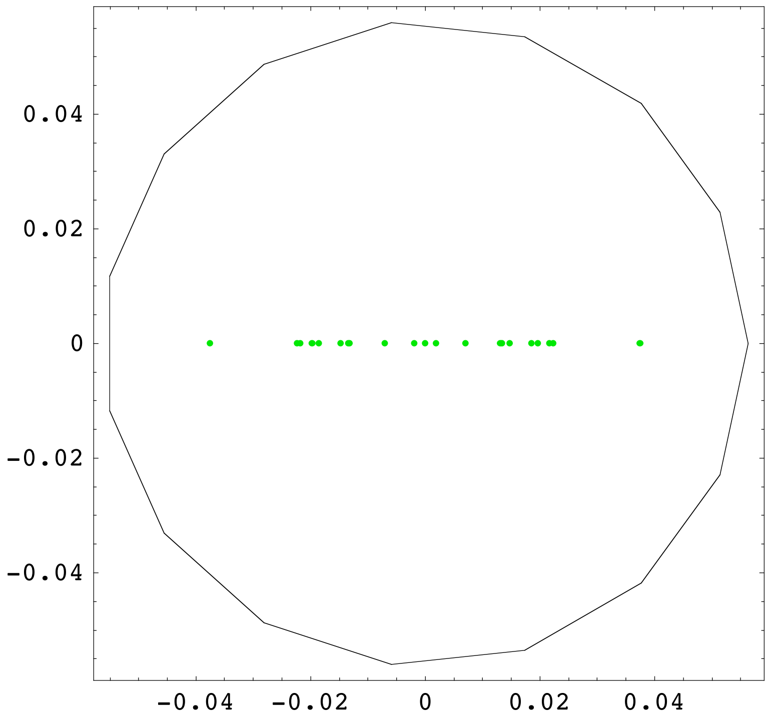
In[123]:=
```
opticalsystem = {Move[LineOfRays[20],{-50,{y,0}}],optics};
```

In[124]:=
```
AnalyzeSystem[opticalsystem,PlotType->TopView];
```



In[125]:=

```
FindFocus[{Move[LineOfRays[20,NumberOfRays->21],-50],optics}]
```



Out[125]=

```
{Screen → Move[Screen[0.112571], 102.638],
 TurboSystem → -traced system-, FocalPoint → {102.638, 0, 0}, FocusType → RMSFocus,
 WeightedSpotSize → 0.01886, SpotSize → 0.01886, BackFocalLength → 92.638,
 FocalPlaneTilt → {1., 0, 0}, TurboRays → -ray intercepts of 1 surfaces-}
```

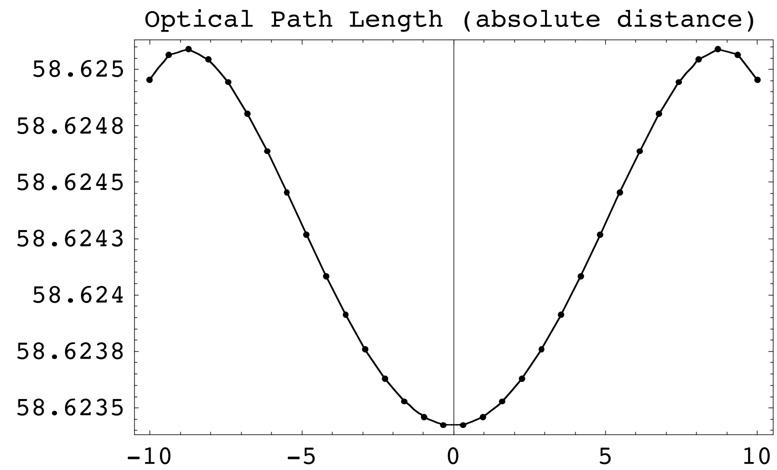
In[126]:=

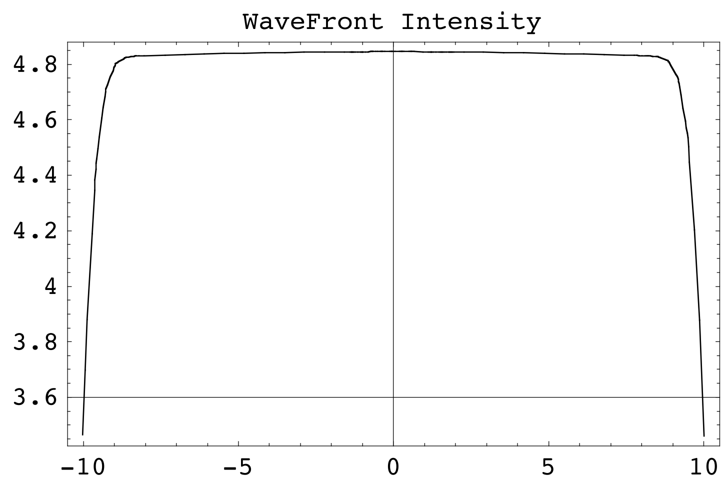
```
lensparameters = FindLensParameters[opticalsystem,FindImagePoint->True]
```

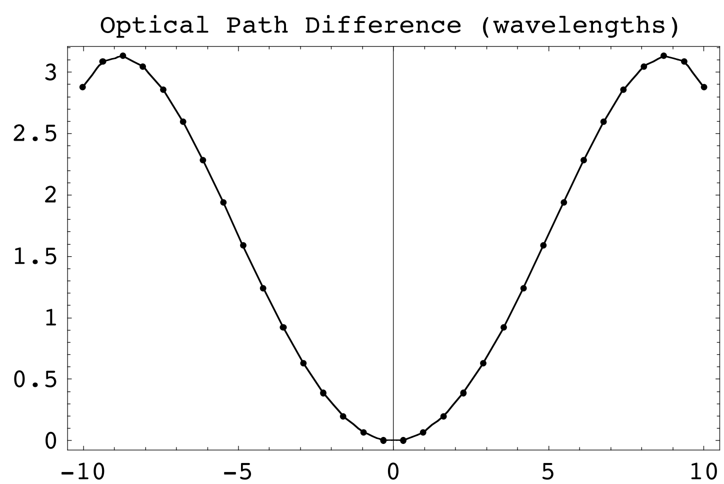
Out[126]=

```
{AngularMagnification → 0.49967, BackFocalLength → 93.351,
 BackFocalPoint → {103.351, 0, 0}, EntrancePupilBoundary → {-50., -50.},
 EntrancePupilDistance → 0, EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {0, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {3.417, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
 FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
 FindImagePoint → True, FocalLength → 99.934, FrontFocalLength → 99.934,
 FrontFocalPoint → {-99.934, 0, 0}, ImagePlaneTilt → {1., 0, 0}, ImagePoint → {102.51, 0, 0},
 ImagingOptics → {PlanoConvexLens[100, 50, 10], Move[Screen[1], {103 + d, 103.}]},
 LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
 ObjectSource → Move[LineOfRays[20], {-50., {y, 0}}], OpticalMedium → Air,
 -Options-, PrinciplePoints → {{2.84217×10^-14, 0, 0}, {3.417, 0, 0}},
 PrinciplePointSeparation → 3.417, SourceID → 17849,
 SystemABCDMatrix → {{0.00351187, 99.7586}, {-0.0100066, 0.49967}},
 TransverseMagnification → 0.00351187, WaveFrontID → 1, WaveLength → 0.532}
```

```
In[127]:=
    pupilfunction = PupilFunction[lensparameters]
```







Out[127]=

```
{AngularMagnification → 0.49967, BackFocalLength → 93.351,
 BackFocalPoint → {103.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
 ComponentNumber → 3, CosineCompensation → True, Energy → 100.,
 EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {0, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {3.417, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
 FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
 FilterTrace → True, FindImagePoint → True, FindPupils → True,
 FocalFraction → Automatic, FocalLength → 99.934, FrontFocalLength → 99.934,
 FrontFocalPoint → {-99.934, 0, 0}, GeometricPointSpreadFunction → False,
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {102.51, 0, 0},
 ImagingOptics → {PlanoConvexLens[100, 50, 10], Move[Screen[1], {103 + d, 103.}]},
 IntensityScale → 1, IntensitySetting → Automatic,
 InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
 LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
 ObjectSource → Move[LineOfRays[20], {-50., {y, 0}}], Offset → 58.6234,
 OpticalLengthFunction → InterpolatingFunction[{{-10.0153, 10.0153}}, <>],
 OpticalMedium → Air,
 OpticalPathDifference → InterpolatingFunction[{{-10.0153, 10.0153}}, <>],
 OpticalPathRange → {58.6234, 58.6251}, -Options-,
 OutputGraphics → {OpticalLengthFunction → (- Graphics -),
   WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
 ParaxialReductionRatio → 0.002, Plot2D → True, PlotPoints → 32,
 PrinciplePoints → {{2.84217×10^-14, 0, 0}, {3.417, 0, 0}},
 PrinciplePointSeparation → 3.417, RayBoundary → {-10.0153, 10.0153},
 RefractiveIndex → 1.00027, RenderedParameters → {OpticalPathDifference},
 ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
 SampleFactor → 2, SeidelAberrations → True, SmoothKernelRange → 3,
 SmoothKernelSize → 1.25, SourceID → 17849, SpatialScale → 1, SurfaceNumber → 1,
 SystemABCDMatrix → {{0.00351187, 99.7586}, {-0.0100066, 0.49967}},
 TransverseMagnification → 0.00351187, WaveFrontID → 1,
 WaveFrontIntensity → InterpolatingFunction[{{-12.4453, 12.3546}}, <>],
 WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

In[128]:=

?PSF

```
PSF[system, options] is an alias to PointSpreadFunction[system, options].
```

In[129]:=

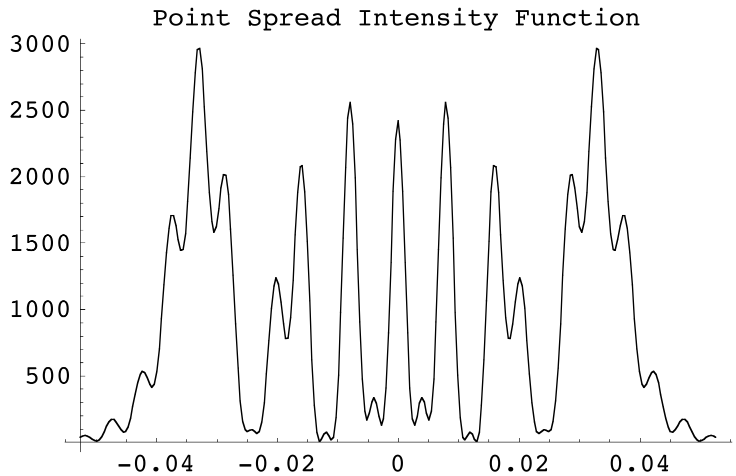
Options[PointSpreadFunction]

Out[129]=

```
{SpatialScale → 1, NumberOfPoints → 128, NumberOfRays → Automatic,
 SignalPlotCutoff → 0.01, PaddingFactor → Automatic, FocalFraction → Automatic,
 FindImagePoint → False, FindPupils → True, FieldStopPosition → Automatic,
 ShowPerfectCase → False, NormalizePlot → False, GeometricPointSpreadFunction → False,
 InterpolationOrder → 1, IntensityTransform → True, PlotPoints → 64,
 FilterTrace → True, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
 RenderedParameters → {PointSpreadFunction, OpticalPathDifference}, Plot2D → True}
```

In[130]:=

```
psf =
PointSpreadFunction[pupilfunction,NumberOfPoints->512,RenderedParameters->PointSpreadFu
nction,PaddingFactor->8]
```



Out[130]=

```
{AngularMagnification → 0.49967, BackFocalLength → 93.351,
 BackFocalPoint → {103.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
 ComponentNumber → 3, CosineCompensation → True, DiffractionSpotSize → 0.105152,
 Energy → 100., EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {0, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {3.417, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
 FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
 FilterTrace → True, FindImagePoint → True, FindPupils → True, FocalFraction → Automatic,
 FocalLength → 99.934, FrequencyCutoff → 378.037, FrontFocalLength → 99.934,
 FrontFocalPoint → {-99.934, 0, 0}, GeometricPointSpreadFunction → False,
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {102.51, 0, 0}, ImageSampleSize → 0.000395309,
 ImagingOptics → {PlanoConvexLens[100, 50, 10], Move[Screen[1], {103 + d, 103.}]},
 IntensityScale → 1, IntensitySetting → Automatic,
 InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
 LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}}, NumberOfPoints → 512,
 ObjectSource → Move[LineOfRays[20], {-50., {y, 0}}], Offset → 58.6234,
 OpticalLengthFunction → InterpolatingFunction[{{-10.0153, 10.0153}}, <>],
 OpticalMedium → Air,
 OpticalPathDifference → InterpolatingFunction[{{-10.0153, 10.0153}}, <>],
 OpticalPathRange → {58.6234, 58.6251}, -Options-, OutputGraphics →
  {OpticalLengthFunction → (- Graphics -), WaveFrontIntensity → (- Graphics -),
   OpticalPathDifference → (- Graphics -), PointSpreadFunction → (- Graphics -)},
 PaddingFactor → 8, ParaxialReductionRatio → 0.002, PerfectPointSpreadFunction →
  InterpolatingFunction[{{-0.100804, 0.101199}}, <>], Plot2D → True, PlotPoints → 32,
 PointSpreadFunction → InterpolatingFunction[{{-0.100804, 0.101199}}, <>],
 PrinciplePoints → {{2.84217 × 10^-14, 0, 0}, {3.417, 0, 0}},
 PrinciplePointSeparation → 3.417, RayBoundary → {-10.0153, 10.0153},
 RefractiveIndex → 1.00027, RenderedParameters → PointSpreadFunction,
 ReportedParameters → {OpticalPathDifference, WaveFrontIntensity}, SampleFactor → 2,
 SeidelAberrations → True, SmoothKernelRange → 3, SmoothKernelSize → 1.25,
 SourceID → 17849, SpatialScale → 1, StrehlRatio → 0.109783, SurfaceNumber → 1,
 SystemABCDMatrix → {{0.00351187, 99.7586}, {-0.0100066, 0.49967}},
 TransverseMagnification → 0.00351187, WaveFrontID → 1,
 WaveFrontIntensity → InterpolatingFunction[{{-12.4453, 12.3546}}, <>],
 WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

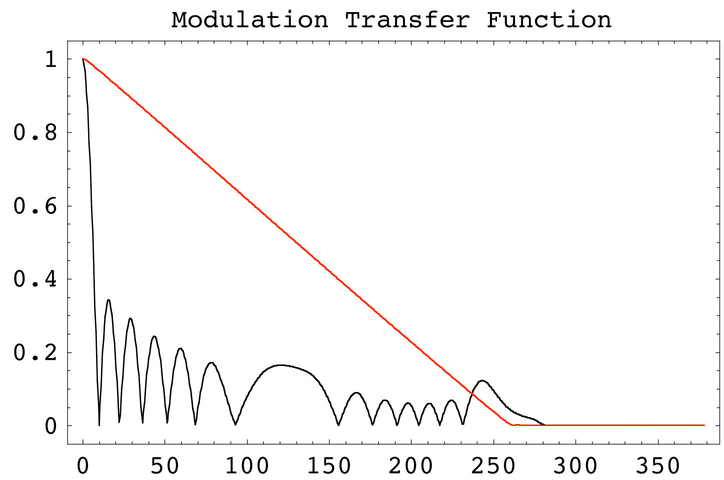
In[131]:=

?MTF

```
MTF[system, options] is an alias to ModulationTransferFunction[system, options].
```

In[132]:=

ModulationTransferFunction[psf]

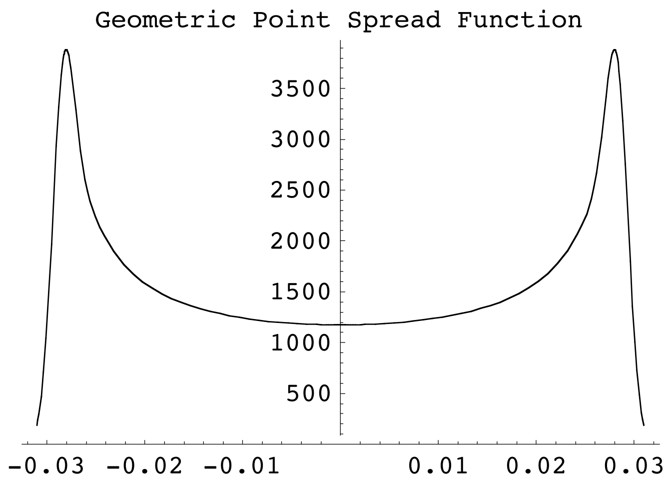

Modulation Transfer Function
1
0.8
0.6
0.4
0.2
0
0 50 100 150 200 250 300 350

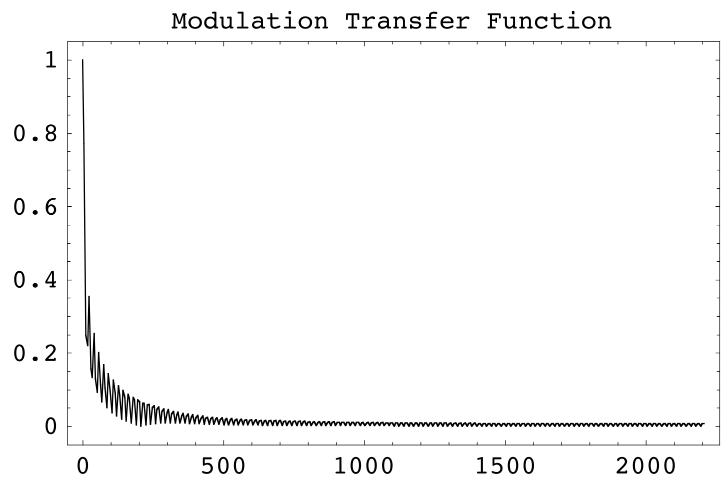
Out[132]=

```
{AngularMagnification → 0.49967, BackFocalLength → 93.351,
 BackFocalPoint → {103.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
 ComponentNumber → 3, CosineCompensation → True, DiffractionSpotSize → 0.105152,
 Energy → 100., EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {0, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {3.417, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
 FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
 FilterTrace → True, FindImagePoint → True, FindPupils → True, FocalFraction → Automatic,
 FocalLength → 99.934, FrequencyCutoff → 378.037, FrontFocalLength → 99.934,
 FrontFocalPoint → {-99.934, 0, 0}, GeometricPointSpreadFunction → False,
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {102.51, 0, 0}, ImageSampleSize → 0.000395309,
 ImagingOptics → {PlanoConvexLens[100, 50, 10], Move[Screen[1], {103 + d, 103.}]},
 IntensityScale → 1, IntensitySetting → Automatic,
 InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
 LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
 ModulationTransferFunction → InterpolatingFunction[{{0., 378.199}}, <>],
 NumberOfPoints → 154, ObjectSource → Move[LineOfRays[20], {-50., {y, 0}}],
 Offset → 58.6234, OpticalLengthFunction →
  InterpolatingFunction[{{-10.0153, 10.0153}}, <>], OpticalMedium → Air,
 OpticalPathDifference → InterpolatingFunction[{{-10.0153, 10.0153}}, <>],
 OpticalPathRange → {58.6234, 58.6251},
 OutputGraphics → {OpticalLengthFunction → (- Graphics -),
   WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -),
   PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
 PaddingFactor → 8, ParaxialReductionRatio → 0.002,
 PerfectModulationTransferFunction → InterpolatingFunction[{{0., 378.199}}, <>],
 PerfectPhaseTransferFunction → InterpolatingFunction[{{0., 378.199}}, <>],
 PerfectPointSpreadFunction → InterpolatingFunction[{{-0.100804, 0.101199}}, <>],
 PhaseTransferFunction → InterpolatingFunction[{{0., 378.199}}, <>],
 Plot2D → True, PlotPoints → 32,
 PointSpreadFunction → InterpolatingFunction[{{-0.100804, 0.101199}}, <>],
 PrinciplePoints → {{2.84217×10^-14, 0, 0}, {3.417, 0, 0}},
 PrinciplePointSeparation → 3.417, RayBoundary → {-10.0153, 10.0153},
 RefractiveIndex → 1.00027, RenderedParameters → PointSpreadFunction,
 ReportedParameters → {OpticalPathDifference, WaveFrontIntensity}, SampleFactor → 2,
 SeidelAberrations → True, SmoothKernelRange → 3, SmoothKernelSize → 1.25,
 SourceID → 17849, SpatialScale → 1, StrehlRatio → 0.109783, SurfaceNumber → 1,
 SystemABCDMatrix → {{0.00351187, 99.7586}, {-0.0100066, 0.49967}},
 TransverseMagnification → 0.00351187, WaveFrontID → 1,
 WaveFrontIntensity → InterpolatingFunction[{{-12.4453, 12.3546}}, <>],
 WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

In[133]:=

```
MTF[opticalsystem,NumberOfRays->256,FindImagePoint->True,GeometricPointSpreadFunction->
True]
```


Geometric Point Spread Function
3500
3000
2500
2000
1500
1000
500
-0.03 -0.02 -0.01
0.01 0.02 0.03


Modulation Transfer Function
1
0.8
0.6
0.4
0.2
0
0 500 1000 1500 2000

```
Out[133]=
    {AngularMagnification → 0.49967, BackFocalLength → 93.351,
     BackFocalPoint → {103.351, 0, 0}, ComponentNumber → 2., Energy → 100.,
     EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
     EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {0, 0, 0},
     EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
     ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
     ExitPupilOffset → {0, 0}, ExitPupilPosition → {3.417, 0, 0},
     ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
     FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
     FindImagePoint → True, FocalLength → 99.934, FrequencyCutoff → 2203.,
     FrontFocalLength → 99.934, FrontFocalPoint → {-99.934, 0, 0}, Full3D → False,
     GeometricPointSpreadFunction → True, ImagePlaneTilt → {1., 0, 0},
     ImagePoint → {102.51, 0, 0}, ImageSampleSize → 0.000151309,
     ImagingOptics → {PlanoConvexLens[100, 50, 10], Move[Screen[1], {103 + d, 103.}]},
     LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
     ModulationTransferFunction → InterpolatingFunction[{{0., 2205.91}}, <>],
     NumberOfPoints → 256, NumberOfRays → 256,
     ObjectSource → Move[LineOfRays[20], {-50., {y, 0}}], OpticalMedium → Air, OutputGraphics →
      {PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
     PerfectModulationTransferFunction → PerfectModulationTransferFunction,
     PerfectPhaseTransferFunction → PerfectPhaseTransferFunction,
     PhaseTransferFunction → InterpolatingFunction[{{0., 2205.91}}, <>],
     PointSpreadFunction → CompiledFunction[-intensity data-],
     PrinciplePoints → {{2.84217×10^-14, 0, 0}, {3.417, 0, 0}},
     PrinciplePointSeparation → 3.417, RayBoundary → {-0.0288244, 0.0288244},
     RayTraceFunction → RayTraceFunction[{y}, -raytrace code: 945016 Bytes- ],
     RotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, SmoothKernelSize → 0.00147213,
     SourceID → 17849, SurfaceNormalFunction → ({{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} &),
     SurfaceNumber → 1., SymbolicValues → {y → 0.},
     SystemABCDMatrix → {{0.00351187, 99.7586}, {-0.0100066, 0.49967}},
     TransverseMagnification → 0.00351187, WaveFrontID → 1., WaveLength → 0.532}
```

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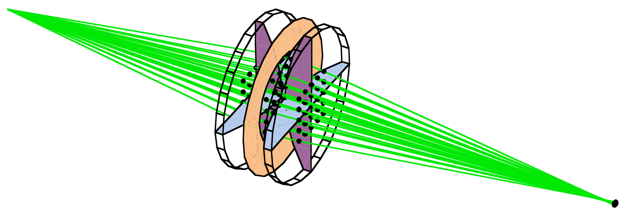
Define Systems

This section serves to define the various optical systems used in later in imaging calculations.

```
In[134]:=
    sys3D =
        {PointOfRays[{10,10}, NumberOfRays -> 5],
        Move[PlanoConvexLens[{f1,100}, 50, 9, CurvatureDirection -> Back],90.5],
        Move[ApertureStop[50,30],100],
        Move[PlanoConvexLens[{f2,100}, 50, 9], 100.5],
        Boundary[208, GraphicDesign -> Off]};
```

In[135]:=

```
AnalyzeSystem[sys3D];
```

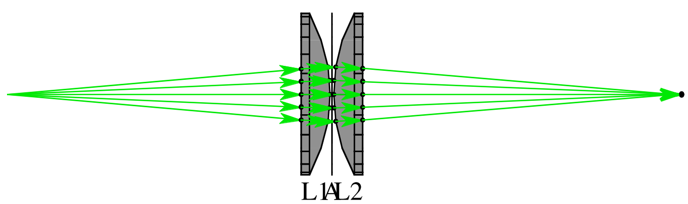


In[136]:=

```
sys =
    {WedgeOfRays[10, NumberOfRays -> 5],
    Move[PlanoConvexLens[{f1,100}, 50, 9, CurvatureDirection -> Back],90.5],
    Move[ApertureStop[50,30],100],
    Move[PlanoConvexLens[{f2,100}, 50, 9], 100.5],
    Boundary[208, GraphicDesign -> Off]};
```

In[137]:=

```
AnalyzeSystem[sys,PlotType->TopView];
```

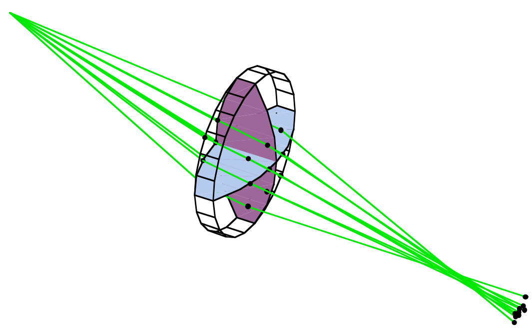


In[138]:=

```
offaxis3D =
    {MoveDirected[PointOfRays[{10,10}],{-90.5,30},{0,0},SideOfObject->After],
    BiConvexLens[50, 50, 20],
    Boundary[108, GraphicDesign -> Off]};
```

In[139]:=

```
AnalyzeSystem[offaxis3D];
```

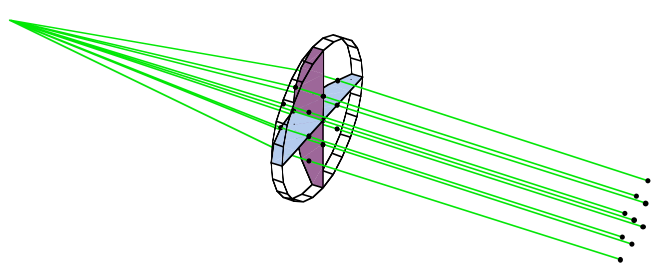


In[140]:=

```
colsys3D =
    {Move[PointOfRays[{10,10}],-100],
    PlanoConvexLens[100, 50, 10],
    Boundary[108, GraphicDesign -> Off]};
```

In[141]:=

```
AnalyzeSystem[colsys3D];
```



In[142]:=

```
colsys32 =
    {Move[PointOfRays[{10,10}, NumberOfRays->32],-100],
    PlanoConvexLens[100, 50, 10],
    Boundary[108, GraphicDesign -> Off]};
```

Go to list of topics

5.2 FindLensParameters

In[143]:=

```
?FindLensParameters
```

```
FindLensParameters[opticalsystem, options] is a function that
  determines the paraxial lens characteristics of an imaging system.

FindLensParameters returns complete information about focal length,
  principle planes, field stop, and pupils. In addition to the necessary
  lens elements, the opticalsystem parameter should include a light
  source. FindLensParameters is also capable of analyzing asigmatic
  imaging systems with the ABCDContruction->FullForm option setting.

See also: FieldStopSurface, FocalLength, BackFocalLength,
  BackFocalPoint, FrontFocalLength, FrontFocalPoint, EntrancePupilDistance,
  EntrancePupilPosition, EntrancePupilBoundary, EntrancePupilOffset,
  EntrancePupilRotationMatrix, ExitPupilDistance, ExitPupilPosition,
  ExitPupilBoundary, ExitPupilOffset, ExitPupilRotationMatrix,
  PrinciplePointSeparation, PrinciplePoints, and ABCDMatrix.
```

In[144]:=

```
Options[FindLensParameters]
```

Out[144]=

```
{SpatialScale → 1, ParentFunction → FindLensParameters,
 RunningCommentary → False, TimeConstraint → 10, ABCDConstruction → Horizontal,
 SkipSimplify → {}, FocalFraction → Automatic, WeightingFunction → 1,
 FindPupils → True, FieldStopPosition → Automatic, FocusType → FocusDiameter,
 FindImagePoint → False, FocalPoint → Automatic, NumberOfFilledRays → 7,
 InfinityFocusAngle → 0., FilterTrace → True, ParaxialReductionRatio → 0.002}
```

In[145]:=

```
FindLensParameters[sys]
```

Out[145]=

```
{AngularMagnification → -0.93012,
 BackFocalLength → 43.791, BackFocalPoint → {153.291, 0, 0},
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FocalLength → 50.2182, FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {208., 0., 0.}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
 ObjectSource → WedgeOfRays[10, NumberOfRays → 5], OpticalMedium → Air,
 -Options-, PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
 PrinciplePointSeparation → 6.14556, SourceID → 19920,
 SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
 TransverseMagnification → -1.08942, WaveFrontID → 1, WaveLength → 0.532}
```

`FindLensParameters` can also be used with astigmatic imaging systems. For example, you can check the focal length along each astigmatic axis.

In[146]:=

```
FindLensParameters[offaxis3D,ABCDConstruction->Full3D,FindPupils->False]
```

Out[146]=

```
{ABCDConstruction → Full3D,
 AngularMagnification → {-1.24643, -1.0916}, BackFocalLength → {39.485, 41.7086},
 BackFocalPoint → {{57.8471, -14.8154, 0}, {59.9887, -15.4135, 0}},
 ExitPupilBoundary → {{-50., -50.}, {-50., -50.}},
 ExitPupilDistance → {0, 0}, ExitPupilOffset → {{0, 0}, {0, 0}},
 ExitPupilPosition → {{19.817, -4.19583, 0}, {19.817, -4.19583, 0}},
 ExitPupilRotationMatrix → {{{0.963153, -0.268952, 0}, {0.268952, 0.963153, 0}, {0, 0, 1}},
   {{0.963153, -0.268952, 0}, {0.268952, 0.963153, 0}, {0, 0, 1}}},
 FieldStopPosition → {2, 2}, FieldStopSurface →
  {{IntersectionNumber → 2, SurfaceNumber → 2, ComponentNumber → 1},
   {IntersectionNumber → 2, SurfaceNumber → 2, ComponentNumber → 1}}, FindPupils → False,
 FocalLength → {45.7974, 48.9854}, ImagePlaneTilt → {0.963153, -0.268952, 0},
 ImagePoint → {108., -28.8202, 0.}, ImagingOptics → {BiConvexLens[50, 50, 20],
   Boundary[{0, -54, -54}, {108, 54, 54}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{{0.862168, 10.2908}, {-0.0218353, 0.899242}},
   {{0.851449, 13.3348}, {-0.0204142, 0.854756}}},
 ObjectSource → Move[PointOfRays[{10, 10}], {-90.5, 30.}, -18.3399],
 OpticalMedium → Air, -Options-, SourceID → {20240, 20240},
 SystemABCDMatrix → {{{-1.137, -19.1063}, {-0.0218353, -1.24643}},
   {{-1.01761, -5.42844}, {-0.0204142, -1.0916}}},
 TransverseMagnification → {-1.137, -1.01761}, WaveFrontID → {1, 1},
 WaveLength → {0.532, 0.532}}
```

Here we can see that it has two focal lengths for each direction.

In[147]:=

```
FocalLength/.%
```

Out[147]=

```
{45.7974, 48.9854}
```

In[148]:=

```
FindLensParameters[sys,FindImagePoint->True]
```

Out[148]=

```
{AngularMagnification → -0.93012,
 BackFocalLength → 43.791, BackFocalPoint → {153.291, 0, 0},
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FindImagePoint → True, FocalLength → 50.2182,
 FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {206.012, 0, 0}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
 ObjectSource → WedgeOfRays[10, NumberOfRays → 5], OpticalMedium → Air,
 -Options-, PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
 PrinciplePointSeparation → 6.14556, SourceID → 19920,
 SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
 TransverseMagnification → -1.08942, WaveFrontID → 1, WaveLength → 0.532}
```

In[149]:=

```
ImagePoint/.%
```

Out[149]=

```
{206.012, 0, 0}
```

In[150]:=

```
FindLensParameters[sys,FindImagePoint->True,FocalFraction->0]
```

Out[150]=

```
{AngularMagnification → -0.93012,
 BackFocalLength → 43.791, BackFocalPoint → {153.291, 0, 0},
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FindImagePoint → True, FocalFraction → 0, FocalLength → 50.2182,
 FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {207.282, 0, 0}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
 ObjectSource → WedgeOfRays[10, NumberOfRays → 5], OpticalMedium → Air, -Options-,
 PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}}, PrinciplePointSeparation → 6.14556,
 SourceID → 19920, SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
 TransverseMagnification → -1.08942, WaveFrontID → 1, WaveLength → 0.532}
```

In[151]:=
```
ImagePoint/.%
```

Out[151]=
```
{207.282, 0, 0}
```

In[152]:=
```
FindLensParameters[sys,FindImagePoint->True,FocalFraction->1]
```

Out[152]=
```
{AngularMagnification → -0.93012,
 BackFocalLength → 43.791, BackFocalPoint → {153.291, 0, 0},
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FindImagePoint → True, FocalFraction → 1, FocalLength → 50.2182,
 FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {205.653, 0, 0}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
 ObjectSource → WedgeOfRays[10, NumberOfRays → 5], OpticalMedium → Air, -Options-,
 PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}}, PrinciplePointSeparation → 6.14556,
 SourceID → 19920, SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
 TransverseMagnification → -1.08942, WaveFrontID → 1, WaveLength → 0.532}
```

In[153]:=
```
ImagePoint/.%
```

Out[153]=
```
{205.653, 0, 0}
```

In[154]:=

```
FindLensParameters[sys,FindImagePoint->True,FocalFraction->.5]
```

Out[154]=

```
{AngularMagnification → -0.93012,
 BackFocalLength → 43.791, BackFocalPoint → {153.291, 0, 0},
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FindImagePoint → True, FocalFraction → 0.5, FocalLength → 50.2182,
 FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {206.468, 0, 0}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
 ObjectSource → WedgeOfRays[10, NumberOfRays → 5], OpticalMedium → Air, -Options-,
 PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}}, PrinciplePointSeparation → 6.14556,
 SourceID → 19920, SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
 TransverseMagnification → -1.08942, WaveFrontID → 1, WaveLength → 0.532}
```

In[155]:=

```
ImagePoint/.%
```

Out[155]=

```
{206.468, 0, 0}
```

Go to list of topics

5.3 PupilFunction

In[156]:=

```
?PupilFunction
```

```
PupilFunction[system, options] is a function designed to
  find and extract the optical path difference and intensity profile
  on a spherical surface at the exit pupil of an imaging system.

PupilFunction first calls FindLensParameters to characterize the system. Then,
  PupilFunction calculates a wavefront function and the intensity mapping
  function at the exit pupil. This information can be used together to
  calculate the complex field at the exit pupil. As input, PupilFunction takes
  either an optical system or the returned output from FindLensParameters.
  The optical system must contain a light source followed by the imaging
  optics with the approximate focal surface as its last element. For
  FindImagePoint->True, the last system element need not be the exact
  focal surface, since the exact focal surface position is automatically
  determined. When the default FindImagePoint->False is given, the last
  system element is assumed to be the exact focal surface. In addition, the
  light source need only contain a small number of rays (since the actual
  number of rays is internally specified). PupilFunction uses information
  from FindLensParameters about the field stop position. When FindPupils->
  False is used, the optical surface preceding the image surface is taken
  as the field stop. The user can also give as input a specific focal point
  or exit pupil position with the FocalPoint and ExitPupilPosition options.

Note: PupilFunction works equally well for both point sources and
  planar sources, as long as the described imaging system contains a
  focus. If a one-dimensional light source is used (ie. WedgeOfRays
  or LineOfRays), then a one-dimensional pupil function is calculated.
  If a two-dimensional light source is used (ie. PointOfRays or
  GridOfRays), then the two-dimensional pupil function is determined.

PupilFunction works together with both PointSpreadFunction
  and OpticalTransferFunction. PupilFunction is also a label used
  with the RenderedParameters option. See also: FindLensParameters,
  PupilIntensityFunction, PointSpreadFunction, PF, and OpticalTransferFunction.
```

In[157]:=

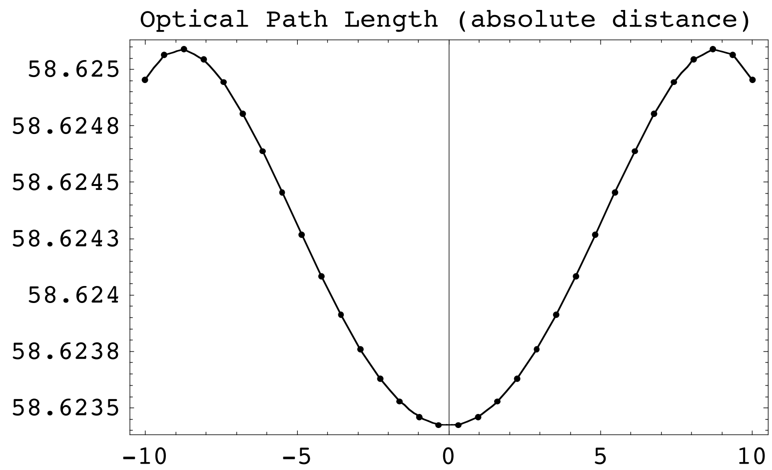
```
Options[PupilFunction]
```

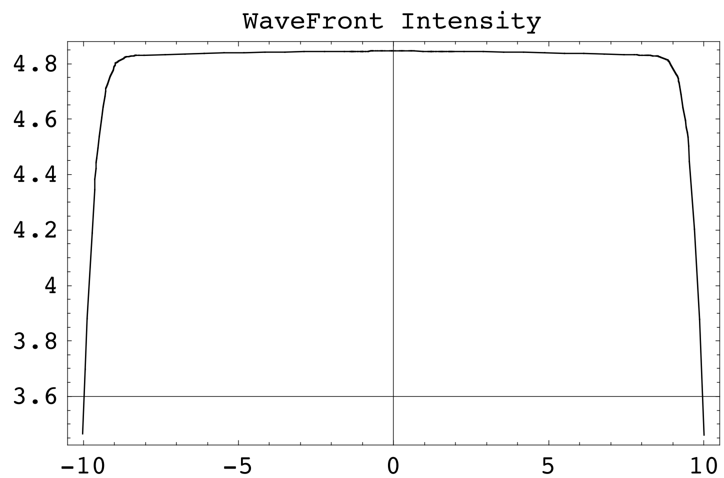
Out[157]=

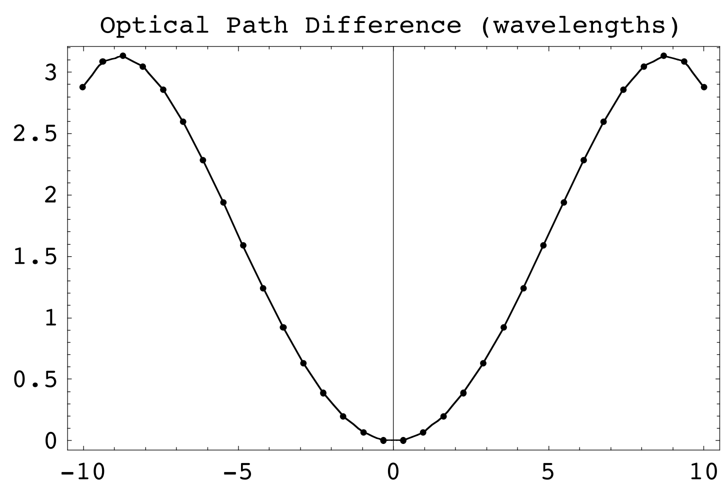
```
{SpatialScale → 1, NumberOfRays → 32, FocalFraction → Automatic,
 FindImagePoint → False, GeometricPointSpreadFunction → False, FindPupils → True,
 ZernikeFit → True, ZernikeOrder → 8, FieldStopPosition → Automatic,
 PlotPoints → 32, InterpolationOrder → 1, FilterTrace → True,
 RenderedParameters → {OpticalPathDifference}, SeidelAberrations → True,
 Plot2D → True, KernelScale → Relative, SmoothKernelSize → 1.25, SmoothKernelRange → 3,
 InterpolatingFunction → True, SampleFactor → 2, IntensitySetting → Automatic,
 Energy → Automatic, IntensityScale → 1, CosineCompensation → True,
 ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &), ParaxialReductionRatio → 0.002}
```

In[158]:=

```
PupilFunction[lensparameters]
```

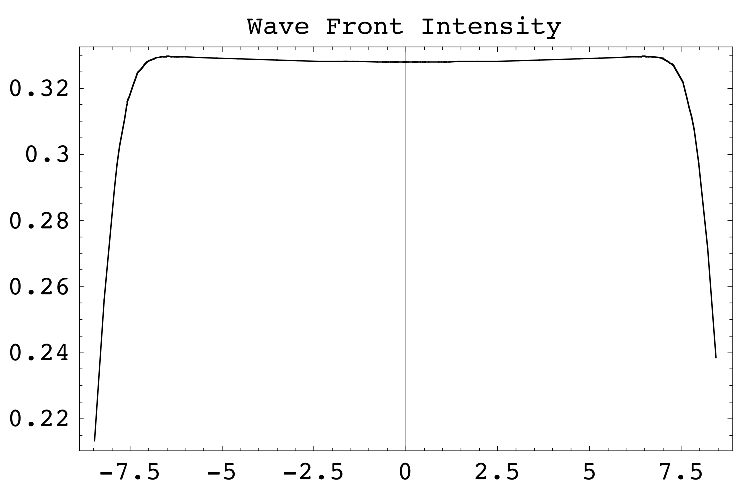

Optical Path Length (absolute distance)
58.625
58.6248
58.6245
58.6243
58.624
58.6238
58.6235
-10
-5
0
5
10

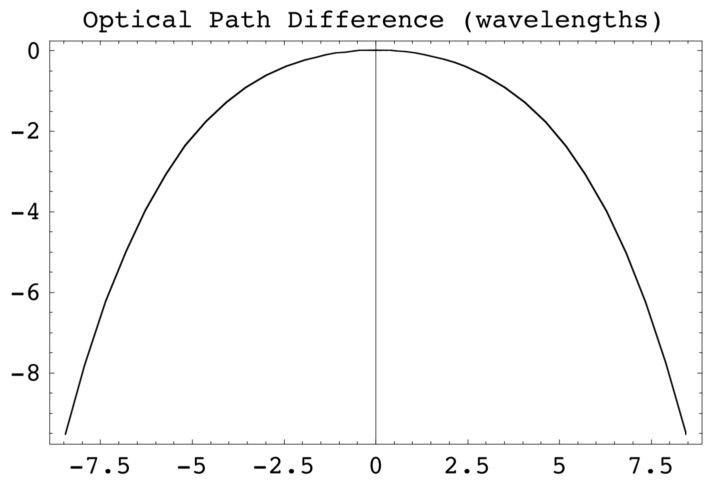

WaveFront Intensity
4.8
4.6
4.4
4.2
4
3.8
3.6
-10
-5
0
5
10


Optical Path Difference (wavelengths)
3
2.5
2
1.5
1
0.5
0
-10
-5
0
5
10

```
Out[158]=
    {AngularMagnification → 0.49967, BackFocalLength → 93.351,
     BackFocalPoint → {103.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
     ComponentNumber → 3, CosineCompensation → True, Energy → 100.,
     EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
     EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {0, 0, 0},
     EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
     ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
     ExitPupilOffset → {0, 0}, ExitPupilPosition → {3.417, 0, 0},
     ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
     FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
     FilterTrace → True, FindImagePoint → True, FindPupils → True,
     FocalFraction → Automatic, FocalLength → 99.934, FrontFocalLength → 99.934,
     FrontFocalPoint → {-99.934, 0, 0}, GeometricPointSpreadFunction → False,
     ImagePlaneTilt → {1., 0, 0}, ImagePoint → {102.51, 0, 0},
     ImagingOptics → {PlanoConvexLens[100, 50, 10], Move[Screen[1], {103 + d, 103.}]},
     IntensityScale → 1, IntensitySetting → Automatic,
     InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
     LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
     ObjectSource → Move[LineOfRays[20], {-50., {y, 0}}], Offset → 58.6234,
     OpticalLengthFunction → InterpolatingFunction[{{-10.0153, 10.0153}}, <>],
     OpticalMedium → Air,
     OpticalPathDifference → InterpolatingFunction[{{-10.0153, 10.0153}}, <>],
     OpticalPathRange → {58.6234, 58.6251}, -Options-,
     OutputGraphics → {OpticalLengthFunction → (- Graphics -),
       WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
     ParaxialReductionRatio → 0.002, Plot2D → True, PlotPoints → 32,
     PrinciplePoints → {{2.84217 × 10^-14, 0, 0}, {3.417, 0, 0}},
     PrinciplePointSeparation → 3.417, RayBoundary → {-10.0153, 10.0153},
     RefractiveIndex → 1.00027, RenderedParameters → {OpticalPathDifference},
     ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
     SampleFactor → 2, SeidelAberrations → True, SmoothKernelRange → 3,
     SmoothKernelSize → 1.25, SourceID → 17849, SpatialScale → 1, SurfaceNumber → 1,
     SystemABCDMatrix → {{0.00351187, 99.7586}, {-0.0100066, 0.49967}},
     TransverseMagnification → 0.00351187, WaveFrontID → 1,
     WaveFrontIntensity → InterpolatingFunction[{{-12.4453, 12.3546}}, <>],
     WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

```
In[159]:=
    PupilFunction[sys3D]
```





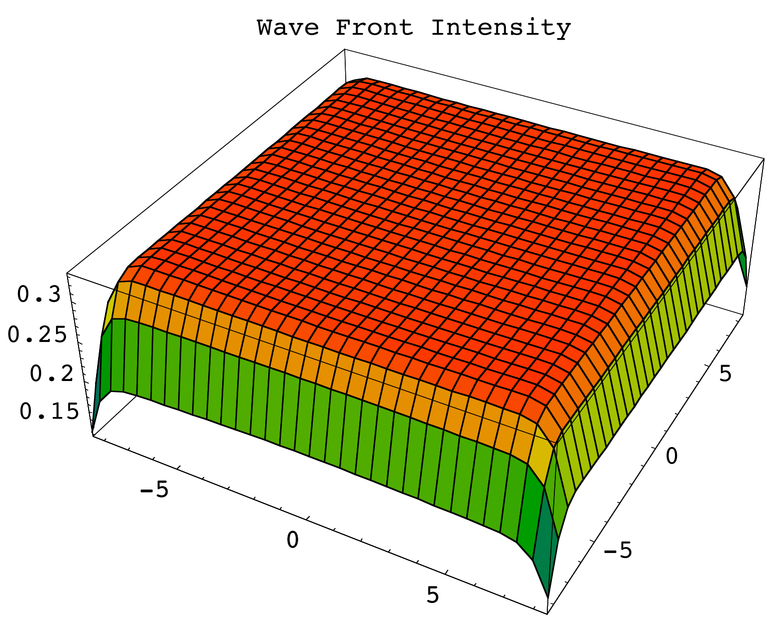
Out[159]=

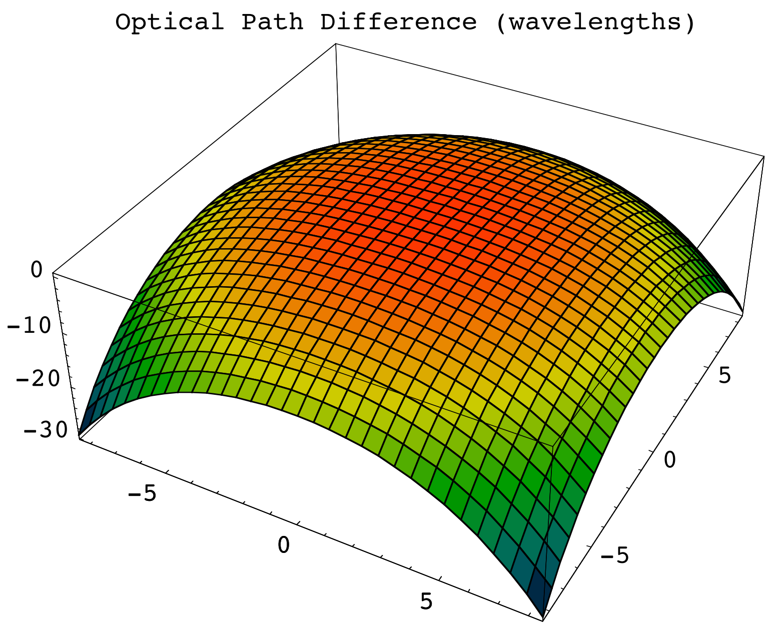
```
{AngularMagnification → -0.93012, BackFocalLength → 43.791,
 BackFocalPoint → {153.291, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
 ComponentNumber → 5, CosineCompensation → True, Energy → 100.,
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FilterTrace → True, FindImagePoint → False, FindPupils → True,
 FocalFraction → Automatic, FocalLength → 50.2182, FrontFocalLength → 43.791,
 FrontFocalPoint → {46.709, 0, 0}, GeometricPointSpreadFunction → False,
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {208., 0., 0.}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]}, IntensityScale → 1,
 IntensitySetting → Automatic, InterceptHole → True, InterpolationOrder → 1,
 KernelScale → Relative, LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
 ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5], Offset → 112.446,
 OpticalLengthFunction → OpticalLengthFunction, OpticalMedium → Air,
 OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0,
    0.00920986, -11.2647 - 3.49288×10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
     1.92707×10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 14.7356 (-1 + 0.0139895 (#1^2 + #2^2)) -
     5.19291×10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209842 (#1^2 + #2^2) + 0.000195705 (#1^2 + #2^2)^2) -
     3.48243 (1 - 0.0419684 (#1^2 + #2^2) + 0.000293557 (#1^2 + #2^2)^2) + 4.22016×10^-6
      Cos[4 ArcTan[#1, #2]] (-0.000244631 (#1^2 + #2^2)^2 + 2.05335×10^-6 (#1^2 + #2^2)^3) -
     3.35835×10^-7 Cos[2 ArcTan[#1, #2]]
      (0.0419684 (#1^2 + #2^2) - 0.000978525 (#1^2 + #2^2)^2 + 5.13338×10^-6 (#1^2 + #2^2)^3) - 0.0208951
      (-1 + 0.0839367 (#1^2 + #2^2) - 0.00146779 (#1^2 + #2^2)^2 + 6.84451×10^-6 (#1^2 + #2^2)^3) -
     7.29982×10^-8 Cos[6 ArcTan[#1, #2]] (-2.39558×10^-6 (#1^2 + #2^2)^3 +
        1.91502×10^-8 (#1^2 + #2^2)^4) + 3.14291×10^-6 Cos[4 ArcTan[#1, #2]]
      (0.000733893 (#1^2 + #2^2)^2 - 0.0000143735 (#1^2 + #2^2)^3 + 6.70257×10^-8 (#1^2 + #2^2)^4) -
     1.27667×10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699473 (#1^2 + #2^2) +
```

```
        0.00293557 (#1^2 + #2^2)^2 - 0.0000359337 (#1^2 + #2^2)^3 + 1.34051×10^-7 (#1^2 + #2^2)^4) -
     0.000204715 (1 - 0.139895 (#1^2 + #2^2) + 0.00440336 (#1^2 + #2^2)^2 -
        0.0000479116 (#1^2 + #2^2)^3 + 1.67564×10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OpticalPathRange → {112.431, 112.446}, -Options-, OutputGraphics →
 {WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
ParaxialReductionRatio → 0.002, Plot2D → True,
PlotPoints → 32,
PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
PrinciplePointSeparation → 6.14556,
RayBoundary → {{-8.45661, 8.45661}, {-8.45284, 8.45284}},
RefractiveIndex → 1.00027,
RenderedParameters → {OpticalPathDifference},
ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
ResidualFitError → 7.23063×10^-7,
SampleFactor → 2,
SeidelAberrations → {SphericalAberration → 83.2032, Astigmatism → 0.0000265544,
  FieldCurvature → 17.3117, Distortion → 5.89832×10^-13, Coma → -8.7573×10^-13},
SmoothKernelRange → 3, SmoothKernelSize → 1.25, SourceID → 19474,
SpatialScale → 1, SurfaceNumber → 1,
SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
TransverseMagnification → -1.08942,
WaveFrontID → 1, WaveFrontIntensity →
 InterpolatingFunction[{{-11.3478, 11.3478}, {-11.344, 11.344}}, <>],
WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

In[161]:=

```
PupilFunction[sys3D,Plot2D->False]
```



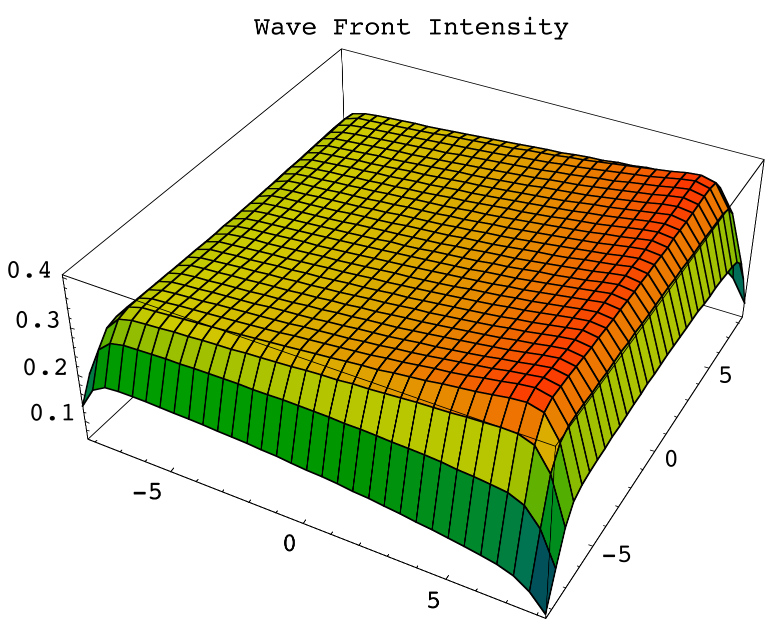


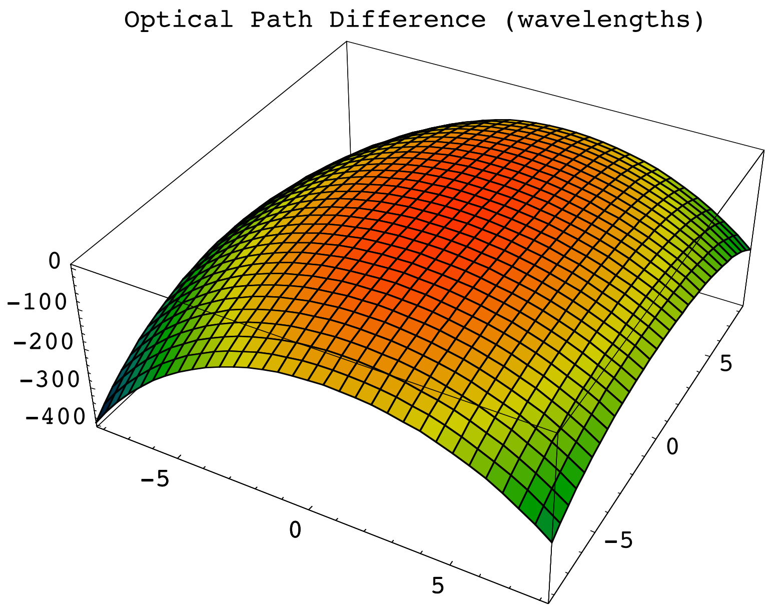
Out[161]=

```
{AngularMagnification → -0.93012, BackFocalLength → 43.791,
 BackFocalPoint → {153.291, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
 ComponentNumber → 5, CosineCompensation → True, Energy → 100.,
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FilterTrace → True, FindImagePoint → False, FindPupils → True,
 FocalFraction → Automatic, FocalLength → 50.2182, FrontFocalLength → 43.791,
 FrontFocalPoint → {46.709, 0, 0}, GeometricPointSpreadFunction → False,
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {208., 0., 0.}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]}, IntensityScale → 1,
 IntensitySetting → Automatic, InterceptHole → True, InterpolationOrder → 1,
 KernelScale → Relative, LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
 ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5], Offset → 112.446,
 OpticalLengthFunction → OpticalLengthFunction, OpticalMedium → Air,
 OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0,
    0.00920986, -11.2647 - 3.49288 × 10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
     1.92707 × 10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 14.7356 (-1 + 0.0139895 (#1^2 + #2^2)) -
     5.19291 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209842 (#1^2 + #2^2) + 0.000195705 (#1^2 + #2^2)^2) -
     3.48243 (1 - 0.0419684 (#1^2 + #2^2) + 0.000293557 (#1^2 + #2^2)^2) + 4.22016 × 10^-6
      Cos[4 ArcTan[#1, #2]] (-0.000244631 (#1^2 + #2^2)^2 + 2.05335 × 10^-6 (#1^2 + #2^2)^3) -
     3.35835 × 10^-7 Cos[2 ArcTan[#1, #2]]
      (0.0419684 (#1^2 + #2^2) - 0.000978525 (#1^2 + #2^2)^2 + 5.13338 × 10^-6 (#1^2 + #2^2)^3) - 0.0208951
      (-1 + 0.0839367 (#1^2 + #2^2) - 0.00146779 (#1^2 + #2^2)^2 + 6.84451 × 10^-6 (#1^2 + #2^2)^3) -
     7.29982 × 10^-8 Cos[6 ArcTan[#1, #2]] (-2.39558 × 10^-6 (#1^2 + #2^2)^3 +
```

```
        1.91502×10^-8 (#1^2 + #2^2)^4) + 3.14291×10^-6 Cos[4 ArcTan[#1, #2]]
      (0.000733893 (#1^2 + #2^2)^2 - 0.0000143735 (#1^2 + #2^2)^3 + 6.70257×10^-8 (#1^2 + #2^2)^4) -
     1.27667×10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699473 (#1^2 + #2^2) +
        0.00293557 (#1^2 + #2^2)^2 - 0.0000359337 (#1^2 + #2^2)^3 + 1.34051×10^-7 (#1^2 + #2^2)^4) -
     0.000204715 (1 - 0.139895 (#1^2 + #2^2) + 0.00440336 (#1^2 + #2^2)^2 -
        0.0000479116 (#1^2 + #2^2)^3 + 1.67564×10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OpticalPathRange → {112.431, 112.446}, -Options-, OutputGraphics →
 {WaveFrontIntensity → (- SurfaceGraphics -),
  OpticalPathDifference → (- SurfaceGraphics -)},
ParaxialReductionRatio → 0.002, Plot2D → False,
PlotPoints → 32,
PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
PrinciplePointSeparation → 6.14556,
RayBoundary → {{-8.45661, 8.45661}, {-8.45284, 8.45284}},
RefractiveIndex → 1.00027,
RenderedParameters → {OpticalPathDifference},
ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
ResidualFitError → 7.23063×10^-7,
SampleFactor → 2,
SeidelAberrations → {SphericalAberration → 83.2032, Astigmatism → 0.0000265544,
  FieldCurvature → 17.3117, Distortion → 5.89832×10^-13, Coma → -8.7573×10^-13},
SmoothKernelRange → 3, SmoothKernelSize → 1.25, SourceID → 19474,
SpatialScale → 1, SurfaceNumber → 1,
SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
TransverseMagnification → -1.08942,
WaveFrontID → 1, WaveFrontIntensity →
 InterpolatingFunction[{{-11.3478, 11.3478}, {-11.344, 11.344}}, <>],
WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

In[18]:= **PupilFunction[offaxis3D,Plot2D->False]**






```
Out[18]= {AngularMagnification → -1.24643,
  BackFocalLength → 39.485, BackFocalPoint → {57.8471, -14.8154, 0},
  ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &), ComponentNumber → 3,
  CosineCompensation → True, Energy → 100., EntrancePupilBoundary → {-57.4255, -57.4255},
  EntrancePupilDistance → -14.8595, EntrancePupilOffset → {0, 0},
  EntrancePupilPosition → {14.5372, -3.07794, 0}, EntrancePupilRotationMatrix →
   {{-0.978312, 0.207137, 0}, {-0.207137, -0.978312, 0}, {0, 0, 1}},
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → 0, ExitPupilOffset → {0, 0},
  ExitPupilPosition → {19.817, -4.19583, 0}, ExitPupilRotationMatrix →
   {{0.963153, -0.268952, 0}, {0.268952, 0.963153, 0}, {0, 0, 1}}, FieldStopPosition → 2,
  FieldStopSurface → {IntersectionNumber → 2, SurfaceNumber → 2, ComponentNumber → 1},
  FilterTrace → True, FindImagePoint → False, FindPupils → True,
  FocalFraction → Automatic, FocalLength → 45.7974,
  FrontFocalLength → 38.2594, FrontFocalPoint → {-36.3161, 12.0385, 0},
  GeometricPointSpreadFunction → False, ImagePlaneTilt → {0.963153, -0.268952, 0},
  ImagePoint → {108., -28.8202, 0.}, ImagingOptics → {BiConvexLens[50, 50, 20],
    Boundary[{0, -54, -54}, {108, 54, 54}, {GraphicDesign → Off}]}, IntensityScale → 1,
  IntensitySetting → Automatic, InterceptHole → True, InterpolationOrder → 1,
  KernelScale → Relative, LensABCDMatrix → {{0.862168, 10.2908}, {-0.0218353, 0.899242}},
  ObjectSource → Move[PointOfRays[{10, 10}], {-90.5, 30.}, -18.3399], Offset → 126.147,
  OpticalLengthFunction → OpticalLengthFunction, OpticalMedium → Air,
  OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0, -0.222399,
```

$$-138.233 + 6.52614\,\mathrm{Cos}[\mathrm{ArcTan}[\#1,\#2]]\sqrt{\#1^2+\#2^2} - 0.824572\,\mathrm{Cos}[2\,\mathrm{ArcTan}[\#1,\#2]]\,(\#1^2+\#2^2) - 0.000622481\,\mathrm{Cos}[3\,\mathrm{ArcTan}[\#1,\#2]]\,(\#1^2+\#2^2)^{3/2} - 2.63431\times10^{-6}\,\mathrm{Cos}[4\,\mathrm{ArcTan}[\#1,\#2]]\,(\#1^2+\#2^2)^2 - 6.37486\times10^{-8}\,\mathrm{Cos}[5\,\mathrm{ArcTan}[\#1,\#2]]\,(\#1^2+\#2^2)^{5/2} +$$

$$4.4463\times10^{-10}\,\mathrm{Cos}[6\,\mathrm{ArcTan}[\#1,\#2]]\,(\#1^2+\#2^2)^3 - 160.664\,(-1+0.0143501\,(\#1^2+\#2^2)) +$$

$$26.7247\,\mathrm{Cos}[\mathrm{ArcTan}[\#1,\#2]]\left(-0.169411\sqrt{\#1^2+\#2^2}+0.0018233\,(\#1^2+\#2^2)^{3/2}\right) -$$

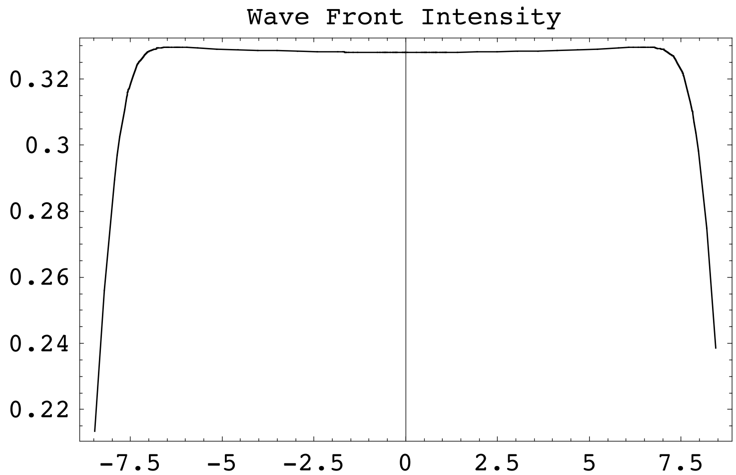
$$3.43272\,\mathrm{Cos}[2\,\mathrm{ArcTan}[\#1,\#2]]\left(-0.0215251\,(\#1^2+\#2^2)+0.000205925\,(\#1^2+\#2^2)^2\right) -$$

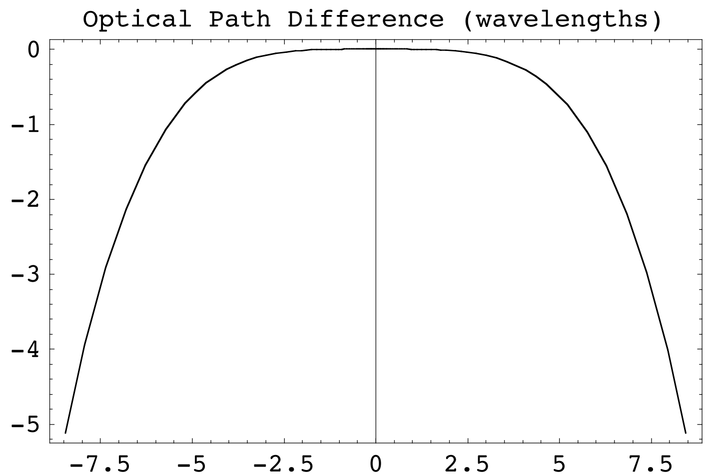
$$23.6014\left(1-0.0430502\,(\#1^2+\#2^2)+0.000308887\,(\#1^2+\#2^2)^2\right) - 0.231619\,\mathrm{Cos}[3\,\mathrm{ArcTan}[\#1,\#2]]\left(-0.00243106\,(\#1^2+\#2^2)^{3/2}+0.0000218037\,(\#1^2+\#2^2)^{5/2}\right) -$$

$$0.711421\,\mathrm{Cos}[\mathrm{ArcTan}[\#1,\#2]]\left(0.254117\sqrt{\#1^2+\#2^2}-0.00729319\,(\#1^2+\#2^2)^{3/2}+\right.$$

```
      0.0000436075 (#1^2 + #2^2)^(5/2)) - 0.00361769 Cos[4 ArcTan[#1, #2]] (-0.000257406
       (#1^2 + #2^2)^2 + 2.21628×10^-6 (#1^2 + #2^2)^3) - 0.184499 Cos[2 ArcTan[#1, #2]]
     (0.0430502 (#1^2 + #2^2) - 0.00102962 (#1^2 + #2^2)^2 + 5.54069×10^-6 (#1^2 + #2^2)^3) -
    1.00162 (-1 + 0.0861005 (#1^2 + #2^2) - 0.00154444 (#1^2 + #2^2)^2 + 7.38759×10^-6 (#1^2 + #2^2)^3) +
    0.00205003 Cos[5 ArcTan[#1, #2]] (-0.0000261645 (#1^2 + #2^2)^(5/2) +
      2.1902×10^-7 (#1^2 + #2^2)^(7/2)) - 0.0149609 Cos[3 ArcTan[#1, #2]]
     (0.00607766 (#1^2 + #2^2)^(3/2) - 0.000130822 (#1^2 + #2^2)^(5/2) + 6.57059×10^-7 (#1^2 + #2^2)^(7/2)) -
    0.113687 Cos[ArcTan[#1, #2]] (-0.338822 Sqrt[#1^2 + #2^2] + 0.018233 (#1^2 + #2^2)^(3/2) -
      0.000261645 (#1^2 + #2^2)^(5/2) + 1.0951×10^-6 (#1^2 + #2^2)^(7/2)) + 0.00077334
     Cos[6 ArcTan[#1, #2]] (-2.58566×10^-6 (#1^2 + #2^2)^3 + 2.12025×10^-8 (#1^2 + #2^2)^4) +
    0.00270727 Cos[4 ArcTan[#1, #2]] (0.000772218 (#1^2 + #2^2)^2 - 0.0000155139 (#1^2 + #2^2)^3 +
      7.42088×10^-8 (#1^2 + #2^2)^4) - 0.0135007 Cos[2 ArcTan[#1, #2]] (-0.0717504 (#1^2 + #2^2) +
      0.00308887 (#1^2 + #2^2)^2 - 0.0000387849 (#1^2 + #2^2)^3 + 1.48418×10^-7 (#1^2 + #2^2)^4) -
    0.0536951 (1 - 0.143501 (#1^2 + #2^2) + 0.00463331 (#1^2 + #2^2)^2 -
      0.0000517131 (#1^2 + #2^2)^3 + 1.85522×10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OpticalPathRange → {125.937, 126.147}, -Options-, OutputGraphics →
 {WaveFrontIntensity → (- SurfaceGraphics -),
  OpticalPathDifference → (- SurfaceGraphics -)},
ParaxialReductionRatio → 0.001,
Plot2D → False,
PlotPoints → 32,
PrinciplePoints →
 {{7.15509, -2.37185, 0}, {13.7372, -2.49811, 0}},
PrinciplePointSeparation → 6.58337,
RayBoundary →
 {{-8.28887, 8.28887}, {-8.40634, 8.40634}},
RefractiveIndex → 1.00027,
RenderedParameters → {OpticalPathDifference},
ReportedParameters →
 {OpticalPathDifference, WaveFrontIntensity},
ResidualFitError → 0.00155077,
SampleFactor → 2,
SeidelAberrations →
 {SphericalAberration → 533.242, Astigmatism → 221.488,
  FieldCurvature → 152.349, Distortion → 22.3014, Coma → 83.3934},
SmoothKernelRange → 3, SmoothKernelSize → 1.25,
SourceID → 1351,
SpatialScale → 1,
SurfaceNumber → 1,
SystemABCDMatrix → {{-1.137, -19.1063}, {-0.0218353, -1.24643}},
TransverseMagnification → -1.137,
WaveFrontID → 1,
WaveFrontIntensity →
 InterpolatingFunction[{{-11.1862, 11.1862}, {-11.3036, 11.3036}}, <>],
WaveLength → 0.532, ZernikeFit → True,
ZernikeOrder → 8}
```

In[19]:= **pupfun = PupilFunction[sys3D, FindImagePoint->True, FocalFraction->0]**

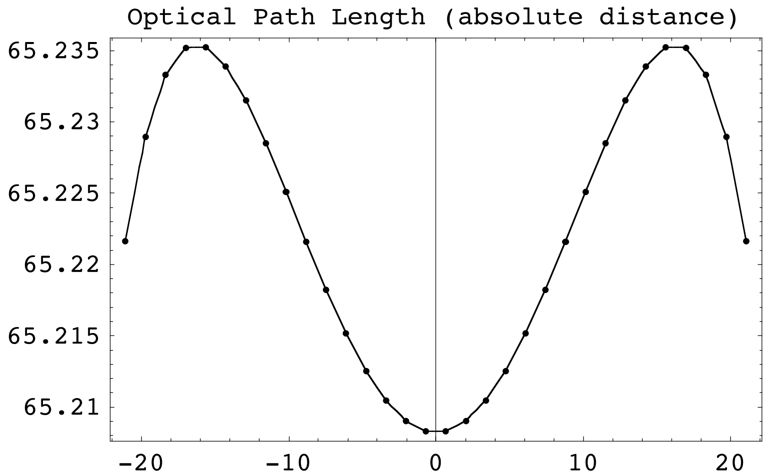


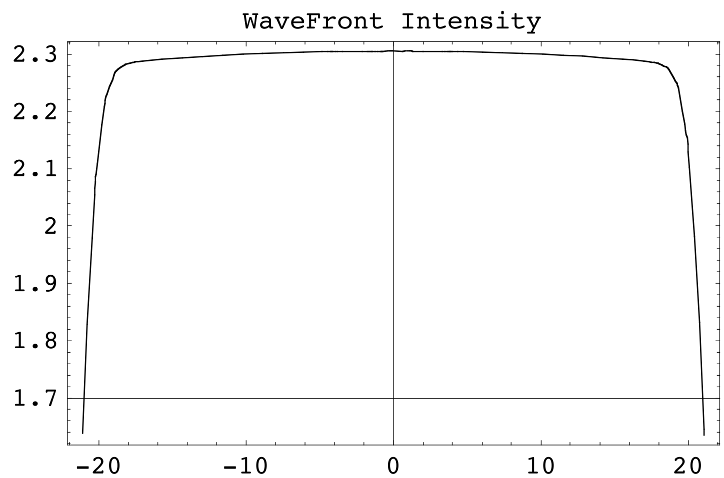


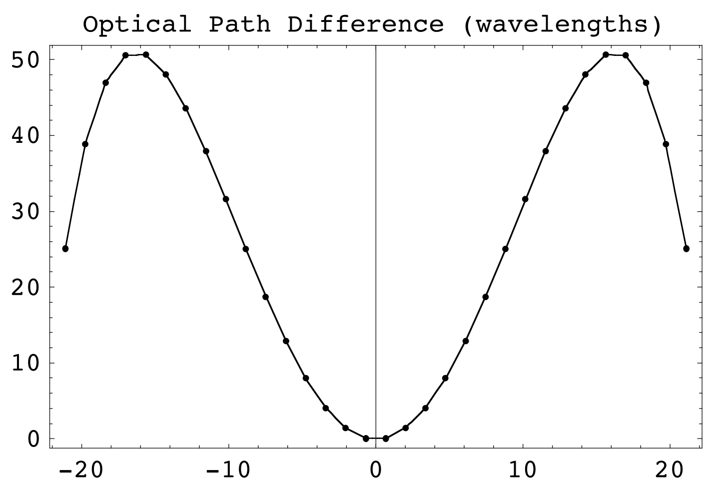

```
Out[19]= {AngularMagnification → -0.93012, BackFocalLength → 43.791,
  BackFocalPoint → {153.291, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 5, CosineCompensation → True, Energy → 100.,
  EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
  FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
  FilterTrace → True, FindImagePoint → True, FindPupils → True,
  FocalFraction → 0, FocalLength → 50.2182, FrontFocalLength → 43.791,
  FrontFocalPoint → {46.709, 0, 0}, GeometricPointSpreadFunction → False,
  ImagePlaneTilt → {1., 0, 0}, ImagePoint → {207.282, 0, 0}, ImagingOptics →
   {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
    Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
    Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]}, IntensityScale → 1,
  IntensitySetting → Automatic, InterceptHole → True, InterpolationOrder → 1,
  KernelScale → Relative, LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
  ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5], Offset → 112.446,
  OpticalLengthFunction → OpticalLengthFunction, OpticalMedium → Air,
  OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0,
     0.0000222711, -6.85363 - 3.53211 × 10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
      1.95153 × 10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 10.3109 (-1 + 0.0139904 (#1^2 + #2^2)) -
```

```
      5.2511 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209856 (#1^2 + #2^2) + 0.000195732 (#1^2 + #2^2)^2) -
      3.47812 (1 - 0.0419713 (#1^2 + #2^2) + 0.000293598 (#1^2 + #2^2)^2) + 4.27139 × 10^-6
       Cos[4 ArcTan[#1, #2]] (-0.000244665 (#1^2 + #2^2)^2 + 2.05378 × 10^-6 (#1^2 + #2^2)^3) -
      3.39626 × 10^-7 Cos[2 ArcTan[#1, #2]]
       (0.0419713 (#1^2 + #2^2) - 0.00097866 (#1^2 + #2^2)^2 + 5.13445 × 10^-6 (#1^2 + #2^2)^3) - 0.0210822
       (-1 + 0.0839425 (#1^2 + #2^2) - 0.00146799 (#1^2 + #2^2)^2 + 6.84593 × 10^-6 (#1^2 + #2^2)^3) -
      7.3801 × 10^-8 Cos[6 ArcTan[#1, #2]] (-2.39608 × 10^-6 (#1^2 + #2^2)^3 +
         1.91555 × 10^-8 (#1^2 + #2^2)^4) + 3.18005 × 10^-6 Cos[4 ArcTan[#1, #2]]
       (0.000733995 (#1^2 + #2^2)^2 - 0.0000143765 (#1^2 + #2^2)^3 + 6.70443 × 10^-8 (#1^2 + #2^2)^4) -
      1.29139 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699521 (#1^2 + #2^2) +
         0.00293598 (#1^2 + #2^2)^2 - 0.0000359412 (#1^2 + #2^2)^3 + 1.34089 × 10^-7 (#1^2 + #2^2)^4) -
      0.000207406 (1 - 0.139904 (#1^2 + #2^2) + 0.00440397 (#1^2 + #2^2)^2 -
         0.0000479215 (#1^2 + #2^2)^3 + 1.67611 × 10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
  OpticalPathRange → {112.435, 112.446}, -Options-, OutputGraphics →
   {WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
  ParaxialReductionRatio → 0.001, Plot2D → True,
  PlotPoints → 32,
  PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
  PrinciplePointSeparation → 6.14556,
  RayBoundary → {{-8.45622, 8.45622}, {-8.45265, 8.45265}},
  RefractiveIndex → 1.00027,
  RenderedParameters → {OpticalPathDifference},
  ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
  ResidualFitError → 7.31386 × 10^-7,
  SampleFactor → 2,
  SeidelAberrations → {SphericalAberration → 82.9255, Astigmatism → 0.0000417324,
    FieldCurvature → -0.246833, Distortion → -3.77796 × 10^-13, Coma → 1.34603 × 10^-12},
  SmoothKernelRange → 3, SmoothKernelSize → 1.25, SourceID → 584,
  SpatialScale → 1, SurfaceNumber → 1,
  SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
  TransverseMagnification → -1.08942,
  WaveFrontID → 1, WaveFrontIntensity →
   InterpolatingFunction[{{-11.3473, 11.3473}, {-11.3437, 11.3437}}, <>],
  WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

In[20]:= **PupilFunction[sys,FindPupils->False]**

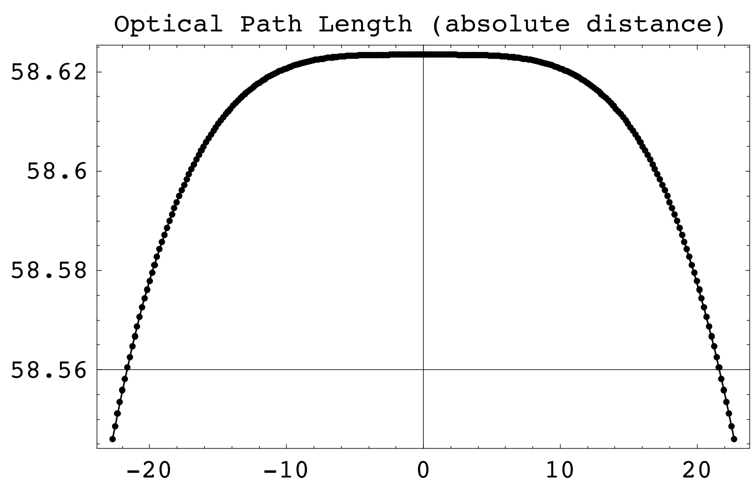

Optical Path Length (absolute distance)
65.235
65.23
65.225
65.22
65.215
65.21
-20
-10
0
10
20

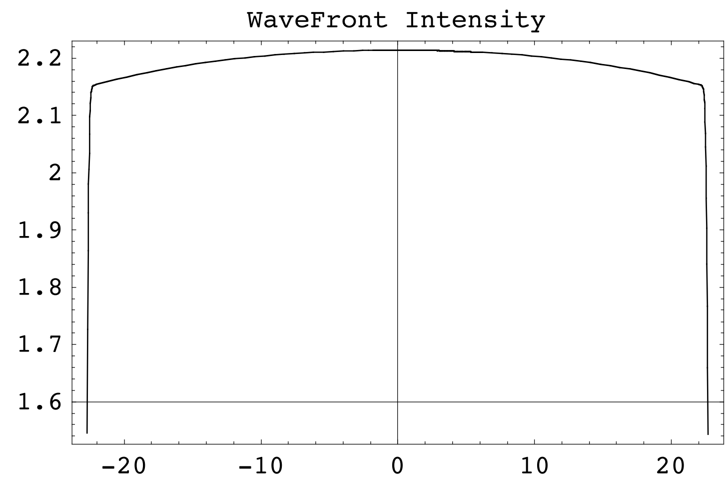

WaveFront Intensity
2.3
2.2
2.1
2
1.9
1.8
1.7
-20
-10
0
10
20

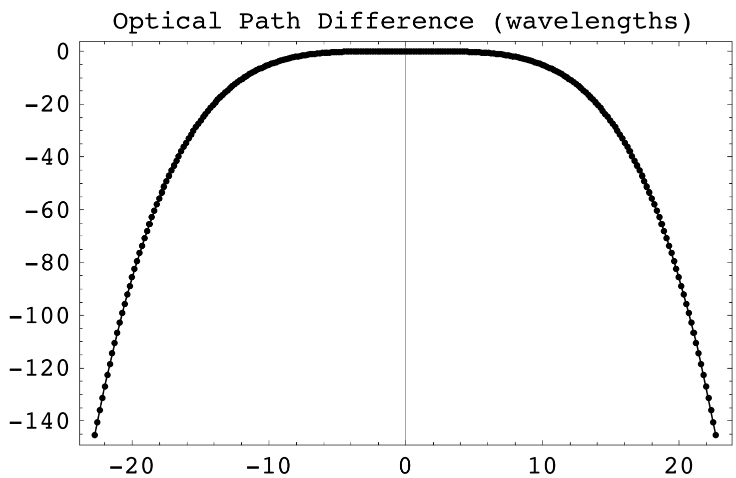

Optical Path Difference (wavelengths)
50
40
30
20
10
0
-20
-10
0
10
20

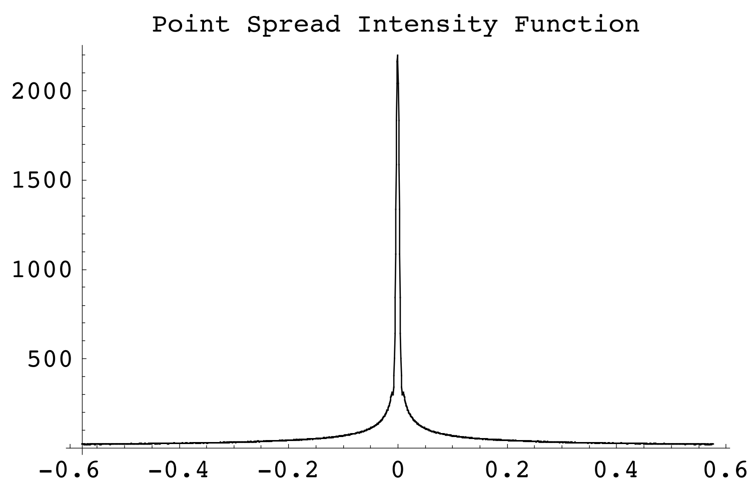
```
Out[20]= {AngularMagnification → 0.49967, BackFocalLength → 93.351,
  BackFocalPoint → {153.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 3, CosineCompensation → True, Energy → 100.,
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → 0,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {60., 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 2,
  FieldStopSurface → {IntersectionNumber → 2, SurfaceNumber → 2, ComponentNumber → 1},
  FilterTrace → True, FindImagePoint → False, FindPupils → False,
  FocalFraction → Automatic, FocalLength → 99.934, GeometricPointSpreadFunction → False,
  ImagePlaneTilt → {1., 0, 0}, ImagePoint → {150., 0., 0.},
  ImagingOptics → {Move[PlanoConvexLens[100, 50, 10], 50.], Move[Screen[50], {x, 150.}]},
  IntensityScale → 1, IntensitySetting → Automatic,
  InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
  ObjectSource → LineOfRays[45, NumberOfRays → 11], Offset → 65.2083,
  OpticalLengthFunction → InterpolatingFunction[{{-21.0975, 21.0975}}, <>],
  OpticalMedium → Air,
  OpticalPathDifference → InterpolatingFunction[{{-21.0975, 21.0975}}, <>],
  OpticalPathRange → {65.2083, 65.2352}, -Options-,
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
  ParaxialReductionRatio → 0.001, Plot2D → True, PlotPoints → 32,
  RayBoundary → {-21.0975, 21.0975}, RefractiveIndex → 1.00027,
  RenderedParameters → {OpticalPathDifference},
  ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
  SampleFactor → 2, SeidelAberrations → True, SmoothKernelRange → 3,
  SmoothKernelSize → 1.25, SourceID → 2190, SpatialScale → 1, SurfaceNumber → 1,
  SystemABCDMatrix → {{0.0335317, 98.2596}, {-0.0100066, 0.49967}},
  TransverseMagnification → 0.0335317, WaveFrontID → 1,
  WaveFrontIntensity → InterpolatingFunction[{{-26.2438, 25.9977}}, <>],
  WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

```
In[26]:= PSF[sys, NumberOfPoints->2048, NumberOfRays->256, FocalFraction->0]
```




WaveFront Intensity
2.2
2.1
2
1.9
1.8
1.7
1.6
-20
-10
0
10
20


Optical Path Difference (wavelengths)
0
-20
-40
-60
-80
-100
-120
-140
-20
-10
0
10
20


Point Spread Intensity Function
2000
1500
1000
500
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

```
Out[26]= {AngularMagnification → 0.49967, BackFocalLength → 93.351,
  BackFocalPoint → {153.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 3, CosineCompensation → True, DiffractionSpotSize → 1.15487,
  Energy → 100., EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {50., 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {53.417, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
  FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
  FilterTrace → True, FindImagePoint → False, FindPupils → True, FocalFraction → 0,
  FocalLength → 99.934, FrequencyCutoff → 832.853, FrontFocalLength → 99.934,
  FrontFocalPoint → {-49.934, 0, 0}, GeometricPointSpreadFunction → False,
  ImagePlaneTilt → {1., 0, 0}, ImagePoint → {153.352, 0, 0}, ImageSampleSize → 0.000643026,
  ImagingOptics → {Move[PlanoConvexLens[100, 50, 10], 50.], Move[Screen[50], {x, 150.}]},
  IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
  InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
  NormalizePlot → False, NumberOfPoints → 2048, NumberOfRays → 256,
  ObjectSource → LineOfRays[45, NumberOfRays → 11], Offset → 58.6234,
  OpticalLengthFunction → InterpolatingFunction[{{-22.7035, 22.7035}}, <>],
  OpticalMedium → Air,
  OpticalPathDifference → InterpolatingFunction[{{-22.7035, 22.7035}}, <>],
  OpticalPathRange → {58.5461, 58.6234}, -Options-, OutputGraphics →
   {OpticalLengthFunction → (- Graphics -), WaveFrontIntensity → (- Graphics -),
    OpticalPathDifference → (- Graphics -), PointSpreadFunction → (- Graphics -)},
  PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
  PerfectPointSpreadFunction → InterpolatingFunction[{{-0.657816, 0.658459}}, <>],
  Plot2D → True, PlotPoints → 64,
  PointSpreadFunction → InterpolatingFunction[{{-0.657816, 0.658459}}, <>],
  PrinciplePoints → {{50., 0, 0}, {53.417, 0, 0}}, PrinciplePointSeparation → 3.417,
  RayBoundary → {-22.7035, 22.7035}, RefractiveIndex → 1.00027,
  RenderedParameters → {PointSpreadFunction, OpticalPathDifference},
  ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
  SampleFactor → 2, SeidelAberrations → True, ShowPerfectCase → False,
  SignalPlotCutoff → 0.01, SmoothKernelRange → 3, SmoothKernelSize → 1.25,
  SourceID → 2190, SpatialScale → 1, StrehlRatio → 0.0284075, SurfaceNumber → 1,
  SystemABCDMatrix → {{0.0335317, 98.2596}, {-0.0100066, 0.49967}},
  TransverseMagnification → 0.0335317, WaveFrontID → 1,
  WaveFrontIntensity → InterpolatingFunction[{{-23.385, 23.355}}, <>],
  WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

Go to list of topics

5.4 Zernike Polynomials and Seidel Aberrations

In[27]:= **?ZernikeFit**

```
ZernikeFit[{{coordinatepoint, opticalpathlength},...}, options] is a
  utility function that attempts to find a Zernike polynomial equation
  fit to the wavefront data as given by a list of two-dimensional
  coordinates and optical path length values (in wavelength units).

ZernikeFit->True is also a option that indicates the use of a
  Zernike polynomial equation fit to a wavefront. ZernikeFit->False
  usually indicates the use of an Interpolation function instead.

 See also: ZernikePolynomial,
  ZernikeOrder, SeidelAberrations,  and ResidualFitError.
```

In[28]:= **?SeidelAberrations**

```
SeidelAberrations is a rule returned by ZernikeFit that
  gives the primary-Seidel aberration terms for the given wavefront.

 These Seidel aberration terms are called SphericalAberration (B),  Astigmatism (
  C), FieldCurvature (D), Distortion (E), and Coma (F), and are given by
  the equation: phasedifference = -1/4 B rho^4 - C yo^2 rho^2 Cos[theta]^
  2 - 1/2 D yo^2 rho^2 + E yo^3 rho Cos[theta] + F yo rho^3 Cos[theta]. See
  also: ZernikeFit, ZernikePolynomial, ResidualFitError, and ZernikeOrder.
```

In[73]:=

ZernikeFit may be called directly. **SeidelAberrations** is one of the results returned by **ZernikeFit**.

```
In[29]:= ZernikeFit[ReadTurboRays[TurboTrace[colsys32,
         ReportedSurfaces->Last],{SurfaceCoordinates,OpticalLength}]]

Out[29]= {ResidualFitError → 2.52814 × 10^-7,
          SeidelAberrations → {SphericalAberration → 4199.57, Astigmatism → -0.367482,
            FieldCurvature → -2066.07, Distortion → -6.29718 × 10^-12, Coma → 1.10389 × 10^-11},
          ZernikePolynomial → CompiledFunction[If[#1 == 0 && #2 == 0, 213.248,
             213.239 - 1.3282 × 10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) + 1.46598 × 10^-10
               Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 0.0131517 (-1 + 0.0138361 (#1^2 + #2^2)) -
              1.9907 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0207542 (#1^2 + #2^2) + 0.000191439 (#1^2 + #2^2)^2) -
              0.00473615 (1 - 0.0415084 (#1^2 + #2^2) + 0.000287158 (#1^2 + #2^2)^2) + 2.55301 × 10^-6
               Cos[4 ArcTan[#1, #2]] (-0.000239298 (#1^2 + #2^2)^2 + 1.98658 × 10^-6 (#1^2 + #2^2)^3) -
              1.27777 × 10^-7 Cos[2 ArcTan[#1, #2]] (0.0415084 (#1^2 + #2^2) -
                 0.000957194 (#1^2 + #2^2)^2 + 4.96645 × 10^-6 (#1^2 + #2^2)^3) - 0.000240399
               (-1 + 0.0830168 (#1^2 + #2^2) - 0.00143579 (#1^2 + #2^2)^2 + 6.62193 × 10^-6 (#1^2 + #2^2)^3) -
              2.10154 × 10^-8 Cos[6 ArcTan[#1, #2]] (-2.31768 × 10^-6 (#1^2 + #2^2)^3 +
                 1.83244 × 10^-8 (#1^2 + #2^2)^4) + 1.44644 × 10^-6 Cos[4 ArcTan[#1, #2]]
               (0.000717895 (#1^2 + #2^2)^2 - 0.0000139061 (#1^2 + #2^2)^3 + 6.41354 × 10^-8 (#1^2 + #2^2)^4) -
              4.75096 × 10^-8 Cos[2 ArcTan[#1, #2]] (-0.0691807 (#1^2 + #2^2) +
                 0.00287158 (#1^2 + #2^2)^2 - 0.0000347652 (#1^2 + #2^2)^3 + 1.28271 × 10^-7 (#1^2 + #2^2)^4) -
              0.0000156084 (1 - 0.138361 (#1^2 + #2^2) + 0.00430737 (#1^2 + #2^2)^2 -
                 0.0000463535 (#1^2 + #2^2)^3 + 1.60339 × 10^-7 (#1^2 + #2^2)^4)], -CompiledCode-]}

In[73]:=
```

When **Compiled->False** is given, the output is left uncompiled.

```
In[30]:= ZernikeFit[ReadTurboRays[TurboTrace[colsys32,
         ReportedSurfaces->Last],{SurfaceCoordinates,OpticalLength}],Compiled->False]
```

```
Out[30]= {ResidualFitError → 2.52814 × 10^-7, SeidelAberrations →
   {SphericalAberration → 4199.57, Astigmatism → -0.367482, FieldCurvature → -2066.07,
    Distortion → -6.29718 × 10^-12, Coma → 1.10389 × 10^-11}, ZernikePolynomial →
   If[#1 == 0 && #2 == 0, 213.248, 213.239 - 1.3282 × 10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
     1.46598 × 10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 0.0131517 (-1 + 0.0138361 (#1^2 + #2^2)) -
     1.9907 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0207542 (#1^2 + #2^2) + 0.000191439 (#1^2 + #2^2)^2) -
     0.00473615 (1 - 0.0415084 (#1^2 + #2^2) + 0.000287158 (#1^2 + #2^2)^2) + 2.55301 × 10^-6
      Cos[4 ArcTan[#1, #2]] (-0.000239298 (#1^2 + #2^2)^2 + 1.98658 × 10^-6 (#1^2 + #2^2)^3) -
     1.27777 × 10^-7 Cos[2 ArcTan[#1, #2]] (0.0415084 (#1^2 + #2^2) -
        0.000957194 (#1^2 + #2^2)^2 + 4.96645 × 10^-6 (#1^2 + #2^2)^3) - 0.000240399
      (-1 + 0.0830168 (#1^2 + #2^2) - 0.00143579 (#1^2 + #2^2)^2 + 6.62193 × 10^-6 (#1^2 + #2^2)^3) -
     2.10154 × 10^-8 Cos[6 ArcTan[#1, #2]] (-2.31768 × 10^-6 (#1^2 + #2^2)^3 +
        1.83244 × 10^-8 (#1^2 + #2^2)^4) + 1.44644 × 10^-6 Cos[4 ArcTan[#1, #2]]
      (0.000717895 (#1^2 + #2^2)^2 - 0.0000139061 (#1^2 + #2^2)^3 + 6.41354 × 10^-8 (#1^2 + #2^2)^4) -
     4.75096 × 10^-8 Cos[2 ArcTan[#1, #2]] (-0.0691807 (#1^2 + #2^2) +
        0.00287158 (#1^2 + #2^2)^2 - 0.0000347652 (#1^2 + #2^2)^3 + 1.28271 × 10^-7 (#1^2 + #2^2)^4) -
     0.0000156084 (1 - 0.138361 (#1^2 + #2^2) + 0.00430737 (#1^2 + #2^2)^2 -
        0.0000463535 (#1^2 + #2^2)^3 + 1.60339 × 10^-7 (#1^2 + #2^2)^4)]}
```

In[73]:=

ZernikeFit is also used internally by **PupilFunction** for three-dimesional sources when the option **ZernikeFit->True** is set.

```
In[31]:= ZernikeFit/.Options[PupilFunction]
```

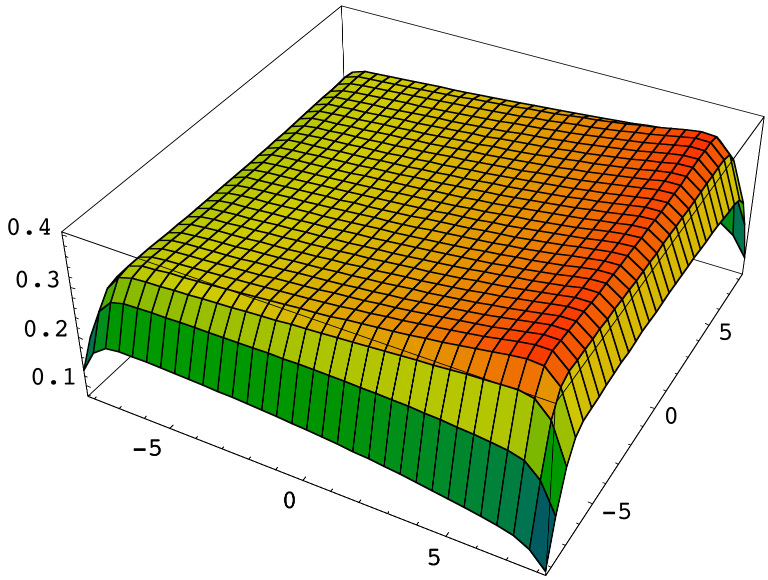
```
Out[31]= True
```

In[73]:=

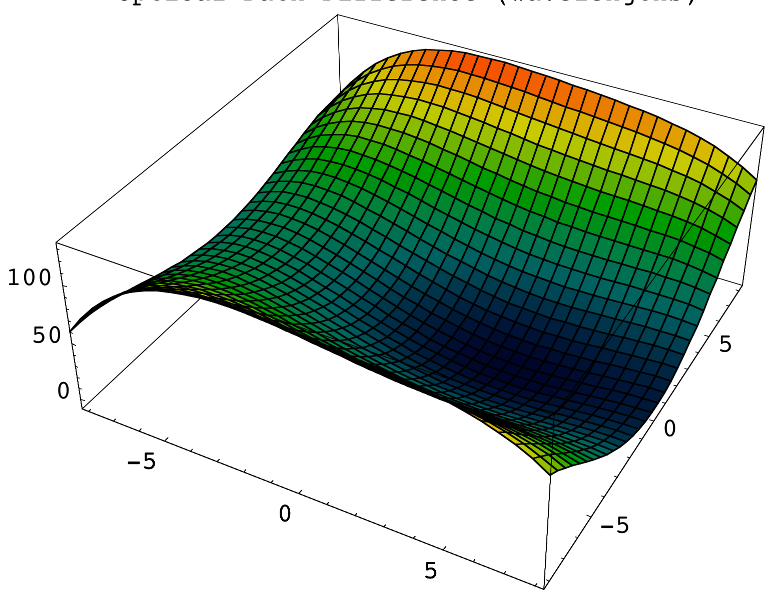
ZernikeFit is also used internally by **PupilFunction** for three-dimensional light sources. **SeidelAberrations** is also returned by **PupilFunction**.

```
In[32]:= {OpticalPathDifference,ResidualFitError,SeidelAberrations}/.
             PupilFunction[offaxis3D,FindImagePoint->True,Plot2D->False,ZernikeFit->True]
```

Wave Front Intensity

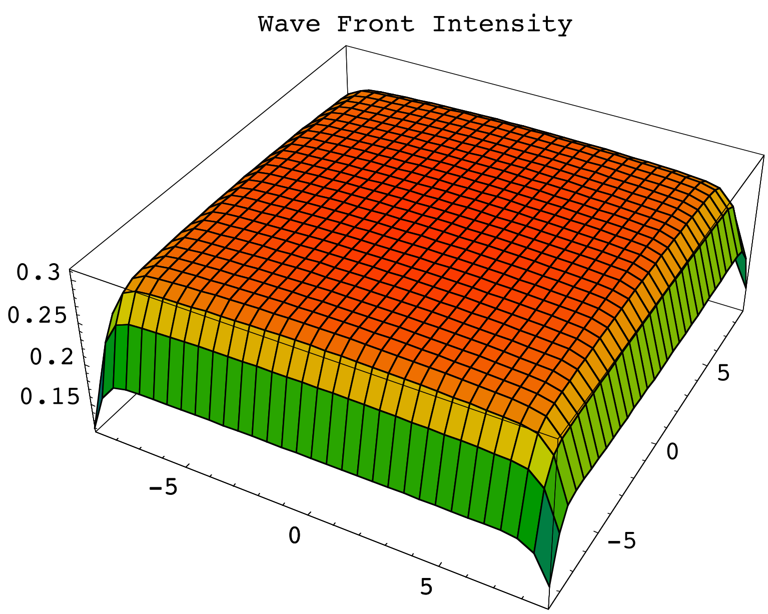


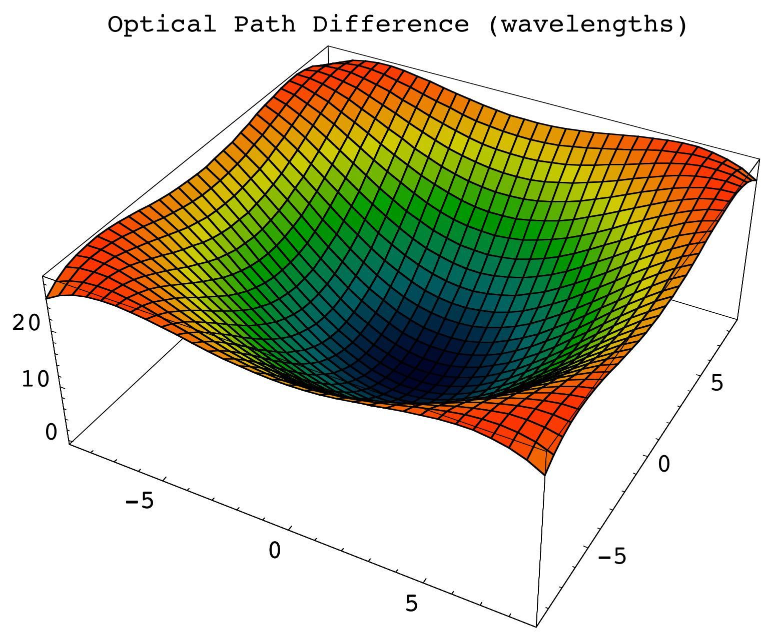
Optical Path Difference (wavelengths)



```
Out[32]= {CompiledFunction[
    If[#1 == 0 && #2 == 0, 3.14985, 61.2636 - 0.599501 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] -
      0.827998 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) - 0.000578093 Cos[3 ArcTan[#1, #2]]
       (#1^2 + #2^2)^(3/2) - 2.78832×10^-6 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 -
      6.16695×10^-8 Cos[5 ArcTan[#1, #2]] (#1^2 + #2^2)^(5/2) +
      5.50593×10^-10 Cos[6 ArcTan[#1, #2]] (#1^2 + #2^2)^3 + 35.7692 (-1 + 0.0144338 (#1^2 + #2^2)) +
      26.6123 Cos[ArcTan[#1, #2]] (-0.169905 Sqrt[#1^2 + #2^2] + 0.00183928 (#1^2 + #2^2)^(3/2)) -
      3.6395 Cos[2 ArcTan[#1, #2]] (-0.0216507 (#1^2 + #2^2) + 0.000208335 (#1^2 + #2^2)^2) -
      23.3707 (1 - 0.0433014 (#1^2 + #2^2) + 0.000312502 (#1^2 + #2^2)^2) -
      0.223812 Cos[3 ArcTan[#1, #2]] (-0.00245237 (#1^2 + #2^2)^(3/2) + 0.0000221232 (#1^2 + #2^2)^(5/2)) -
      0.600685 Cos[ArcTan[#1, #2]]
       (0.254857 Sqrt[#1^2 + #2^2] - 0.00735712 (#1^2 + #2^2)^(3/2) + 0.0000442464 (#1^2 + #2^2)^(5/2)) -
      0.00313181 Cos[4 ArcTan[#1, #2]] (-0.000260419 (#1^2 + #2^2)^2 + 2.2553×10^-6 (#1^2 + #2^2)^3) -
      0.205359 Cos[2 ArcTan[#1, #2]]
       (0.0433014 (#1^2 + #2^2) - 0.00104167 (#1^2 + #2^2)^2 + 5.63825×10^-6 (#1^2 + #2^2)^3) -
      1.08666 (-1 + 0.0866029 (#1^2 + #2^2) - 0.00156251 (#1^2 + #2^2)^2 + 7.51767×10^-6 (#1^2 + #2^2)^3) +
      0.00223709 Cos[5 ArcTan[#1, #2]] (-0.0000265478 (#1^2 + #2^2)^(5/2) +
         2.23525×10^-7 (#1^2 + #2^2)^(7/2)) - 0.015043 Cos[3 ArcTan[#1, #2]]
       (0.00613093 (#1^2 + #2^2)^(3/2) - 0.000132739 (#1^2 + #2^2)^(5/2) + 6.70576×10^-7 (#1^2 + #2^2)^(7/2)) -
      0.10943 Cos[ArcTan[#1, #2]] (-0.339809 Sqrt[#1^2 + #2^2] + 0.0183928 (#1^2 + #2^2)^(3/2) -
         0.000265478 (#1^2 + #2^2)^(5/2) + 1.11763×10^-6 (#1^2 + #2^2)^(7/2)) + 0.000850661
       Cos[6 ArcTan[#1, #2]] (-2.63118×10^-6 (#1^2 + #2^2)^3 + 2.17017×10^-8 (#1^2 + #2^2)^4) +
      0.00324143 Cos[4 ArcTan[#1, #2]]
       (0.000781256 (#1^2 + #2^2)^2 - 0.0000157871 (#1^2 + #2^2)^3 + 7.5956×10^-8 (#1^2 + #2^2)^4) -
      0.0148225 Cos[2 ArcTan[#1, #2]] (-0.0721691 (#1^2 + #2^2) + 0.00312502 (#1^2 + #2^2)^2 -
         0.0000394678 (#1^2 + #2^2)^3 + 1.51912×10^-7 (#1^2 + #2^2)^4) -
      0.0604679 (1 - 0.144338 (#1^2 + #2^2) + 0.00468754 (#1^2 + #2^2)^2 -
         0.0000526237 (#1^2 + #2^2)^3 + 1.8989×10^-7 (#1^2 + #2^2)^4)],
    -CompiledCode-], 0.00161023, {SphericalAberration → 591.972,
    Astigmatism → 220.653,
    FieldCurvature → -661.599,
    Distortion → -61.0418,
    Coma → 81.7115}}
```

```
In[33]:= {OpticalPathDifference,ResidualFitError,SeidelAberrations}/.
           PupilFunction[colsys3D,FindImagePoint->True,Plot2D->False,ZernikeFit->True]
```


Wave Front Intensity
0.3
0.25
0.2
0.15
-5
0
5
5
0
-5


Optical Path Difference (wavelengths)
20
10
0
-5
0
5
5
0
-5

```
Out[33]= {CompiledFunction[
  If[#1 == 0 && #2 == 0, -0.0762772, 21.1301 + 5.69705 × 10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) -
    1.75741 × 10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 + 12.7585 (-1 + 0.0126531 (#1^2 + #2^2)) +
    9.3165 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0189797 (#1^2 + #2^2) + 0.000160102 (#1^2 + #2^2)^2) -
    8.46092 (1 - 0.0379594 (#1^2 + #2^2) + 0.000240153 (#1^2 + #2^2)^2) - 5.85561 × 10^-6
     Cos[4 ArcTan[#1, #2]] (-0.000200128 (#1^2 + #2^2)^2 + 1.51935 × 10^-6 (#1^2 + #2^2)^3) +
    5.9751 × 10^-7 Cos[2 ArcTan[#1, #2]]
     (0.0379594 (#1^2 + #2^2) - 0.00080051 (#1^2 + #2^2)^2 + 3.79836 × 10^-6 (#1^2 + #2^2)^3) - 0.0125625
     (-1 + 0.0759189 (#1^2 + #2^2) - 0.00120077 (#1^2 + #2^2)^2 + 5.06449 × 10^-6 (#1^2 + #2^2)^3) +
    1.39899 × 10^-7 Cos[6 ArcTan[#1, #2]] (-1.77257 × 10^-6 (#1^2 + #2^2)^3 +
       1.28163 × 10^-8 (#1^2 + #2^2)^4) - 4.94921 × 10^-6 Cos[4 ArcTan[#1, #2]]
     (0.000600383 (#1^2 + #2^2)^2 - 0.0000106354 (#1^2 + #2^2)^3 + 4.48572 × 10^-8 (#1^2 + #2^2)^4) +
    2.24126 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0632657 (#1^2 + #2^2) +
       0.00240153 (#1^2 + #2^2)^2 - 0.0000265886 (#1^2 + #2^2)^3 + 8.97143 × 10^-8 (#1^2 + #2^2)^4) +
    0.000456829 (1 - 0.126531 (#1^2 + #2^2) + 0.0036023 (#1^2 + #2^2)^2 -
       0.0000354514 (#1^2 + #2^2)^3 + 1.12143 × 10^-7 (#1^2 + #2^2)^4)],
  -CompiledCode-], 1.22831 × 10^-6, {SphericalAberration → 201.316,
  Astigmatism → 0.000145395,
  FieldCurvature → -151.692,
  Distortion → -1.65204 × 10^-14,
  Coma → 5.31271 × 10^-14}}
```

Go to list of topics

5.5 PointSpreadFunction

In[34]:= **?PointSpreadFunction**

```
PointSpreadFunction[system, options] calculates the point
  spread function of the optical system at a particular wavelength.

PointSpreadFunction first calls PupilFunction to calculate the exit pupil
  function. Finally, PointSpreadFunction calculates a point spread function
  by using the Fourier transform of the calculated complex field. As input,
  PointSpreadFunction takes either an optical system or the returned output
  from PupilFunction. The optical system must contain a light source
  followed by the imaging optics with the approximate focal surface as its
  last element. For FindImagePoint->True, the last system element need
  not be the exact focal surface, since the exact focal surface position
  is automatically determined. When the default FindImagePoint->False is
  given, the last system element is assumed to be the exact focal surface.
  In addition, the light source need only contain a small number of rays (
  since the actual number of rays is internally specified). When FindPupils->
  False is used, the optical surface preceding the image surface is taken
  as the field stop. The user can also give as input a specific focal point
  or exit pupil position with the FocalPoint and ExitPupilPosition options.

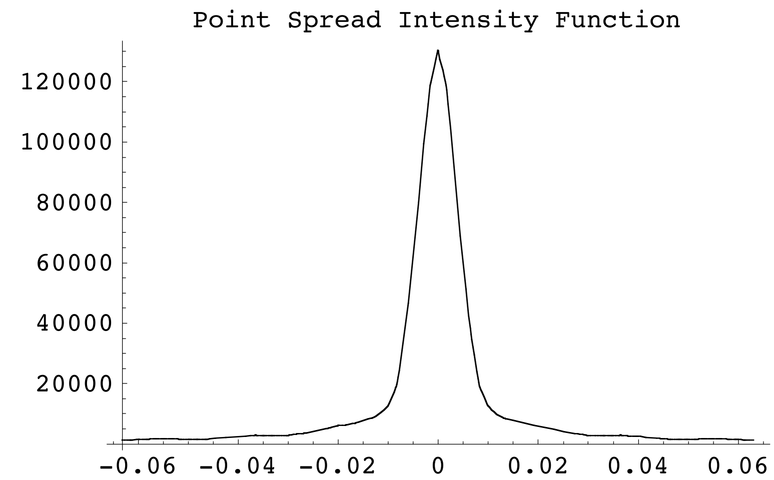
Note: PointSpreadFunction works equally well for both point sources and planar
  sources, as long as the described imaging system contains a focus. If a one-
  dimensional light source is used (ie. WedgeOfRays or LineOfRays), then a
  one-dimensional point spread function is calculated. If a two-dimensional
  light source is used (ie. PointOfRays or GridOfRays), then the point spread
  calculations are carried out in two-dimensions. Warning: in general, one-
  dimensional and two-dimensional results are not fully equivalent to each other.

PointSpreadFunction is also a label used with the RenderedParameters
  option. PointSpreadFunction is also the name of a returned rule
  from PointSpreadFunction that gives the point spread function
  result for a specific image point. See also: IntensityTransform,
  FindLensParameters, PupilFunction, PSF, and OpticalTransferFunction.
```

In[35]:= **Options[PointSpreadFunction]**

Out[35]=
```
{SpatialScale → 1, NumberOfPoints → 128, NumberOfRays → Automatic,
 SignalPlotCutoff → 0.01, PaddingFactor → Automatic, FocalFraction → Automatic,
 FindImagePoint → False, FindPupils → True, FieldStopPosition → Automatic,
 ShowPerfectCase → False, NormalizePlot → False, GeometricPointSpreadFunction → False,
 InterpolationOrder → 1, IntensityTransform → True, PlotPoints → 64,
 FilterTrace → True, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
 RenderedParameters → {PointSpreadFunction, OpticalPathDifference}, Plot2D → True}
```

In[36]:= **PSF[pupfun,NumberOfPoints->128]**




```
Out[36]= {AngularMagnification → -0.93012, BackFocalLength → 43.791,
  BackFocalPoint → {153.291, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 5, CosineCompensation → True,
  DiffractionSpotSize → {0.126195, 0.126247}, Energy → 100.,
  EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
  FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
  FilterTrace → True, FindImagePoint → True, FindPupils → True,
  FocalFraction → 0, FocalLength → 50.2182, FrequencyCutoff → {304.062, 303.934},
  FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
  GeometricPointSpreadFunction → False, ImagePlaneTilt → {1., 0, 0},
  ImagePoint → {207.282, 0, 0}, ImageSampleSize → {0.00166046, 0.00166114}, ImagingOptics →
   {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
    Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
    Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]}, IntensityScale → 1,
  IntensitySetting → Automatic, InterceptHole → True, InterpolationOrder → 1,
  KernelScale → Relative, LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
  NumberOfPoints → 128, ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5],
  Offset → 112.446, OpticalLengthFunction → OpticalLengthFunction,
  OpticalMedium → Air, OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0,
     0.0000222711, -6.85363 - 3.53211 × 10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
      1.95153 × 10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 10.3109 (-1 + 0.0139904 (#1^2 + #2^2)) -
      5.2511 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209856 (#1^2 + #2^2) + 0.000195732 (#1^2 + #2^2)^2) -
      3.47812 (1 - 0.0419713 (#1^2 + #2^2) + 0.000293598 (#1^2 + #2^2)^2) + 4.27139 × 10^-6
       Cos[4 ArcTan[#1, #2]] (-0.000244665 (#1^2 + #2^2)^2 + 2.05378 × 10^-6 (#1^2 + #2^2)^3) -
      3.39626 × 10^-7 Cos[2 ArcTan[#1, #2]]
       (0.0419713 (#1^2 + #2^2) - 0.00097866 (#1^2 + #2^2)^2 + 5.13445 × 10^-6 (#1^2 + #2^2)^3) - 0.0210822
       (-1 + 0.0839425 (#1^2 + #2^2) - 0.00146799 (#1^2 + #2^2)^2 + 6.84593 × 10^-6 (#1^2 + #2^2)^3) -
      7.3801 × 10^-8 Cos[6 ArcTan[#1, #2]] (-2.39608 × 10^-6 (#1^2 + #2^2)^3 +
         1.91555 × 10^-8 (#1^2 + #2^2)^4) + 3.18005 × 10^-6 Cos[4 ArcTan[#1, #2]]
       (0.000733995 (#1^2 + #2^2)^2 - 0.0000143765 (#1^2 + #2^2)^3 + 6.70443 × 10^-8 (#1^2 + #2^2)^4) -
      1.29139 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699521 (#1^2 + #2^2) +
```

```
        0.00293598 (#1^2 + #2^2)^2 - 0.0000359412 (#1^2 + #2^2)^3 + 1.34089×10^-7 (#1^2 + #2^2)^4) -
      0.000207406 (1 - 0.139904 (#1^2 + #2^2) + 0.00440397 (#1^2 + #2^2)^2 -
        0.0000479215 (#1^2 + #2^2)^3 + 1.67611×10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
 OpticalPathRange → {112.435, 112.446}, -Options-, OutputGraphics →
  {WaveFrontIntensity → (- Graphics -),
   OpticalPathDifference → (- Graphics -), PointSpreadFunction → (- Graphics -)},
 ParaxialReductionRatio → 0.001, PerfectPointSpreadFunction →
  InterpolatingFunction[{{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>],
 Plot2D → True, PlotPoints → 32, PointSpreadFunction →
  InterpolatingFunction[{{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>],
 PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
 PrinciplePointSeparation → 6.14556,
 RayBoundary → {{-8.45622, 8.45622}, {-8.45265, 8.45265}},
 RefractiveIndex → 1.00027,
 RenderedParameters → {OpticalPathDifference},
 ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
 ResidualFitError → 7.31386×10^-7,
 SampleFactor → 2,
 SeidelAberrations → {SphericalAberration → 82.9255, Astigmatism → 0.0000417324,
   FieldCurvature → -0.246833, Distortion → -3.77796×10^-13, Coma → 1.34603×10^-12},
 SmoothKernelRange → 3, SmoothKernelSize → 1.25, SourceID → 584,
 SpatialScale → 1, StrehlRatio → 0.0156867,
 SurfaceNumber → 1,
 SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
 TransverseMagnification → -1.08942,
 WaveFrontID → 1, WaveFrontIntensity →
  InterpolatingFunction[{{-11.3473, 11.3473}, {-11.3437, 11.3437}}, <>],
 WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

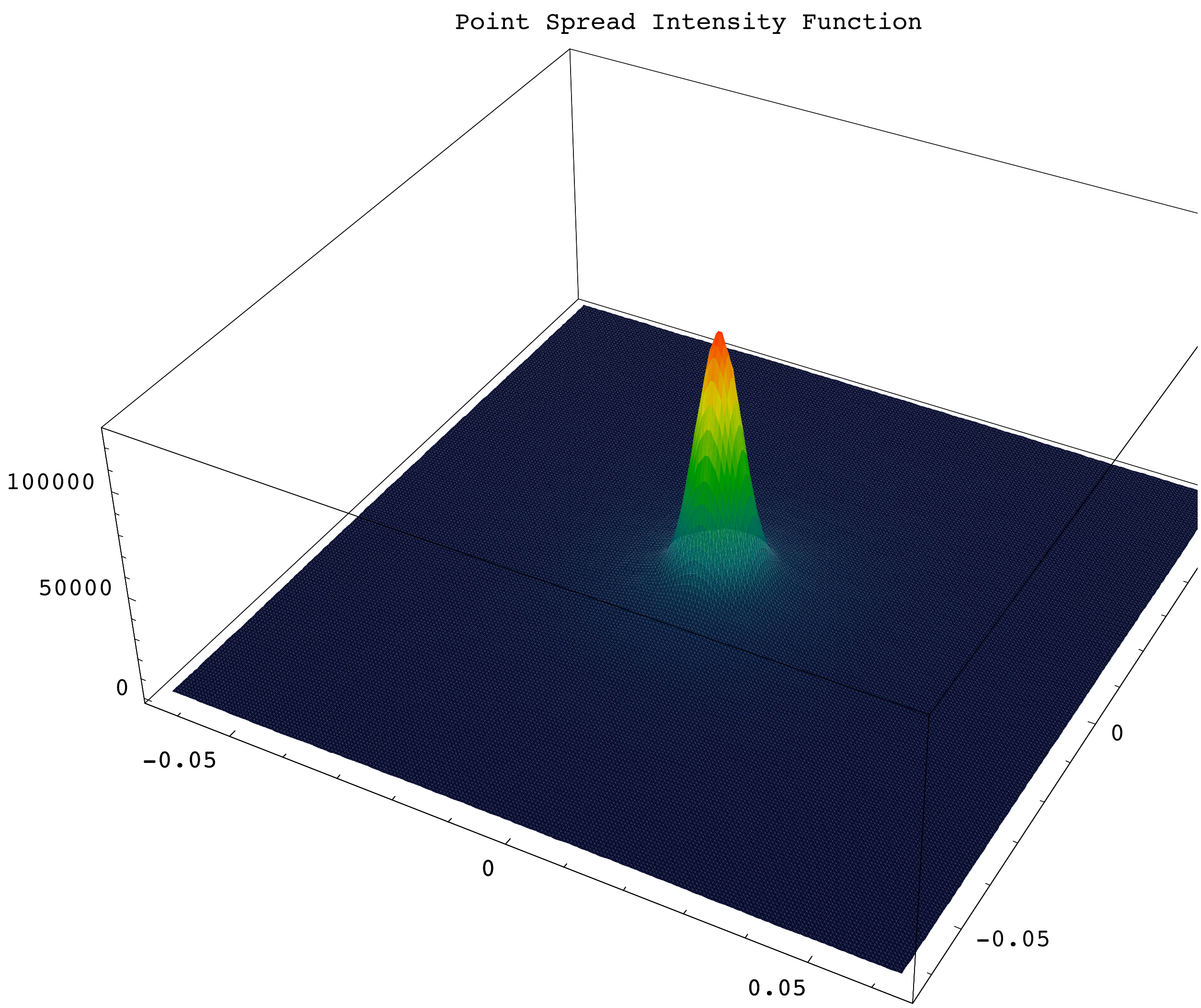
In[38]:= **Options[Graphics3D]**

```
Out[38]= {AmbientLight → GrayLevel[0], AspectRatio → Automatic, Axes → False,
 AxesEdge → Automatic, AxesLabel → None, AxesStyle → Automatic,
 Background → Automatic, Boxed → True, BoxRatios → Automatic,
 BoxStyle → Automatic, ColorOutput → Automatic, DefaultColor → Automatic,
 Epilog → {}, FaceGrids → None, ImageSize → Automatic, Lighting → True,
 LightSources → {{{1., 0., 1.}, RGBColor[1, 0, 0]}, {{1., 1., 1.}, RGBColor[0, 1, 0]},
   {{0., 1., 1.}, RGBColor[0, 0, 1]}}, Plot3Matrix → Automatic, PlotLabel → None,
 PlotRange → Automatic, PlotRegion → Automatic, PolygonIntersections → True,
 Prolog → {}, RenderAll → True, Shading → True, SphericalRegion → False,
 Ticks → Automatic, ViewCenter → Automatic, ViewPoint → {1.3, -2.4, 2.},
 ViewVertical → {0., 0., 1.}, DefaultFont :→ $DefaultFont,
 DisplayFunction :→ $DisplayFunction, FormatType :→ $FormatType, TextStyle :→ $TextStyle}
```

```
In[45]:= Options[Plot3D]

Out[45]= {AmbientLight → GrayLevel[0], AspectRatio → Automatic, Axes → True,
  AxesEdge → Automatic, AxesLabel → None, AxesStyle → Automatic,
  Background → Automatic, Boxed → True, BoxRatios → {1, 1, 0.4}, BoxStyle → Automatic,
  ClipFill → Automatic, ColorFunction → Automatic, ColorFunctionScaling → True,
  ColorOutput → Automatic, Compiled → True, DefaultColor → Automatic,
  Epilog → {}, FaceGrids → None, HiddenSurface → True, ImageSize → Automatic,
  Lighting → True, LightSources → {{{1., 0., 1.}, RGBColor[1, 0, 0]},
    {{1., 1., 1.}, RGBColor[0, 1, 0]}, {{0., 1., 1.}, RGBColor[0, 0, 1]}},
  Mesh → True, MeshStyle → Automatic, Plot3Matrix → Automatic, PlotLabel → None,
  PlotPoints → 15, PlotRange → Automatic, PlotRegion → Automatic, Prolog → {},
  Shading → True, SphericalRegion → False, Ticks → Automatic, ViewCenter → Automatic,
  ViewPoint → {1.3, -2.4, 2.}, ViewVertical → {0., 0., 1.}, DefaultFont :→ $DefaultFont,
  DisplayFunction :→ $DisplayFunction, FormatType :→ $FormatType, TextStyle :→ $TextStyle}

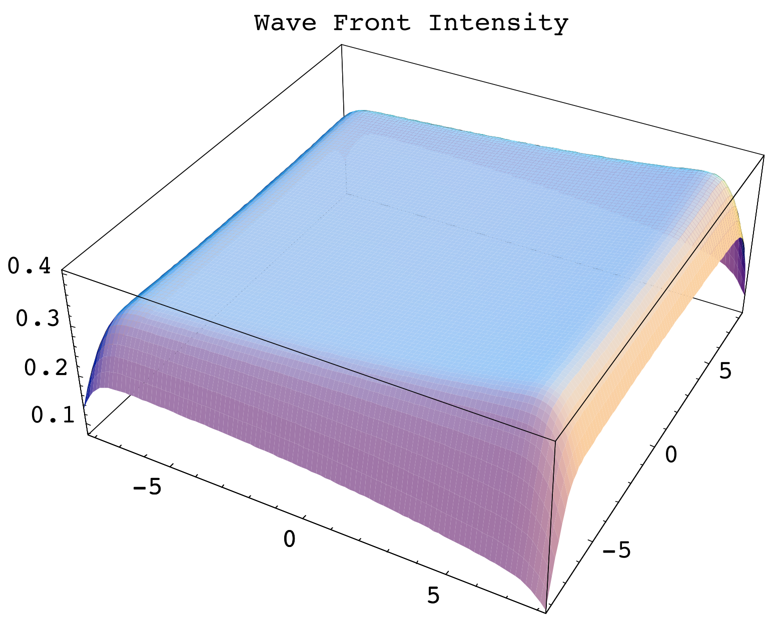
In[46]:= PSF[pupfun, NumberOfPoints->128, Plot2D->False, PlotRange->All, PlotPoints->200,
  Mesh->False]
```

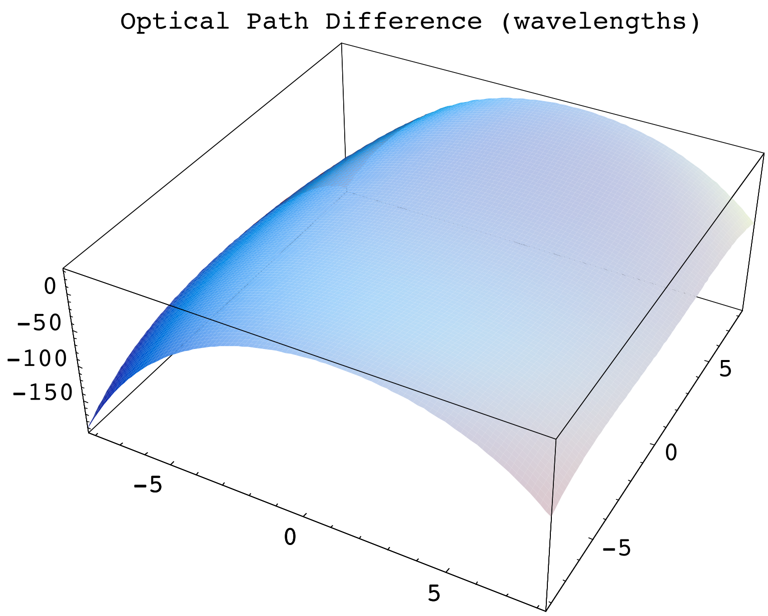


```
Out[46]= {AngularMagnification → -0.93012, BackFocalLength → 43.791,
  BackFocalPoint → {153.291, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 5, CosineCompensation → True,
  DiffractionSpotSize → {0.126195, 0.126247}, Energy → 100.,
  EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
  FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
  FilterTrace → True, FindImagePoint → True, FindPupils → True,
  FocalFraction → 0, FocalLength → 50.2182, FrequencyCutoff → {304.062, 303.934},
  FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
  GeometricPointSpreadFunction → False, ImagePlaneTilt → {1., 0, 0},
  ImagePoint → {207.282, 0, 0}, ImageSampleSize → {0.00166046, 0.00166114}, ImagingOptics →
   {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
    Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
    Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
  IntensityScale → 1, IntensitySetting → Automatic, InterceptHole → True,
  InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}}, Mesh → False,
  NumberOfPoints → 128, ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5],
  Offset → 112.446, OpticalLengthFunction → OpticalLengthFunction,
  OpticalMedium → Air, OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0,
     0.0000222711, -6.85363 - 3.53211 × 10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
      1.95153 × 10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 10.3109 (-1 + 0.0139904 (#1^2 + #2^2)) -
      5.2511 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209856 (#1^2 + #2^2) + 0.000195732 (#1^2 + #2^2)^2) -
      3.47812 (1 - 0.0419713 (#1^2 + #2^2) + 0.000293598 (#1^2 + #2^2)^2) + 4.27139 × 10^-6
       Cos[4 ArcTan[#1, #2]] (-0.000244665 (#1^2 + #2^2)^2 + 2.05378 × 10^-6 (#1^2 + #2^2)^3) -
      3.39626 × 10^-7 Cos[2 ArcTan[#1, #2]]
       (0.0419713 (#1^2 + #2^2) - 0.00097866 (#1^2 + #2^2)^2 + 5.13445 × 10^-6 (#1^2 + #2^2)^3) - 0.0210822
       (-1 + 0.0839425 (#1^2 + #2^2) - 0.00146799 (#1^2 + #2^2)^2 + 6.84593 × 10^-6 (#1^2 + #2^2)^3) -
      7.3801 × 10^-8 Cos[6 ArcTan[#1, #2]] (-2.39608 × 10^-6 (#1^2 + #2^2)^3 +
         1.91555 × 10^-8 (#1^2 + #2^2)^4) + 3.18005 × 10^-6 Cos[4 ArcTan[#1, #2]]
       (0.000733995 (#1^2 + #2^2)^2 - 0.0000143765 (#1^2 + #2^2)^3 + 6.70443 × 10^-8 (#1^2 + #2^2)^4) -
      1.29139 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699521 (#1^2 + #2^2) +
         0.00293598 (#1^2 + #2^2)^2 - 0.0000359412 (#1^2 + #2^2)^3 + 1.34089 × 10^-7 (#1^2 + #2^2)^4) -
      0.000207406 (1 - 0.139904 (#1^2 + #2^2) + 0.00440397 (#1^2 + #2^2)^2 -
         0.0000479215 (#1^2 + #2^2)^3 + 1.67611 × 10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
  OpticalPathRange → {112.435, 112.446}, -Options-, OutputGraphics →
   {WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -),
    PointSpreadFunction → (- Graphics3D -)},
  ParaxialReductionRatio → 0.001, PerfectPointSpreadFunction →
   InterpolatingFunction[{{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>],
  Plot2D → False, PlotPoints → 200, PlotRange → All,
  PointSpreadFunction →
   InterpolatingFunction[{{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>],
  PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
  PrinciplePointSeparation → 6.14556,
  RayBoundary → {{-8.45622, 8.45622}, {-8.45265, 8.45265}},
```

```
RefractiveIndex → 1.00027,
RenderedParameters → {OpticalPathDifference},
ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
ResidualFitError → 7.31386 × 10^-7,
SampleFactor → 2,
SeidelAberrations → {SphericalAberration → 82.9255, Astigmatism → 0.0000417324,
  FieldCurvature → -0.246833, Distortion → -3.77796 × 10^-13, Coma → 1.34603 × 10^-12},
SmoothKernelRange → 3, SmoothKernelSize → 1.25, SourceID → 584,
SpatialScale → 1, StrehlRatio → 0.0156867,
SurfaceNumber → 1,
SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
TransverseMagnification → -1.08942,
WaveFrontID → 1, WaveFrontIntensity →
 InterpolatingFunction[{{-11.3473, 11.3473}, {-11.3437, 11.3437}}, <>],
WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

In[58]:= **PSF[offaxis3D, NumberOfPoints->1024, NumberOfRays->32, Plot2D->False, FocalFraction->0, PlotPoints->100, ColorFunction->Automatic, Mesh->False]**

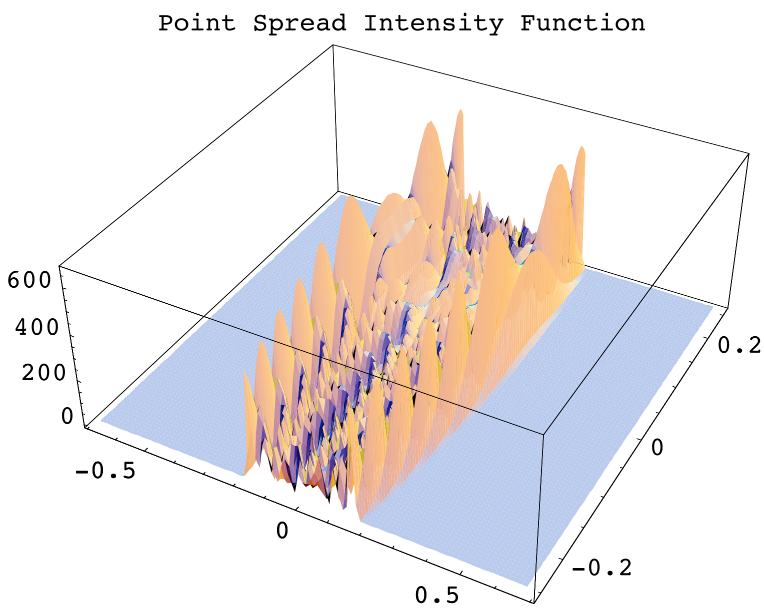





```
Warning: too few sample points.

Need to increase the NumberOfPoints option.

Suggestion: NumberOfPoints -> 2048.
```

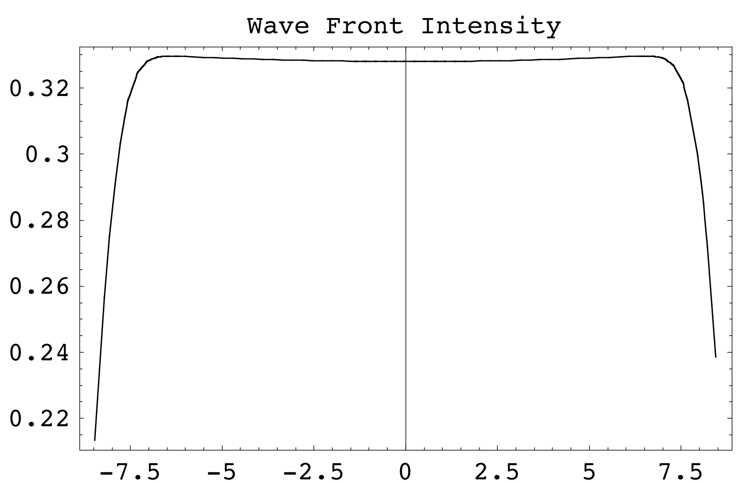


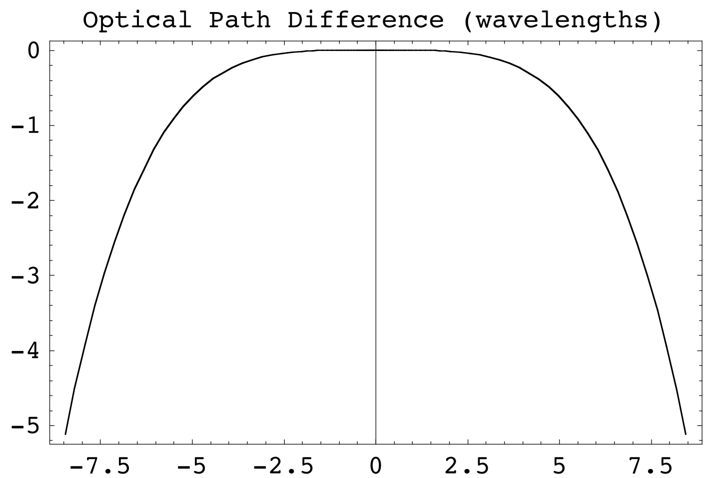

```
Out[58]= {AngularMagnification → -1.24643, BackFocalLength → 39.485,
  BackFocalPoint → {57.8471, -14.8154, 0}, ColorFunction → Automatic, ComponentNumber → 3,
  CosineCompensation → True, DiffractionSpotSize → {1.35425, 0.506901},
  Energy → 100., EntrancePupilBoundary → {-57.4255, -57.4255},
  EntrancePupilDistance → -14.8595, EntrancePupilOffset → {0, 0},
  EntrancePupilPosition → {14.5372, -3.07794, 0}, EntrancePupilRotationMatrix →
   {{-0.978312, 0.207137, 0}, {-0.207137, -0.978312, 0}, {0, 0, 1}},
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → 0, ExitPupilOffset → {0, 0},
  ExitPupilPosition → {19.817, -4.19583, 0}, ExitPupilRotationMatrix →
   {{0.963153, -0.268952, 0}, {0.268952, 0.963153, 0}, {0, 0, 1}}, FieldStopPosition → 2,
```

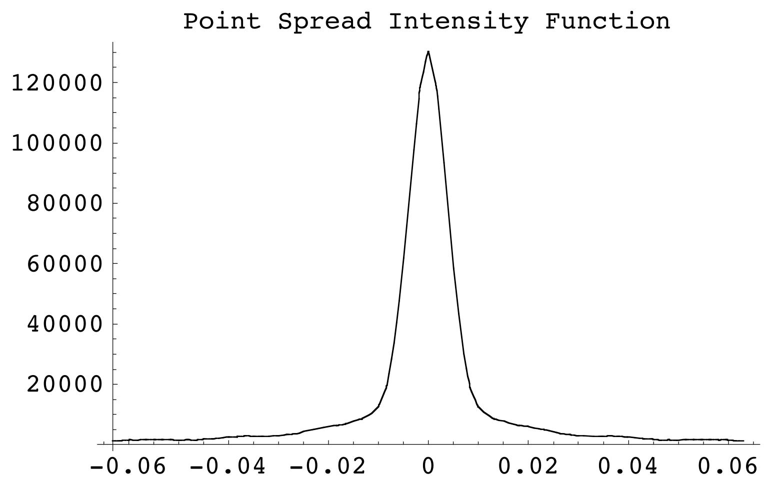
```
FieldStopSurface → {IntersectionNumber → 2, SurfaceNumber → 2, ComponentNumber → 1},
FilterTrace → True, FindImagePoint → False, FindPupils → True, FocalFraction → 0,
FocalLength → 45.7974, FrequencyCutoff → {383.55, 388.81}, FrontFocalLength → 38.2594,
FrontFocalPoint → {-36.3161, 12.0385, 0}, GeometricPointSpreadFunction → False,
ImagePlaneTilt → {0.963153, -0.268952, 0}, ImagePoint → {97.5654, -25.9064, 0},
ImageSampleSize → {0.00132381, 0.00130644}, ImagingOptics → {BiConvexLens[50, 50, 20],
  Boundary[{0, -54, -54}, {108, 54, 54}, {GraphicDesign → Off}]},
IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
LensABCDMatrix → {{0.862168, 10.2908}, {-0.0218353, 0.899242}},
Mesh → False, NormalizePlot → False, NumberOfPoints → 1024, NumberOfRays → 32,
ObjectSource → Move[PointOfRays[{10, 10}], {-90.5, 30.}, -18.3399], Offset → 126.147,
OpticalLengthFunction → OpticalLengthFunction, OpticalMedium → Air,
OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0, -0.14469,
   -41.8122 + 5.28676 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] - 0.826207 Cos[2 ArcTan[#1, #2]]
     (#1^2 + #2^2) - 0.000615403 Cos[3 ArcTan[#1, #2]] (#1^2 + #2^2)^(3/2) - 2.69986 × 10^-6
     Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 6.30976 × 10^-8 Cos[5 ArcTan[#1, #2]] (#1^2 + #2^2)^(5/2) +
    4.88202 × 10^-10 Cos[6 ArcTan[#1, #2]] (#1^2 + #2^2)^3 - 64.237 (-1 + 0.0143892 (#1^2 + #2^2)) +
    26.6882 Cos[ArcTan[#1, #2]] (-0.169642 Sqrt[#1^2 + #2^2] + 0.00183076 (#1^2 + #2^2)^(3/2)) -
    3.53058 Cos[2 ArcTan[#1, #2]] (-0.0215838 (#1^2 + #2^2) + 0.000207049 (#1^2 + #2^2)^2) -
    23.5564 (1 - 0.0431676 (#1^2 + #2^2) + 0.000310574 (#1^2 + #2^2)^2) - 0.229599
     Cos[3 ArcTan[#1, #2]] (-0.00244101 (#1^2 + #2^2)^(3/2) + 0.0000219526 (#1^2 + #2^2)^(5/2)) -
    0.664073 Cos[ArcTan[#1, #2]] (0.254463 Sqrt[#1^2 + #2^2] - 0.00732304 (#1^2 + #2^2)^(3/2) +
       0.0000439053 (#1^2 + #2^2)^(5/2)) - 0.00345903 Cos[4 ArcTan[#1, #2]] (-0.000258811
        (#1^2 + #2^2)^2 + 2.23445 × 10^-6 (#1^2 + #2^2)^3) - 0.194583 Cos[2 ArcTan[#1, #2]]
     (0.0431676 (#1^2 + #2^2) - 0.00103525 (#1^2 + #2^2)^2 + 5.58613 × 10^-6 (#1^2 + #2^2)^3) -
    1.04386 (-1 + 0.0863352 (#1^2 + #2^2) - 0.00155287 (#1^2 + #2^2)^2 + 7.44818 × 10^-6 (#1^2 + #2^2)^3) +
    0.00215237 Cos[5 ArcTan[#1, #2]] (-0.0000263432 (#1^2 + #2^2)^(5/2) +
       2.21117 × 10^-7 (#1^2 + #2^2)^(7/2)) - 0.0150955 Cos[3 ArcTan[#1, #2]]
     (0.00610253 (#1^2 + #2^2)^(3/2) - 0.000131716 (#1^2 + #2^2)^(5/2) + 6.6335 × 10^-7 (#1^2 + #2^2)^(7/2)) -
    0.11238 Cos[ArcTan[#1, #2]] (-0.339284 Sqrt[#1^2 + #2^2] + 0.0183076 (#1^2 + #2^2)^(3/2) -
       0.000263432 (#1^2 + #2^2)^(5/2) + 1.10558 × 10^-6 (#1^2 + #2^2)^(7/2)) + 0.000809293
     Cos[6 ArcTan[#1, #2]] (-2.60686 × 10^-6 (#1^2 + #2^2)^3 + 2.14347 × 10^-8 (#1^2 + #2^2)^4) +
    0.0029519 Cos[4 ArcTan[#1, #2]] (0.000776434 (#1^2 + #2^2)^2 - 0.0000156412 (#1^2 + #2^2)^3 +
       7.50213 × 10^-8 (#1^2 + #2^2)^4) - 0.0141619 Cos[2 ArcTan[#1, #2]] (-0.071946 (#1^2 + #2^2) +
       0.00310574 (#1^2 + #2^2)^2 - 0.0000391029 (#1^2 + #2^2)^3 + 1.50043 × 10^-7 (#1^2 + #2^2)^4) -
    0.0569781 (1 - 0.143892 (#1^2 + #2^2) + 0.0046586 (#1^2 + #2^2)^2 -
       0.0000521372 (#1^2 + #2^2)^3 + 1.87553 × 10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OpticalPathRange → {126.045, 126.163}, -Options-, OutputGraphics →
 {WaveFrontIntensity → (- SurfaceGraphics -),
  OpticalPathDifference → (- SurfaceGraphics -), PointSpreadFunction → (- Graphics3D -)},
PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
PerfectPointSpreadFunction →
 InterpolatingFunction[{{-0.676465, 0.677789}, {-0.667593, 0.6689}}, <>],
```

```
Plot2D → False, PlotPoints → 100,
PointSpreadFunction →
 InterpolatingFunction[{{-0.676465, 0.677789}, {-0.667593, 0.6689}}, <>],
PrinciplePoints → {{7.15509, -2.37185, 0}, {13.7372, -2.49811, 0}},
PrinciplePointSeparation → 6.58337,
RayBoundary → {{-8.27889, 8.27889}, {-8.39364, 8.39364}},
RefractiveIndex → 1.00027,
RenderedParameters →
 {PointSpreadFunction, OpticalPathDifference},
ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
ResidualFitError → 0.00158641,
SampleFactor → 2,
SeidelAberrations → {SphericalAberration → 532.608, Astigmatism → 221.095,
  FieldCurvature → -232.703, Distortion → 7.70783, Coma → 83.1698},
ShowPerfectCase → False, SignalPlotCutoff → 0.01,
SmoothKernelRange → 3,
SmoothKernelSize → 1.25,
SourceID → 1351,
SpatialScale → 1,
StrehlRatio → 0.000051985,
SurfaceNumber → 1,
SystemABCDMatrix → {{-1.137, -19.1063}, {-0.0218353, -1.24643}},
TransverseMagnification → -1.137,
WaveFrontID → 1,
WaveFrontIntensity →
 InterpolatingFunction[{{-11.1694, 11.1694}, {-11.2842, 11.2842}}, <>],
WaveLength → 0.532, ZernikeFit → True,
ZernikeOrder → 8}
```

```
In[59]:= PSF[sys3D, FindImagePoint->True, FocalFraction->0, NumberOfPoints->128]
```



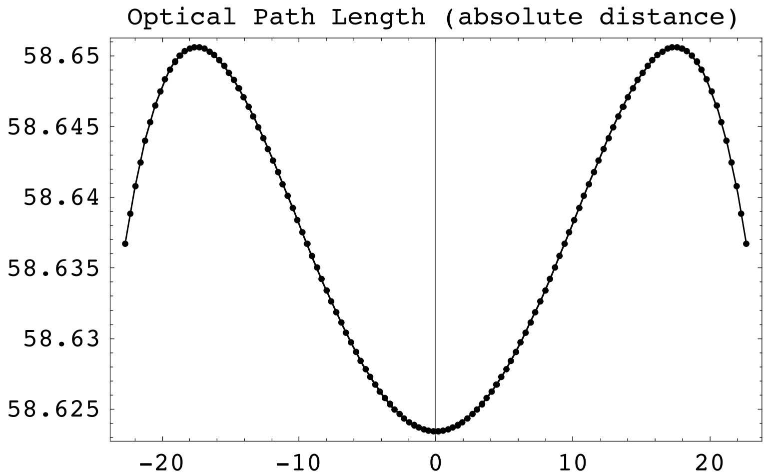


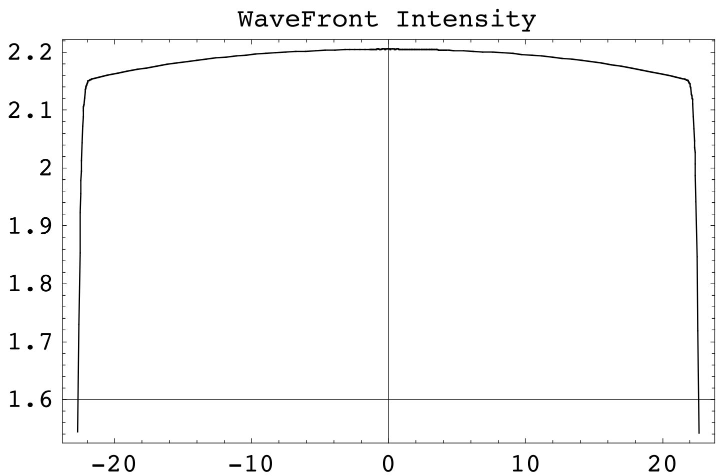


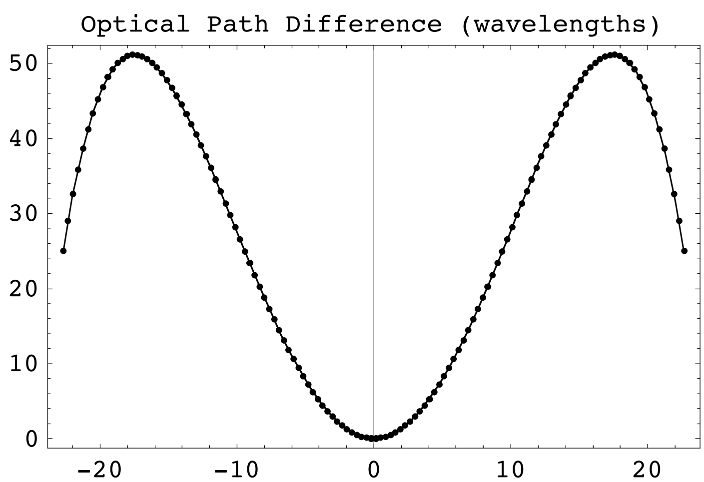

```
Out[59]= {AngularMagnification → -0.93012, BackFocalLength → 43.791,
  BackFocalPoint → {153.291, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 5, CosineCompensation → True,
  DiffractionSpotSize → {0.126195, 0.126247}, Energy → 100.,
  EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
  FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
  FilterTrace → True, FindImagePoint → True, FindPupils → True,
  FocalFraction → 0, FocalLength → 50.2182, FrequencyCutoff → {304.062, 303.934},
  FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
  GeometricPointSpreadFunction → False, ImagePlaneTilt → {1., 0, 0},
  ImagePoint → {207.282, 0, 0}, ImageSampleSize → {0.00166046, 0.00166114}, ImagingOptics →
   {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
    Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
    Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
  IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
  InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}}, NormalizePlot → False,
  NumberOfPoints → 128, ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5],
  Offset → 112.446, OpticalLengthFunction → OpticalLengthFunction,
```

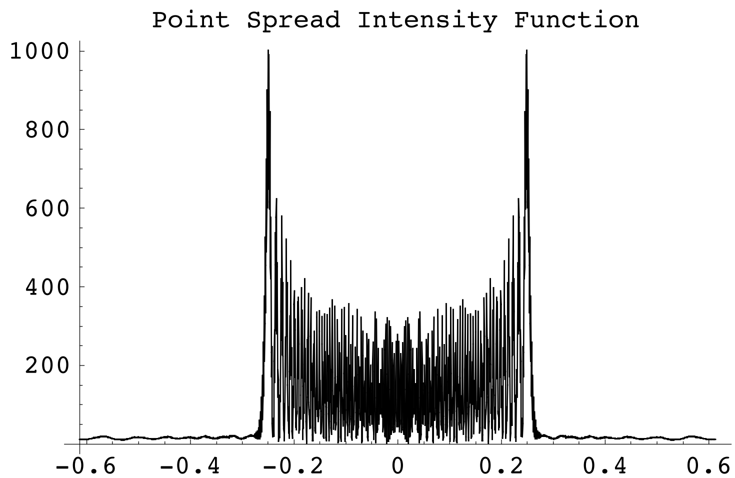
```
OpticalMedium → Air, OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0,
   0.0000222711, -6.85363 - 3.53211 × 10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
    1.95153 × 10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 10.3109 (-1 + 0.0139904 (#1^2 + #2^2)) -
    5.2511 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209856 (#1^2 + #2^2) + 0.000195732 (#1^2 + #2^2)^2) -
    3.47812 (1 - 0.0419713 (#1^2 + #2^2) + 0.000293598 (#1^2 + #2^2)^2) + 4.27139 × 10^-6
     Cos[4 ArcTan[#1, #2]] (-0.000244665 (#1^2 + #2^2)^2 + 2.05378 × 10^-6 (#1^2 + #2^2)^3) -
    3.39626 × 10^-7 Cos[2 ArcTan[#1, #2]]
     (0.0419713 (#1^2 + #2^2) - 0.00097866 (#1^2 + #2^2)^2 + 5.13445 × 10^-6 (#1^2 + #2^2)^3) - 0.0210822
     (-1 + 0.0839425 (#1^2 + #2^2) - 0.00146799 (#1^2 + #2^2)^2 + 6.84593 × 10^-6 (#1^2 + #2^2)^3) -
    7.3801 × 10^-8 Cos[6 ArcTan[#1, #2]] (-2.39608 × 10^-6 (#1^2 + #2^2)^3 +
       1.91555 × 10^-8 (#1^2 + #2^2)^4) + 3.18005 × 10^-6 Cos[4 ArcTan[#1, #2]]
     (0.000733995 (#1^2 + #2^2)^2 - 0.0000143765 (#1^2 + #2^2)^3 + 6.70443 × 10^-8 (#1^2 + #2^2)^4) -
    1.29139 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699521 (#1^2 + #2^2) +
       0.00293598 (#1^2 + #2^2)^2 - 0.0000359412 (#1^2 + #2^2)^3 + 1.34089 × 10^-7 (#1^2 + #2^2)^4) -
    0.000207406 (1 - 0.139904 (#1^2 + #2^2) + 0.00440397 (#1^2 + #2^2)^2 -
       0.0000479215 (#1^2 + #2^2)^3 + 1.67611 × 10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OpticalPathRange → {112.435, 112.446}, -Options-, OutputGraphics →
 {WaveFrontIntensity → (- Graphics -),
  OpticalPathDifference → (- Graphics -), PointSpreadFunction → (- Graphics -)},
PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
PerfectPointSpreadFunction →
 InterpolatingFunction[{{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>],
Plot2D → True, PlotPoints → 64, PointSpreadFunction →
 InterpolatingFunction[{{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>],
PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
PrinciplePointSeparation → 6.14556,
RayBoundary → {{-8.45622, 8.45622}, {-8.45265, 8.45265}},
RefractiveIndex → 1.00027,
RenderedParameters → {PointSpreadFunction, OpticalPathDifference},
ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
ResidualFitError → 7.31386 × 10^-7,
SampleFactor → 2,
SeidelAberrations → {SphericalAberration → 82.9255, Astigmatism → 0.0000417324,
  FieldCurvature → -0.246833, Distortion → -3.77796 × 10^-13, Coma → 1.34603 × 10^-12},
ShowPerfectCase → False, SignalPlotCutoff → 0.01, SmoothKernelRange → 3,
SmoothKernelSize → 1.25,
SourceID → 584, SpatialScale → 1,
StrehlRatio → 0.0156867, SurfaceNumber → 1,
SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
TransverseMagnification → -1.08942,
WaveFrontID → 1, WaveFrontIntensity →
 InterpolatingFunction[{{-11.3473, 11.3473}, {-11.3437, 11.3437}}, <>],
WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

```
In[66]:= PSF[sys, NumberOfPoints->2048, NumberOfRays->128]
```


Optical Path Length (absolute distance)
58.65
58.645
58.64
58.635
58.63
58.625
-20
-10
0
10
20


WaveFront Intensity
2.2
2.1
2
1.9
1.8
1.7
1.6
-20
-10
0
10
20


Optical Path Difference (wavelengths)
50
40
30
20
10
0
-20
-10
0
10
20




```
Out[66]= {AngularMagnification → 0.49967, BackFocalLength → 93.351,
  BackFocalPoint → {153.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 3, CosineCompensation → True, DiffractionSpotSize → 1.22636,
  Energy → 100., EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {50., 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {53.417, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
  FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
  FilterTrace → True, FindImagePoint → False, FindPupils → True, FocalFraction → Automatic,
  FocalLength → 99.934, FrequencyCutoff → 859.481, FrontFocalLength → 99.934,
  FrontFocalPoint → {-49.934, 0, 0}, GeometricPointSpreadFunction → False,
  ImagePlaneTilt → {1., 0, 0}, ImagePoint → {150., 0., 0.}, ImageSampleSize → 0.000625696,
  ImagingOptics → {Move[PlanoConvexLens[100, 50, 10], 50.], Move[Screen[50], {x, 150.}]},
  IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
  InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
  NormalizePlot → False, NumberOfPoints → 2048, NumberOfRays → 128,
  ObjectSource → LineOfRays[45, NumberOfRays → 11], Offset → 58.6234,
  OpticalLengthFunction → InterpolatingFunction[{{-22.6817, 22.6817}}, <>],
  OpticalMedium → Air,
  OpticalPathDifference → InterpolatingFunction[{{-22.6817, 22.6817}}, <>],
  OpticalPathRange → {58.6234, 58.6506}, -Options-, OutputGraphics →
   {OpticalLengthFunction → (- Graphics -), WaveFrontIntensity → (- Graphics -),
    OpticalPathDifference → (- Graphics -), PointSpreadFunction → (- Graphics -)},
  PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
  PerfectPointSpreadFunction → InterpolatingFunction[{{-0.640087, 0.640713}}, <>],
  Plot2D → True, PlotPoints → 64,
  PointSpreadFunction → InterpolatingFunction[{{-0.640087, 0.640713}}, <>],
  PrinciplePoints → {{50., 0, 0}, {53.417, 0, 0}}, PrinciplePointSeparation → 3.417,
  RayBoundary → {-22.6817, 22.6817}, RefractiveIndex → 1.00027,
  RenderedParameters → {PointSpreadFunction, OpticalPathDifference},
  ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
  SampleFactor → 2, SeidelAberrations → True, ShowPerfectCase → False,
  SignalPlotCutoff → 0.01, SmoothKernelRange → 3, SmoothKernelSize → 1.25,
  SourceID → 2190, SpatialScale → 1, StrehlRatio → 0.0126468, SurfaceNumber → 1,
  SystemABCDMatrix → {{0.0335317, 98.2596}, {-0.0100066, 0.49967}},
  TransverseMagnification → 0.0335317, WaveFrontID → 1,
  WaveFrontIntensity → InterpolatingFunction[{{-24.0455, 23.9863}}, <>],
  WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

Go to list of topics

5.6 OpticalTransferFunction

In[63]:= **?OpticalTransferFunction**

```
OpticalTransferFunction[system, options] calculates the modulation and phase
  transfer functions of the optical system for a given object source input.

OpticalTransferFunction works together with either GeometricPointSpreadFunction,
  FindIntensity, and PointSpreadFunction. OpticalTransferFunction first calls
  either GeometricPointSpreadFunction or PointSpreadFunction to find the point
  spread function of the imaging system. Finally, OpticalTransferFunction
  calculates a modulation and phase transfer function by using the Fourier
  transform of the point spread function. As input, OpticalTransferFunction
  takes either an optical system or the returned output from either
  GeometricPointSpreadFunction, FindIntensity, or PointSpreadFunction.
  The optical system must contain a light source followed by the imaging
  optics with the approximate focal surface as its last element. For
  FindImagePoint->True, the last system element need not be the exact
  focal surface, since the exact focal surface position is automatically
  determined. When the default FindImagePoint->False is given, the last
  system element is assumed to be the exact focal surface. In addition,
  the light source need only contain a small number of rays (since the
  actual number of rays is internally specified). When FindPupils->False
  is used, the optical surface preceding the image surface is taken as the
  field stop. The user can also give as input a specific focal point or
  exit pupil position with the FocalPoint and ExitPupilPosition options.

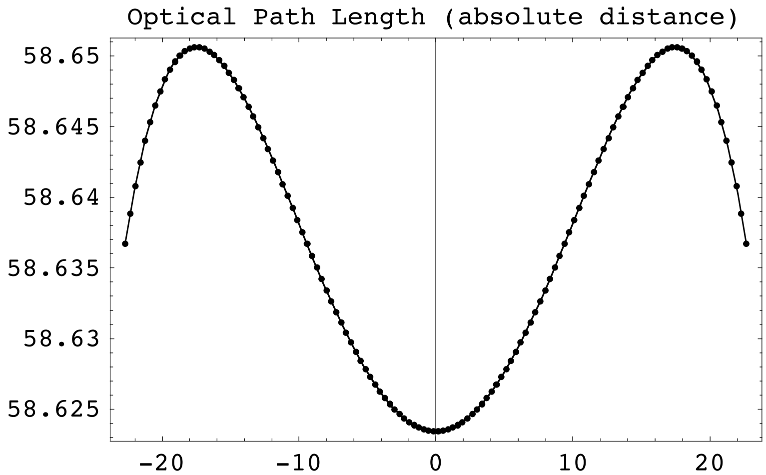
Note: OpticalTransferFunction works equally well for both point sources
  and planar sources, as long as the described imaging system contains
  a focus. If a one-dimensional light source is used (ie. WedgeOfRays or
  LineOfRays), then a one-dimensional modulation transfer function is
  calculated. If a two-dimensional light source is used (ie. PointOfRays
  or GridOfRays), then the optical transfer function calculations are
  carried out in two-dimensions. Warning: in general, most one-dimensional
  and two-dimensional results are not fully equivalent to each other.

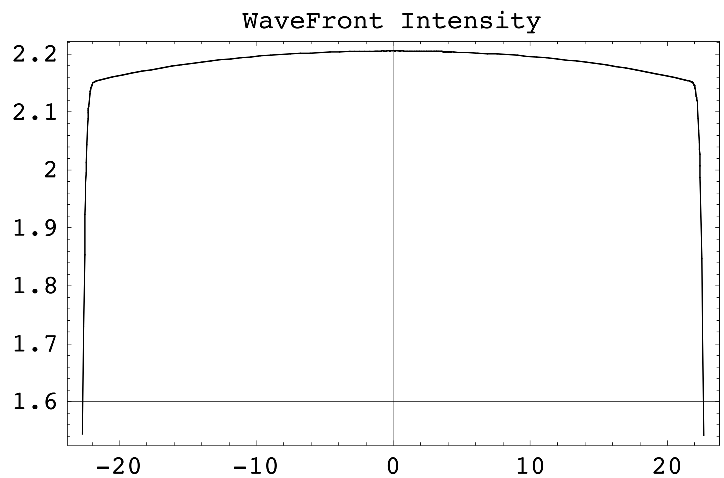
OpticalTransferFunction is also a label used with the RenderedParameters
  option. See also: IntensityTransform, FindLensParameters, PupilFunction,
  PointSpreadFunction, ModulationTransferFunction, PhaseTransferFunction,
  OTF, MTF, FindIntensity, and GeometricPointSpreadFunction.
```

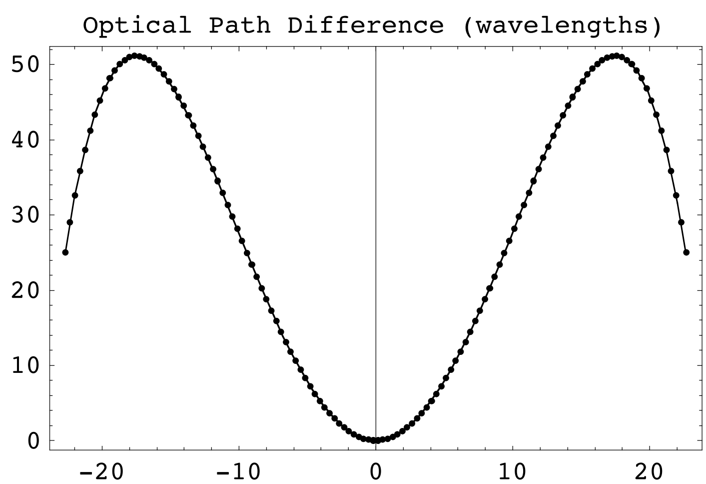
In[64]:= **?MTF**

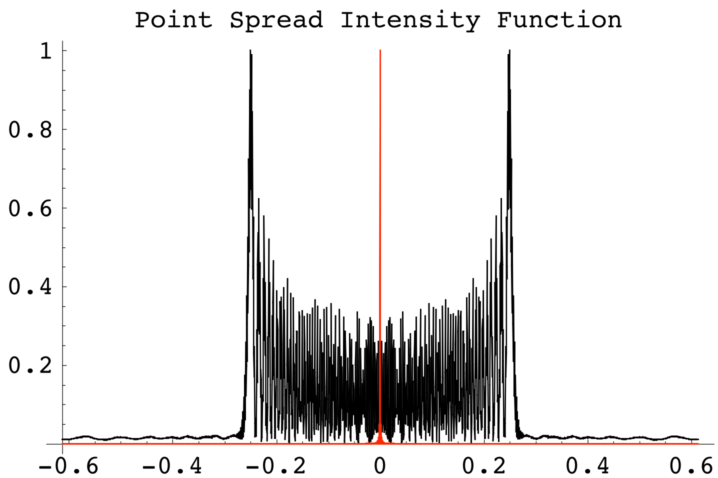
```
MTF[system, options] is an alias to ModulationTransferFunction[system, options].
```

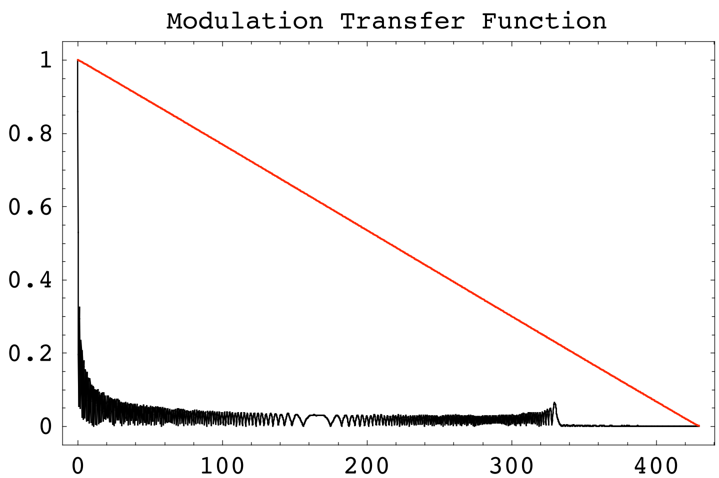
In[68]:= **MTF[sys, NumberOfPoints->2048, NumberOfRays->128]**


Optical Path Length (absolute distance)
58.65
58.645
58.64
58.635
58.63
58.625
-20
-10
0
10
20


WaveFront Intensity
2.2
2.1
2
1.9
1.8
1.7
1.6
-20
-10
0
10
20

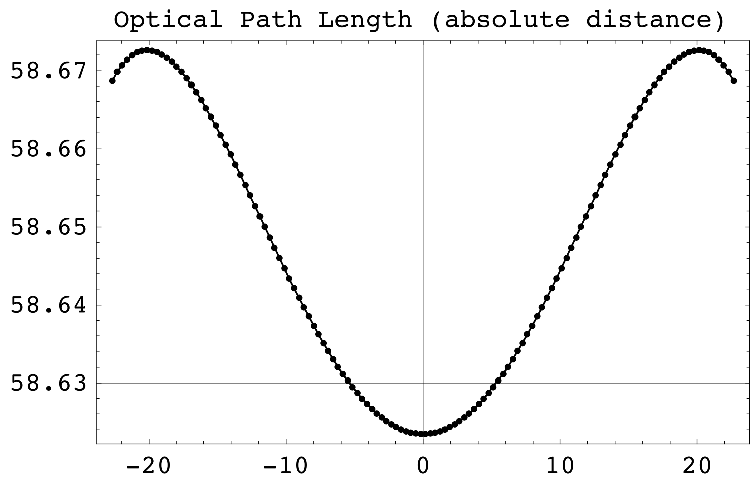

Optical Path Difference (wavelengths)
50
40
30
20
10
0
-20
-10
0
10
20

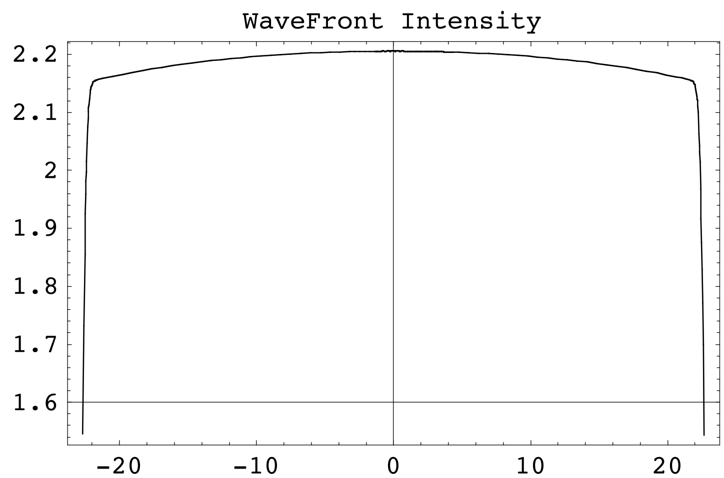

Point Spread Intensity Function
1
0.8
0.6
0.4
0.2
-0.6 -0.4 -0.2 0 0.2 0.4 0.6

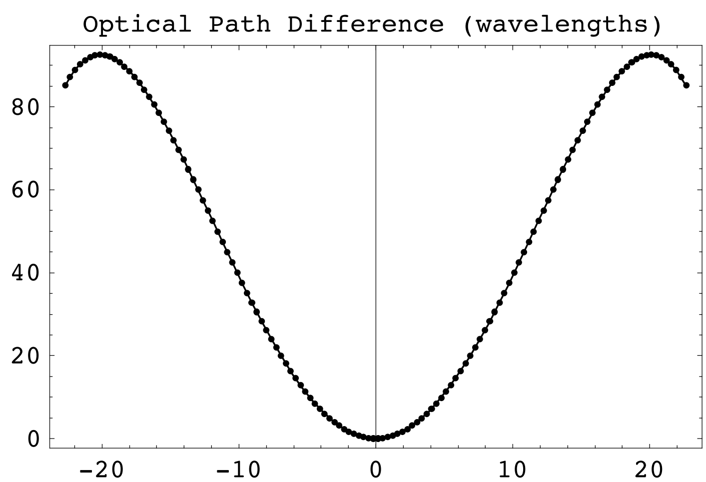

Modulation Transfer Function
1
0.8
0.6
0.4
0.2
0
0 100 200 300 400

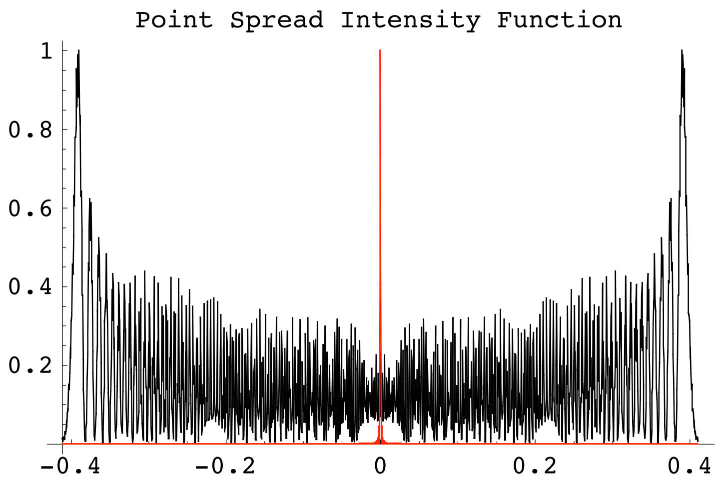
```
Out[68]= {AngularMagnification → 0.49967, BackFocalLength → 93.351,
  BackFocalPoint → {153.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 3, CosineCompensation → True, DiffractionSpotSize → 1.22636,
  Energy → 100., EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {50., 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {53.417, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
  FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
  FilterTrace → True, FindImagePoint → False, FindPupils → Automatic,
  FocalFraction → Automatic, FocalLength → 99.934, FrequencyCutoff → 859.481,
  FrontFocalLength → 99.934, FrontFocalPoint → {-49.934, 0, 0},
  GeometricPointSpreadFunction → False, ImagePlaneTilt → {1., 0, 0},
  ImagePoint → {150., 0., 0.}, ImageSampleSize → 0.000625696,
  ImagingOptics → {Move[PlanoConvexLens[100, 50, 10], 50.], Move[Screen[50], {x, 150.}]},
  IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
  InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
  ModulationTransferFunction → InterpolatingFunction[{{0., 429.74}}, <>],
  NormalizePlot → True, NumberOfPoints → 2048, NumberOfRays → 128,
  ObjectSource → LineOfRays[45, NumberOfRays → 11], Offset → 58.6234,
  OpticalLengthFunction → InterpolatingFunction[{{-22.6817, 22.6817}}, <>],
  OpticalMedium → Air,
  OpticalPathDifference → InterpolatingFunction[{{-22.6817, 22.6817}}, <>],
  OpticalPathRange → {58.6234, 58.6506},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -),
    PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
  PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
  PerfectModulationTransferFunction → InterpolatingFunction[{{0., 429.74}}, <>],
  PerfectPhaseTransferFunction → InterpolatingFunction[{{0., 429.74}}, <>],
  PerfectPointSpreadFunction → InterpolatingFunction[{{-0.640087, 0.640713}}, <>],
  PhaseTransferFunction → InterpolatingFunction[{{0., 429.74}}, <>],
  Plot2D → True, PlotPoints → 64,
  PointSpreadFunction → InterpolatingFunction[{{-0.640087, 0.640713}}, <>],
  PrinciplePoints → {{50., 0, 0}, {53.417, 0, 0}}, PrinciplePointSeparation → 3.417,
  RayBoundary → {-22.6817, 22.6817}, RefractiveIndex → 1.00027, RenderedParameters →
   {ModulationTransferFunction, PointSpreadFunction, OpticalPathDifference},
  ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
  SampleFactor → 2, SeidelAberrations → True, ShowPerfectCase → True,
  SignalPlotCutoff → 0.01, SmoothKernelRange → 3, SmoothKernelSize → 1.25,
  SourceID → 2190, SpatialScale → 1, StrehlRatio → 0.0126468, SurfaceNumber → 1,
  SystemABCDMatrix → {{0.0335317, 98.2596}, {-0.0100066, 0.49967}},
  TransverseMagnification → 0.0335317, WaveFrontID → 1,
  WaveFrontIntensity → InterpolatingFunction[{{-24.0455, 23.9863}}, <>],
  WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

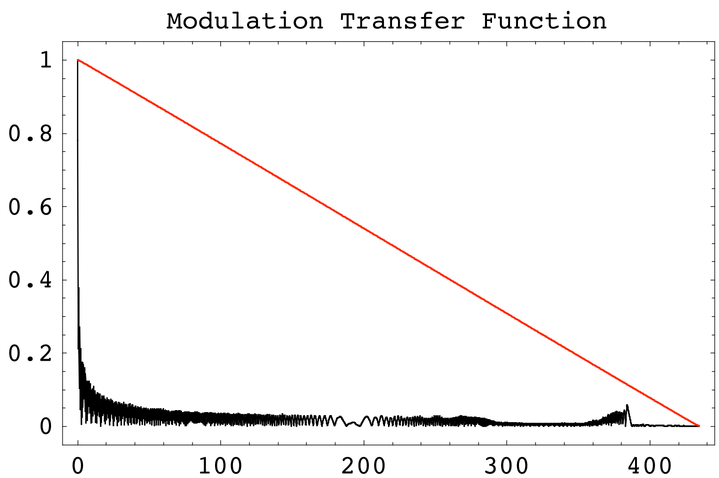
```
In[72]:= MTF[sys, NumberOfPoints->2048, NumberOfRays->128, FindImagePoint->True]
```


Optical Path Length (absolute distance)
58.67
58.66
58.65
58.64
58.63
-20
-10
0
10
20


WaveFront Intensity
2.2
2.1
2
1.9
1.8
1.7
1.6
-20
-10
0
10
20

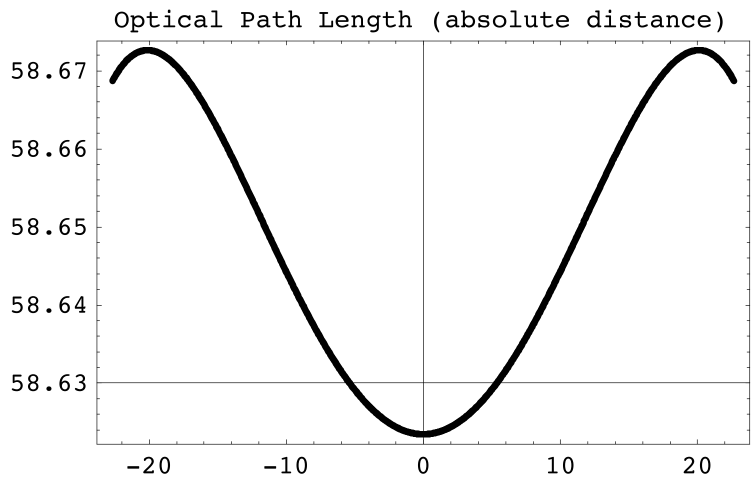

Optical Path Difference (wavelengths)
80
60
40
20
0
-20
-10
0
10
20

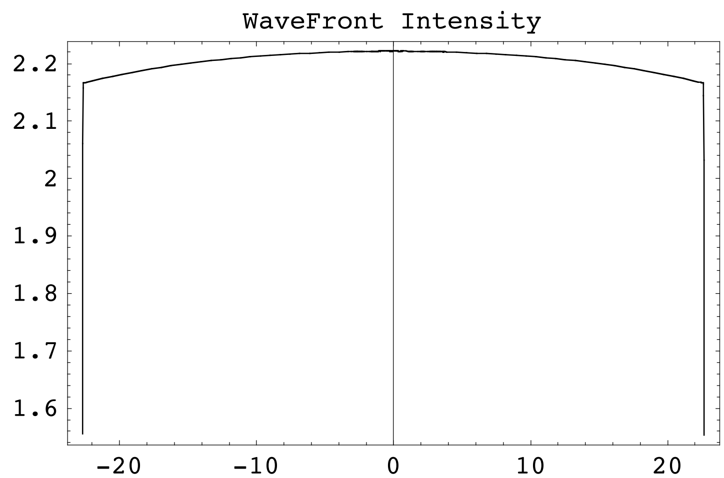

Point Spread Intensity Function
1
0.8
0.6
0.4
0.2
-0.4
-0.2
0
0.2
0.4

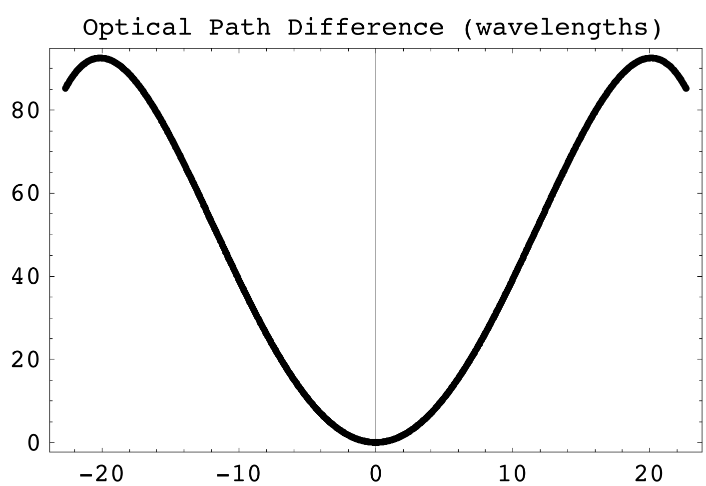

Modulation Transfer Function
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400

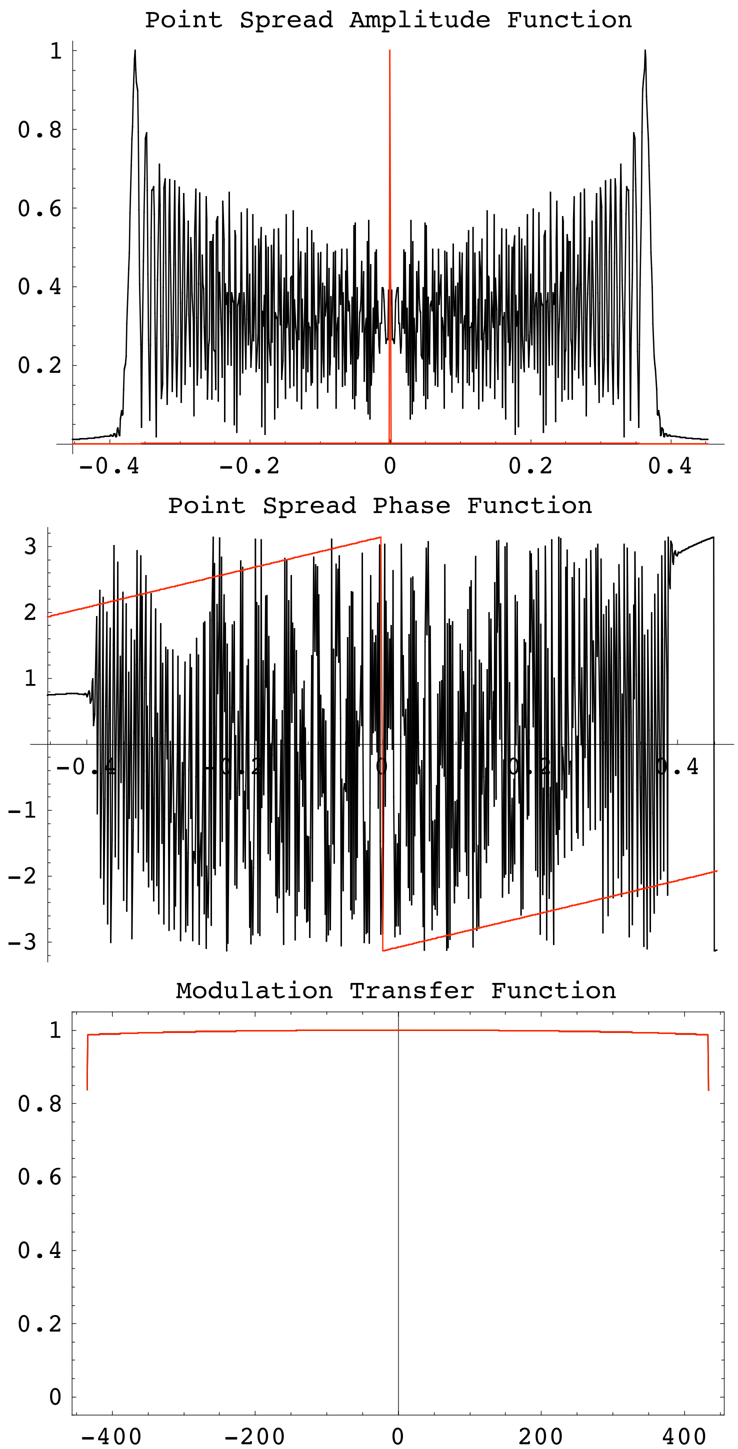
```
Out[72]= {AngularMagnification → 0.49967, BackFocalLength → 93.351,
  BackFocalPoint → {153.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 3, CosineCompensation → True, DiffractionSpotSize → 0.82196,
  Energy → 100., EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {50., 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {53.417, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
  FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
  FilterTrace → True, FindImagePoint → True, FindPupils → Automatic,
  FocalFraction → Automatic, FocalLength → 99.934, FrequencyCutoff → 868.832,
  FrontFocalLength → 99.934, FrontFocalPoint → {-49.934, 0, 0},
  GeometricPointSpreadFunction → False, ImagePlaneTilt → {1., 0, 0},
  ImagePoint → {148.87, 0, 0}, ImageSampleSize → 0.000619879,
  ImagingOptics → {Move[PlanoConvexLens[100, 50, 10], 50.], Move[Screen[50], {x, 150.}]},
  IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
  InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
  ModulationTransferFunction → InterpolatingFunction[{{0., 434.416}}, <>],
  NormalizePlot → True, NumberOfPoints → 2048, NumberOfRays → 128,
  ObjectSource → LineOfRays[45, NumberOfRays → 11], Offset → 58.6234,
  OpticalLengthFunction → InterpolatingFunction[{{-22.674, 22.674}}, <>],
  OpticalMedium → Air, OpticalPathDifference →
   InterpolatingFunction[{{-22.674, 22.674}}, <>], OpticalPathRange → {58.6234, 58.6726},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -),
    PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
  PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
  PerfectModulationTransferFunction → InterpolatingFunction[{{0., 434.416}}, <>],
  PerfectPhaseTransferFunction → InterpolatingFunction[{{0., 434.416}}, <>],
  PerfectPointSpreadFunction → InterpolatingFunction[{{-0.634136, 0.634756}}, <>],
  PhaseTransferFunction → InterpolatingFunction[{{0., 434.416}}, <>],
  Plot2D → True, PlotPoints → 64,
  PointSpreadFunction → InterpolatingFunction[{{-0.634136, 0.634756}}, <>],
  PrinciplePoints → {{50., 0, 0}, {53.417, 0, 0}}, PrinciplePointSeparation → 3.417,
  RayBoundary → {-22.674, 22.674}, RefractiveIndex → 1.00027, RenderedParameters →
   {ModulationTransferFunction, PointSpreadFunction, OpticalPathDifference},
  ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
  SampleFactor → 2, SeidelAberrations → True, ShowPerfectCase → True,
  SignalPlotCutoff → 0.01, SmoothKernelRange → 3, SmoothKernelSize → 1.25,
  SourceID → 2190, SpatialScale → 1, StrehlRatio → 0.00932262, SurfaceNumber → 1,
  SystemABCDMatrix → {{0.0335317, 98.2596}, {-0.0100066, 0.49967}},
  TransverseMagnification → 0.0335317, WaveFrontID → 1,
  WaveFrontIntensity → InterpolatingFunction[{{-24.0364, 23.9792}}, <>],
  WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

```
In[75]:= OpticalTransferFunction[sys, NumberOfPoints->1024, NumberOfRays->2048,
  FindImagePoint->True, IntensityTransform->False]
```


Optical Path Length (absolute distance)
58.67
58.66
58.65
58.64
58.63
-20
-10
0
10
20

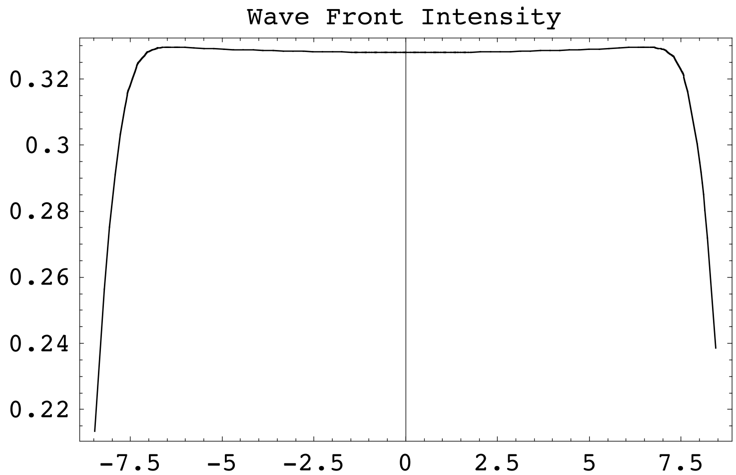

WaveFront Intensity
2.2
2.1
2
1.9
1.8
1.7
1.6
-20
-10
0
10
20

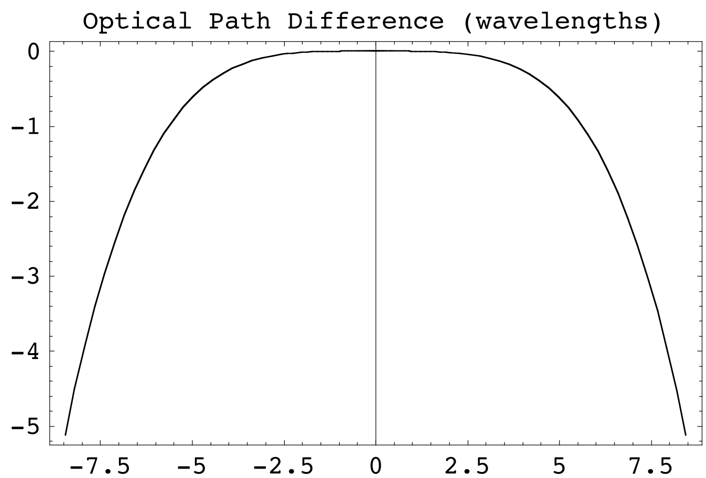

Optical Path Difference (wavelengths)
80
60
40
20
0
-20
-10
0
10
20

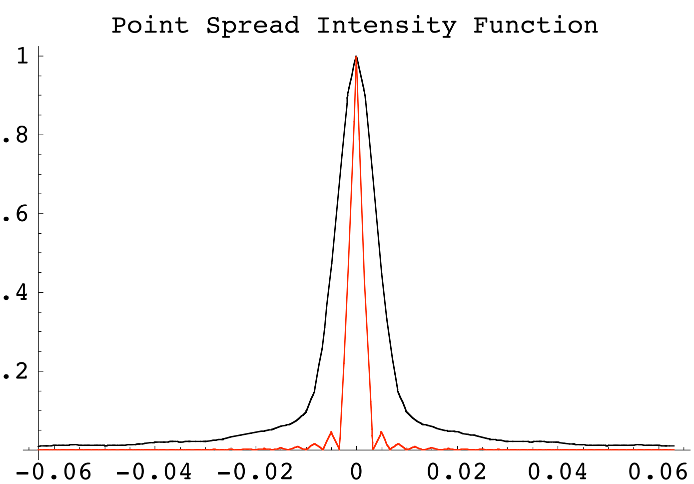

Point Spread Amplitude Function
1
0.8
0.6
0.4
0.2
-0.4
-0.2
0
0.2
0.4
Point Spread Phase Function
3
2
1
-1
-2
-3
Modulation Transfer Function
1
0.8
0.6
0.4
0.2
0
-400
-200
0
200
400

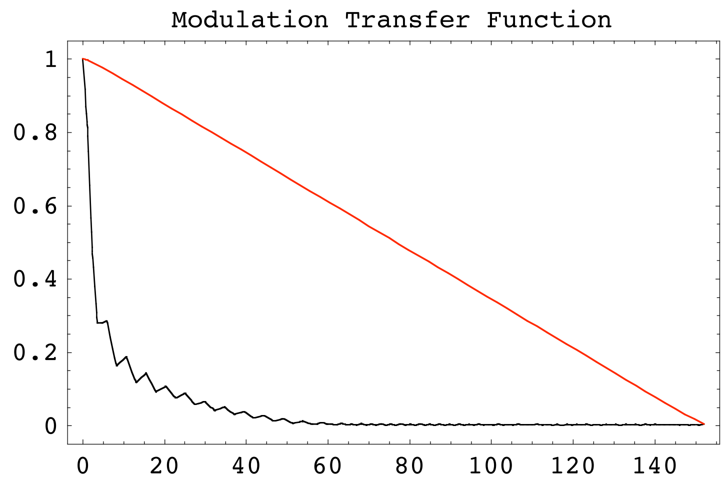
```
Out[75]= {AngularMagnification → 0.49967, BackFocalLength → 93.351,
  BackFocalPoint → {153.351, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 3, CosineCompensation → True, DiffractionSpotSize → 0.906965,
  Energy → 100., EntrancePupilBoundary → {-50., -50.}, EntrancePupilDistance → 0,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {50., 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → -6.583,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {53.417, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 1,
  FieldStopSurface → {IntersectionNumber → 1, SurfaceNumber → 1, ComponentNumber → 1},
  FilterTrace → True, FindImagePoint → True, FindPupils → Automatic,
  FocalFraction → Automatic, FocalLength → 99.934, FrequencyCutoff → 434.416,
  FrontFocalLength → 99.934, FrontFocalPoint → {-49.934, 0, 0},
  GeometricPointSpreadFunction → False, ImagePlaneTilt → {1., 0, 0},
  ImagePoint → {148.87, 0, 0}, ImageSampleSize → 0.00115097,
  ImagingOptics → {Move[PlanoConvexLens[100, 50, 10], 50.], Move[Screen[50], {x, 150.}]},
  IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → False,
  InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.934126, 6.583}, {-0.0100066, 1.}},
  ModulationTransferFunction → InterpolatingFunction[{{-434.841, 433.991}}, <>],
  NormalizePlot → True, NumberOfPoints → 1024, NumberOfRays → 2048,
  ObjectSource → LineOfRays[45, NumberOfRays → 11], Offset → 58.6234,
  OpticalLengthFunction → InterpolatingFunction[{{-22.674, 22.674}}, <>],
  OpticalMedium → Air, OpticalPathDifference →
   InterpolatingFunction[{{-22.674, 22.674}}, <>], OpticalPathRange → {58.6234, 58.6726},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -),
    PointSpreadFunction → (- Graphics -), PointSpreadPhaseFunction → (- Graphics -),
    ModulationTransferFunction → (- Graphics -)},
  PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
  PerfectModulationTransferFunction → InterpolatingFunction[{{-434.841, 433.991}}, <>],
  PerfectPhaseTransferFunction → InterpolatingFunction[{{-434.841, 433.991}}, <>],
  PerfectPointSpreadFunction → InterpolatingFunction[{{-0.588146, 0.589297}}, <>],
  PerfectPointSpreadPhaseFunction → InterpolatingFunction[{{-0.588146, 0.589297}}, <>],
  PhaseTransferFunction → InterpolatingFunction[{{-434.841, 433.991}}, <>],
  Plot2D → True, PlotPoints → 64,
  PointSpreadFunction → InterpolatingFunction[{{-0.588146, 0.589297}}, <>],
  PointSpreadPhaseFunction → InterpolatingFunction[{{-0.588146, 0.589297}}, <>],
  PrinciplePoints → {{50., 0, 0}, {53.417, 0, 0}}, PrinciplePointSeparation → 3.417,
  RayBoundary → {-22.674, 22.674}, RefractiveIndex → 1.00027, RenderedParameters →
   {ModulationTransferFunction, PointSpreadFunction, OpticalPathDifference},
  ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
  SampleFactor → 2, SeidelAberrations → True, ShowPerfectCase → True,
  SignalPlotCutoff → 0.01, SmoothKernelRange → 3, SmoothKernelSize → 1.25,
  SourceID → 2190, SpatialScale → 1, StrehlRatio → 0.00915411, SurfaceNumber → 1,
  SystemABCDMatrix → {{0.0335317, 98.2596}, {-0.0100066, 0.49967}},
  TransverseMagnification → 0.0335317, WaveFrontID → 1,
  WaveFrontIntensity → InterpolatingFunction[{{-22.7586, 22.7556}}, <>],
  WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

```
In[76]:= ModulationTransferFunction[sys3D, FindImagePoint->True, FocalFraction->0,
  NumberOfPoints->128]//Timing
```


Wave Front Intensity
0.32
0.3
0.28
0.26
0.24
0.22
-7.5 -5 -2.5 0 2.5 5 7.5


Optical Path Difference (wavelengths)
0
-1
-2
-3
-4
-5
-7.5 -5 -2.5 0 2.5 5 7.5


Point Spread Intensity Function
1
0.8
0.6
0.4
0.2
-0.06 -0.04 -0.02 0 0.02 0.04 0.06

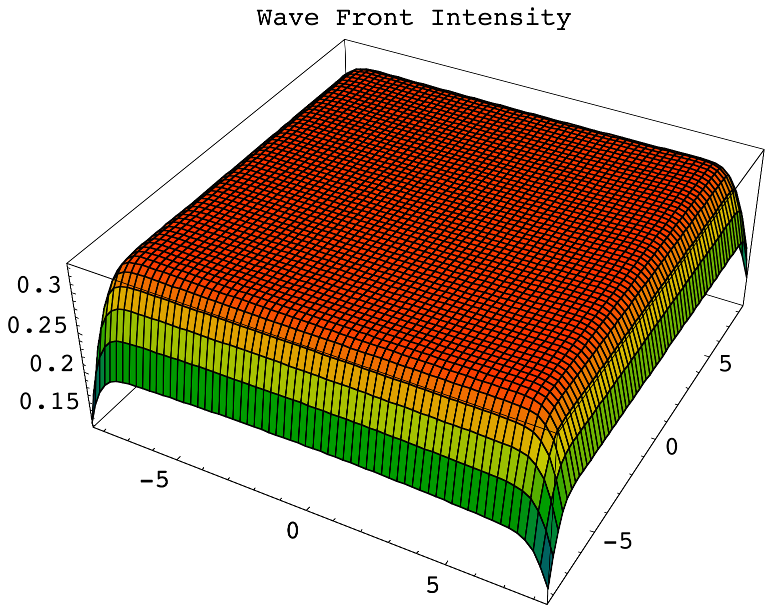


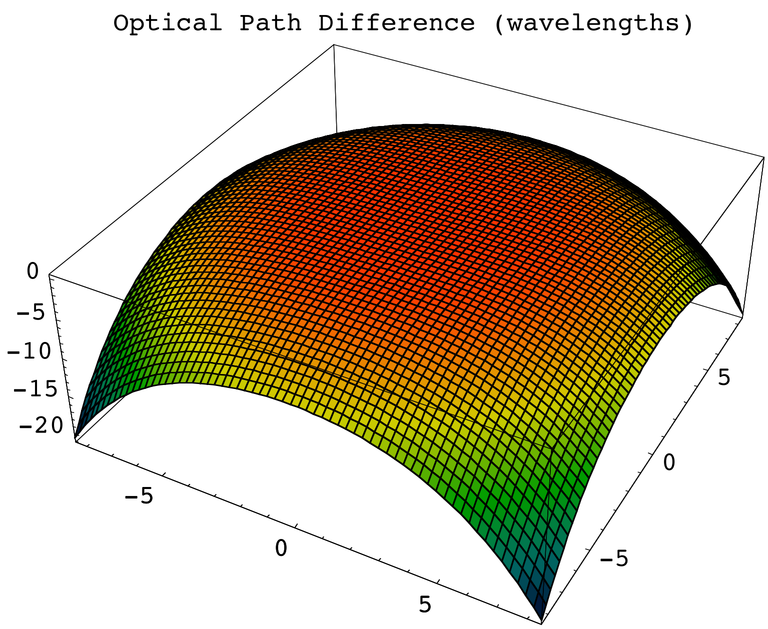

```
Out[76]= {40.38 Second, {AngularMagnification → -0.93012,
   BackFocalLength → 43.791, BackFocalPoint → {153.291, 0, 0},
   ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &), ComponentNumber → 5,
   CosineCompensation → True, DiffractionSpotSize → {0.126195, 0.126247}, Energy → 100.,
   EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
   EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
   EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
   ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
   ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
   ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
   FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
   FilterTrace → True, FindImagePoint → True, FindPupils → Automatic, FocalFraction → 0,
   FocalLength → 50.2182, FrequencyCutoff → {304.062, 303.934}, FrontFocalLength → 43.791,
   FrontFocalPoint → {46.709, 0, 0}, GeometricPointSpreadFunction → False,
   ImagePlaneTilt → {1., 0, 0}, ImagePoint → {207.282, 0, 0},
   ImageSampleSize → {0.00166046, 0.00166114}, ImagingOptics →
    {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
     Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
     Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
   IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
   InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
   LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
   ModulationTransferFunction → InterpolatingFunction[
     {{-153.228, 152.031}, {-153.164, 151.967}}, <>], NormalizePlot → True,
   NumberOfPoints → 128, ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5],
   Offset → 112.446, OpticalLengthFunction → OpticalLengthFunction,
   OpticalMedium → Air, OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0,
      0.0000222711, -6.85363 - 3.53211×10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
       1.95153×10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 10.3109 (-1 + 0.0139904 (#1^2 + #2^2)) -
       5.2511×10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209856 (#1^2 + #2^2) + 0.000195732 (#1^2 + #2^2)^2) -
       3.47812 (1 - 0.0419713 (#1^2 + #2^2) + 0.000293598 (#1^2 + #2^2)^2) + 4.27139×10^-6
        Cos[4 ArcTan[#1, #2]] (-0.000244665 (#1^2 + #2^2)^2 + 2.05378×10^-6 (#1^2 + #2^2)^3) -
       3.39626×10^-7 Cos[2 ArcTan[#1, #2]] (0.0419713 (#1^2 + #2^2) -
          0.00097866 (#1^2 + #2^2)^2 + 5.13445×10^-6 (#1^2 + #2^2)^3) - 0.0210822
        (-1 + 0.0839425 (#1^2 + #2^2) - 0.00146799 (#1^2 + #2^2)^2 + 6.84593×10^-6 (#1^2 + #2^2)^3) -
       7.3801×10^-8 Cos[6 ArcTan[#1, #2]] (-2.39608×10^-6 (#1^2 + #2^2)^3 +
```

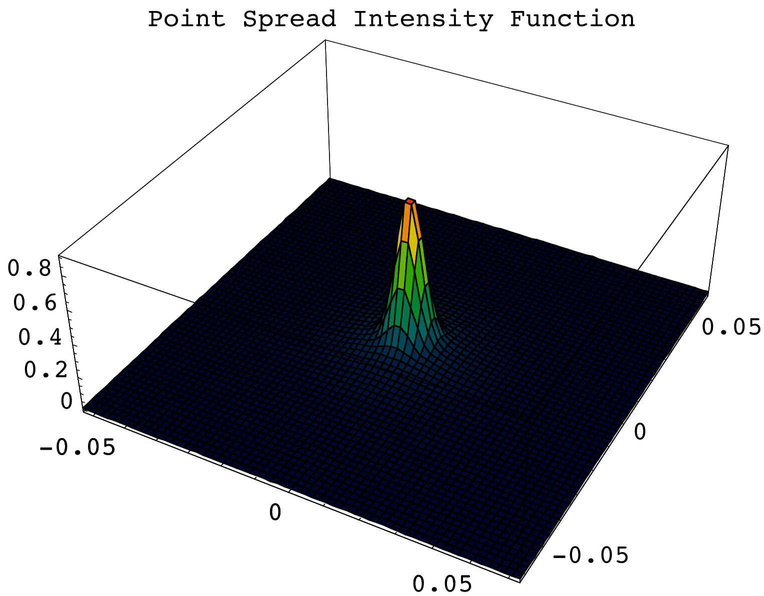
```
        1.91555 × 10^-8 (#1^2 + #2^2)^4) + 3.18005 × 10^-6 Cos[4 ArcTan[#1, #2]]
      (0.000733995 (#1^2 + #2^2)^2 - 0.0000143765 (#1^2 + #2^2)^3 + 6.70443 × 10^-8 (#1^2 + #2^2)^4) -
      1.29139 × 10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699521 (#1^2 + #2^2) +
        0.00293598 (#1^2 + #2^2)^2 - 0.0000359412 (#1^2 + #2^2)^3 + 1.34089 × 10^-7 (#1^2 + #2^2)^4) -
      0.000207406 (1 - 0.139904 (#1^2 + #2^2) + 0.00440397 (#1^2 + #2^2)^2 -
        0.0000479215 (#1^2 + #2^2)^3 + 1.67611 × 10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
  OpticalPathRange → {112.435, 112.446}, OutputGraphics →
   {WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -),
    PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
  PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
  PerfectModulationTransferFunction →
   InterpolatingFunction[{{-153.228, 152.031}, {-153.164, 151.967}}, <>],
  PerfectPhaseTransferFunction →
   InterpolatingFunction[{{-153.228, 152.031}, {-153.164, 151.967}}, <>],
  PerfectPointSpreadFunction → InterpolatingFunction[
    {{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>], PhaseTransferFunction →
   InterpolatingFunction[{{-153.228, 152.031}, {-153.164, 151.967}}, <>],
  Plot2D → True, PlotPoints → 64, PointSpreadFunction →
   InterpolatingFunction[{{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>],
  PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
  PrinciplePointSeparation → 6.14556,
  RayBoundary → {{-8.45622, 8.45622}, {-8.45265, 8.45265}},
  RefractiveIndex → 1.00027, RenderedParameters →
   {ModulationTransferFunction, PointSpreadFunction, OpticalPathDifference},
  ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
  ResidualFitError → 7.31386 × 10^-7, SampleFactor → 2,
  SeidelAberrations → {SphericalAberration → 82.9255, Astigmatism → 0.0000417324,
    FieldCurvature → -0.246833, Distortion → -3.77796 × 10^-13, Coma → 1.34603 × 10^-12},
  ShowPerfectCase → True, SignalPlotCutoff → 0.01, SmoothKernelRange → 3,
  SmoothKernelSize → 1.25, SourceID → 584, SpatialScale → 1,
  StrehlRatio → 0.0156867, SurfaceNumber → 1,
  SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
  TransverseMagnification → -1.08942,
  WaveFrontID → 1, WaveFrontIntensity →
   InterpolatingFunction[{{-11.3473, 11.3473}, {-11.3437, 11.3437}}, <>],
  WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}}
```

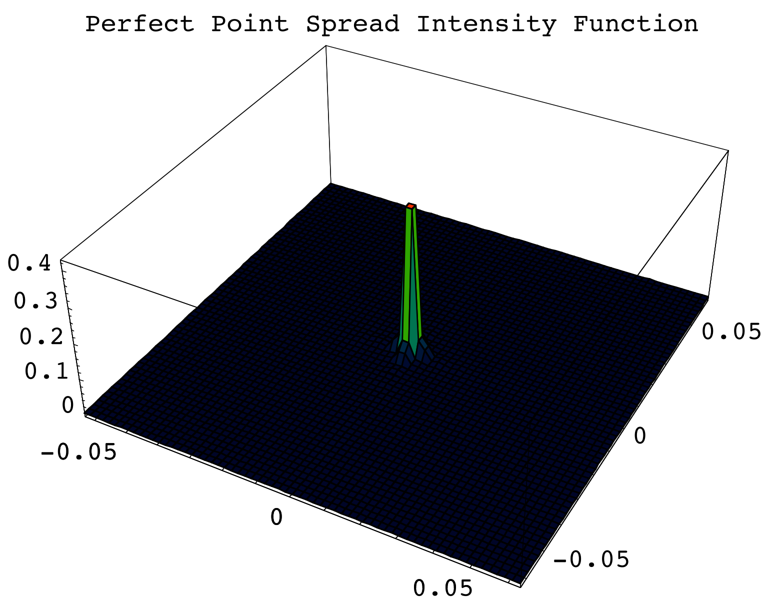
In[77]:=

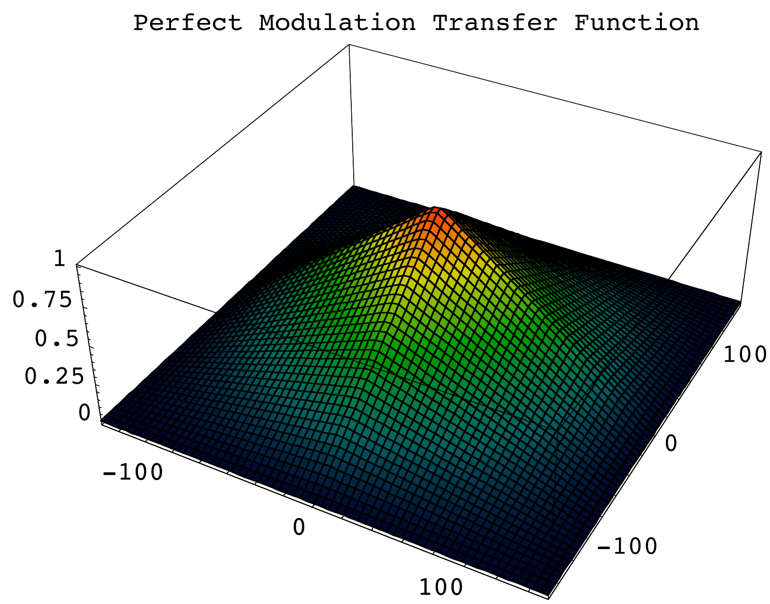
```
MTF[sys3D, FindImagePoint->True, FocalFraction->0, NumberOfPoints->128,
  Plot2D->False]
```

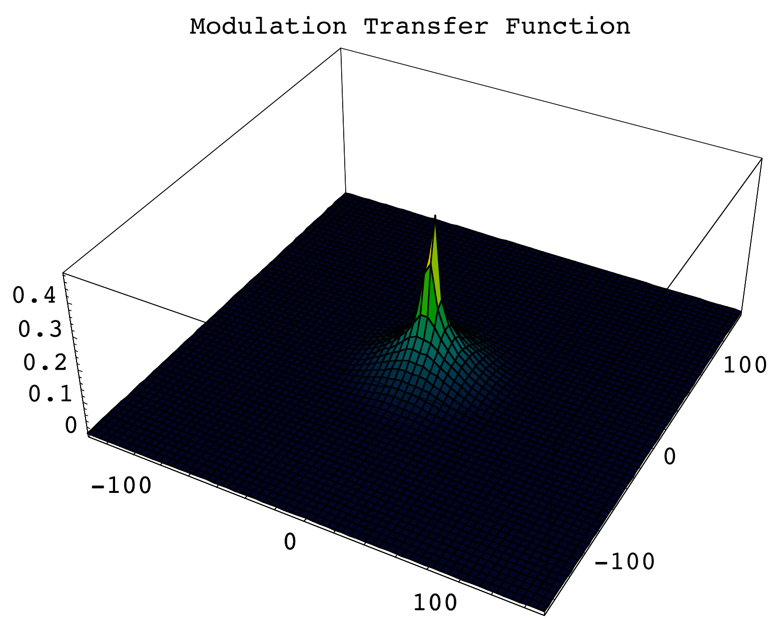

Wave Front Intensity
0.3
0.25
0.2
0.15
-5
0
5
5
0
-5


Optical Path Difference (wavelengths)
0
-5
-10
-15
-20
-5
0
5
5
0
-5


Point Spread Intensity Function
0.8
0.6
0.4
0.2
0
-0.05
0
0.05
0.05
0
-0.05


Perfect Point Spread Intensity Function
0.4
0.3
0.2
0.1
0
-0.05
0
0.05
0.05
0
-0.05



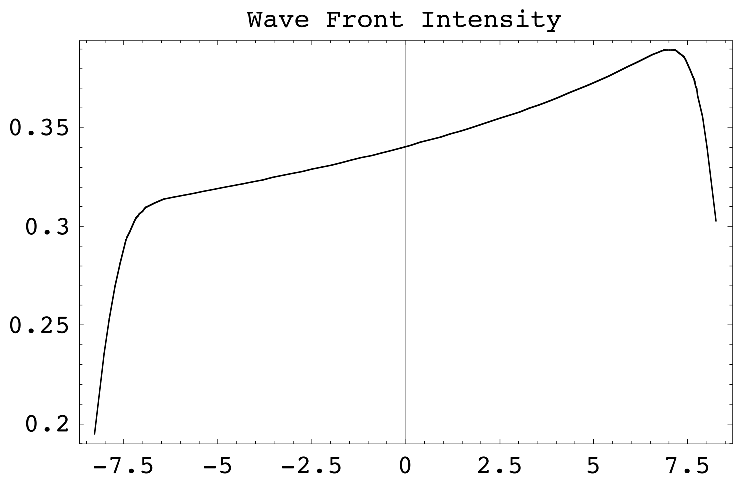


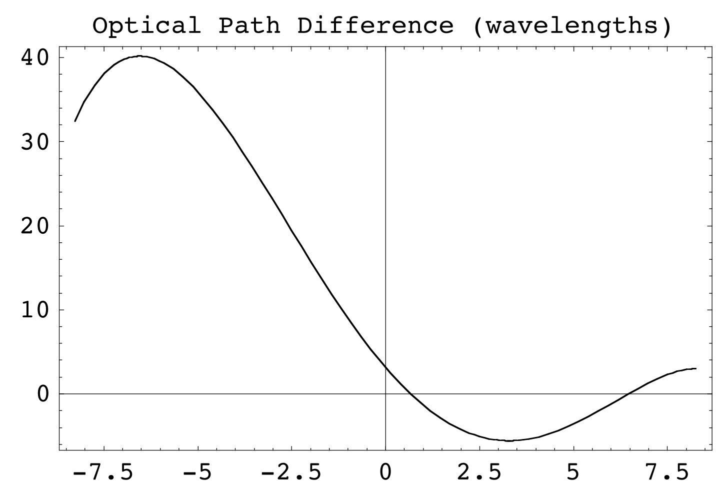

```
Out[77]= {AngularMagnification → -0.93012, BackFocalLength → 43.791,
  BackFocalPoint → {153.291, 0, 0}, ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &),
  ComponentNumber → 5, CosineCompensation → True,
  DiffractionSpotSize → {0.126195, 0.126247}, Energy → 100.,
  EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
  EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
  EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
  ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
  ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
  FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
  FilterTrace → True, FindImagePoint → True, FindPupils → Automatic,
  FocalFraction → 0, FocalLength → 50.2182, FrequencyCutoff → {304.062, 303.934},
  FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
  GeometricPointSpreadFunction → False, ImagePlaneTilt → {1., 0, 0},
```

```
ImagePoint → {207.282, 0, 0}, ImageSampleSize → {0.00166046, 0.00166114}, ImagingOptics →
 {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
  Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
  Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
ModulationTransferFunction →
 InterpolatingFunction[{{-153.228, 152.031}, {-153.164, 151.967}}, <>],
NormalizePlot → True, NumberOfPoints → 128,
ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5], Offset → 112.446,
OpticalLengthFunction → OpticalLengthFunction, OpticalMedium → Air,
OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0,
   0.0000222711, -6.85363 - 3.53211×10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
    1.95153×10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 10.3109 (-1 + 0.0139904 (#1^2 + #2^2)) -
    5.2511×10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209856 (#1^2 + #2^2) + 0.000195732 (#1^2 + #2^2)^2) -
    3.47812 (1 - 0.0419713 (#1^2 + #2^2) + 0.000293598 (#1^2 + #2^2)^2) + 4.27139×10^-6
     Cos[4 ArcTan[#1, #2]] (-0.000244665 (#1^2 + #2^2)^2 + 2.05378×10^-6 (#1^2 + #2^2)^3) -
    3.39626×10^-7 Cos[2 ArcTan[#1, #2]]
     (0.0419713 (#1^2 + #2^2) - 0.00097866 (#1^2 + #2^2)^2 + 5.13445×10^-6 (#1^2 + #2^2)^3) - 0.0210822
     (-1 + 0.0839425 (#1^2 + #2^2) - 0.00146799 (#1^2 + #2^2)^2 + 6.84593×10^-6 (#1^2 + #2^2)^3) -
    7.3801×10^-8 Cos[6 ArcTan[#1, #2]] (-2.39608×10^-6 (#1^2 + #2^2)^3 +
       1.91555×10^-8 (#1^2 + #2^2)^4) + 3.18005×10^-6 Cos[4 ArcTan[#1, #2]]
     (0.000733995 (#1^2 + #2^2)^2 - 0.0000143765 (#1^2 + #2^2)^3 + 6.70443×10^-8 (#1^2 + #2^2)^4) -
    1.29139×10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699521 (#1^2 + #2^2) +
       0.00293598 (#1^2 + #2^2)^2 - 0.0000359412 (#1^2 + #2^2)^3 + 1.34089×10^-7 (#1^2 + #2^2)^4) -
    0.000207406 (1 - 0.139904 (#1^2 + #2^2) + 0.00440397 (#1^2 + #2^2)^2 -
       0.0000479215 (#1^2 + #2^2)^3 + 1.67611×10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OpticalPathRange → {112.435, 112.446}, OutputGraphics →
 {WaveFrontIntensity → (- SurfaceGraphics -),
  OpticalPathDifference → (- SurfaceGraphics -), PointSpreadFunction →
   (- SurfaceGraphics -), PerfectPointSpreadFunction → (- SurfaceGraphics -),
  PerfectModulationTransferFunction → (- SurfaceGraphics -),
  ModulationTransferFunction → (- SurfaceGraphics -)},
PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
PerfectModulationTransferFunction →
 InterpolatingFunction[{{-153.228, 152.031}, {-153.164, 151.967}}, <>],
PerfectPhaseTransferFunction →
 InterpolatingFunction[{{-153.228, 152.031}, {-153.164, 151.967}}, <>],
PerfectPointSpreadFunction → InterpolatingFunction[
  {{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>], PhaseTransferFunction →
 InterpolatingFunction[{{-153.228, 152.031}, {-153.164, 151.967}}, <>],
Plot2D → False, PlotPoints → 64, PointSpreadFunction →
 InterpolatingFunction[{{-0.104609, 0.106269}, {-0.104652, 0.106313}}, <>],
PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
PrinciplePointSeparation → 6.14556,
RayBoundary → {{-8.45622, 8.45622}, {-8.45265, 8.45265}},
RefractiveIndex → 1.00027,
RenderedParameters →
 {ModulationTransferFunction, PointSpreadFunction, OpticalPathDifference},
ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
```

```
ResidualFitError → 7.31386 × 10^-7, SampleFactor → 2,
SeidelAberrations → {SphericalAberration → 82.9255, Astigmatism → 0.0000417324,
  FieldCurvature → -0.246833, Distortion → -3.77796 × 10^-13, Coma → 1.34603 × 10^-12},
ShowPerfectCase → True, SignalPlotCutoff → 0.01, SmoothKernelRange → 3,
SmoothKernelSize → 1.25,
SourceID → 584, SpatialScale → 1,
StrehlRatio → 0.0156867, SurfaceNumber → 1,
SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
TransverseMagnification → -1.08942,
WaveFrontID → 1, WaveFrontIntensity →
 InterpolatingFunction[{{-11.3473, 11.3473}, {-11.3437, 11.3437}}, <>],
WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```

In[81]:= **OpticalTransferFunction[offaxis3D, FindImagePoint->True, NumberOfPoints->512]**

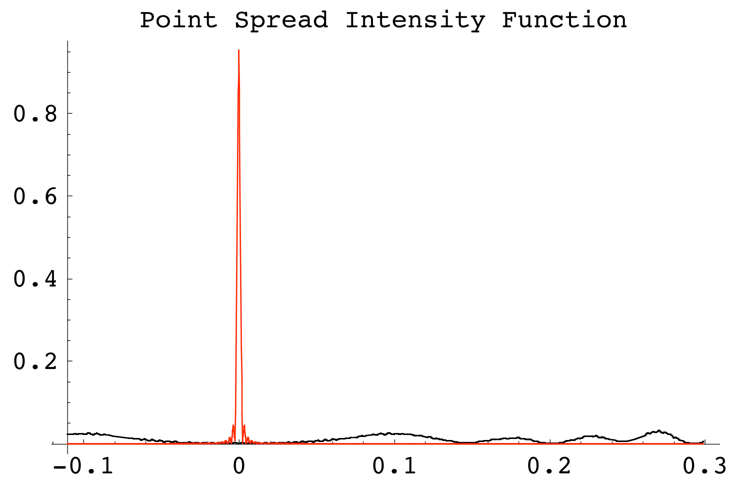


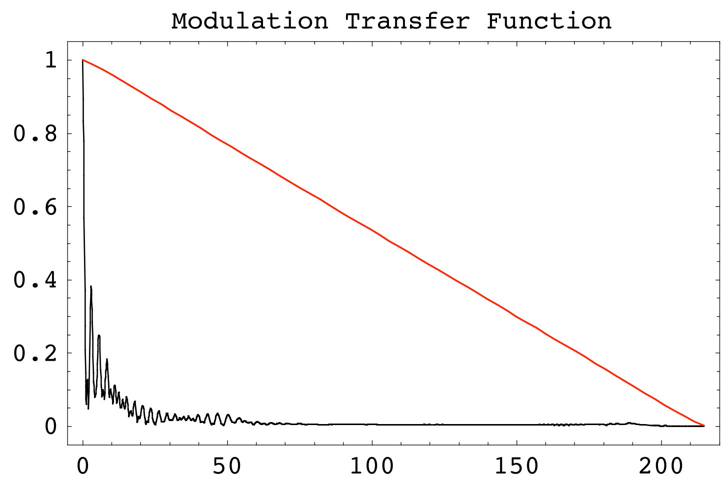



```
Warning: too few sample points.

Need to increase the NumberOfPoints option.

Suggestion: NumberOfPoints -> 1024.
```

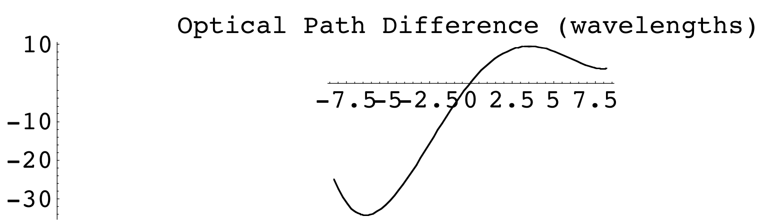




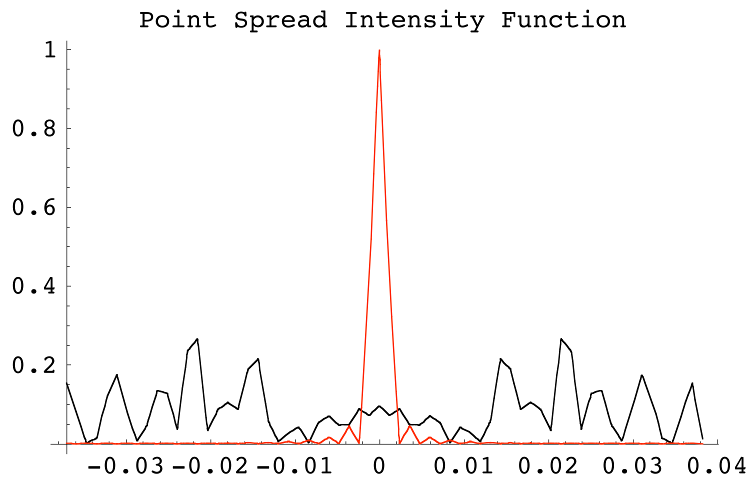

```
Out[81]= {AngularMagnification → -1.24643,
  BackFocalLength → 39.485, BackFocalPoint → {57.8471, -14.8154, 0},
  ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &), ComponentNumber → 3,
  CosineCompensation → True, DiffractionSpotSize → {0.40933, 0.598784},
  Energy → 100., EntrancePupilBoundary → {-57.4255, -57.4255},
  EntrancePupilDistance → -14.8595, EntrancePupilOffset → {0, 0},
  EntrancePupilPosition → {14.5372, -3.07794, 0}, EntrancePupilRotationMatrix →
   {{-0.978312, 0.207137, 0}, {-0.207137, -0.978312, 0}, {0, 0, 1}},
  ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → 0, ExitPupilOffset → {0, 0},
  ExitPupilPosition → {19.817, -4.19583, 0}, ExitPupilRotationMatrix →
   {{0.963153, -0.268952, 0}, {0.268952, 0.963153, 0}, {0, 0, 1}}, FieldStopPosition → 2,
  FieldStopSurface → {IntersectionNumber → 2, SurfaceNumber → 2, ComponentNumber → 1},
  FilterTrace → True, FindImagePoint → True, FindPupils → Automatic,
  FocalFraction → Automatic, FocalLength → 45.7974,
  FrequencyCutoff → {429.595, 435.19}, FrontFocalLength → 38.2594,
  FrontFocalPoint → {-36.3161, 12.0385, 0}, GeometricPointSpreadFunction → False,
  ImagePlaneTilt → {0.963153, -0.268952, 0}, ImagePoint → {89.1021, -23.3597, 0},
  ImageSampleSize → {0.00118646, 0.00117179}, ImagingOptics → {BiConvexLens[50, 50, 20],
    Boundary[{0, -54, -54}, {108, 54, 54}, {GraphicDesign → Off}]},
  IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
  InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
  LensABCDMatrix → {{0.862168, 10.2908}, {-0.0218353, 0.899242}},
  ModulationTransferFunction →
   InterpolatingFunction[{{-215.218, 214.798}, {-218.021, 217.595}}, <>],
  NormalizePlot → True, NumberOfPoints → 512,
```

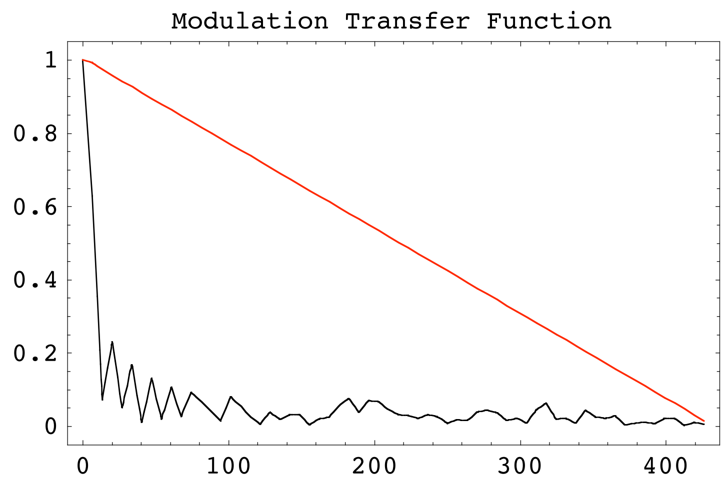
```
ObjectSource → Move[PointOfRays[{10, 10}], {-90.5, 30.}, -18.3399], Offset → 126.147,
OpticalLengthFunction → OpticalLengthFunction, OpticalMedium → Air,
OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0, 3.14985,
   61.2636 - 0.599501 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] - 0.827998 Cos[2 ArcTan[#1, #2]]
     (#1^2 + #2^2) - 0.000578093 Cos[3 ArcTan[#1, #2]] (#1^2 + #2^2)^(3/2) - 2.78832 × 10^-6
     Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 - 6.16695 × 10^-8 Cos[5 ArcTan[#1, #2]] (#1^2 + #2^2)^(5/2) +
    5.50593 × 10^-10 Cos[6 ArcTan[#1, #2]] (#1^2 + #2^2)^3 + 35.7692 (-1 + 0.0144338 (#1^2 + #2^2)) +
    26.6123 Cos[ArcTan[#1, #2]] (-0.169905 Sqrt[#1^2 + #2^2] + 0.00183928 (#1^2 + #2^2)^(3/2)) -
    3.6395 Cos[2 ArcTan[#1, #2]] (-0.0216507 (#1^2 + #2^2) + 0.000208335 (#1^2 + #2^2)^2) -
    23.3707 (1 - 0.0433014 (#1^2 + #2^2) + 0.000312502 (#1^2 + #2^2)^2) - 0.223812
     Cos[3 ArcTan[#1, #2]] (-0.00245237 (#1^2 + #2^2)^(3/2) + 0.0000221232 (#1^2 + #2^2)^(5/2)) -
    0.600685 Cos[ArcTan[#1, #2]] (0.254857 Sqrt[#1^2 + #2^2] - 0.00735712 (#1^2 + #2^2)^(3/2) +
       0.0000442464 (#1^2 + #2^2)^(5/2)) - 0.00313181 Cos[4 ArcTan[#1, #2]]
     (-0.000260419 (#1^2 + #2^2)^2 + 2.2553 × 10^-6 (#1^2 + #2^2)^3) - 0.205359 Cos[2 ArcTan[#1, #2]]
     (0.0433014 (#1^2 + #2^2) - 0.00104167 (#1^2 + #2^2)^2 + 5.63825 × 10^-6 (#1^2 + #2^2)^3) -
    1.08666 (-1 + 0.0866029 (#1^2 + #2^2) - 0.00156251 (#1^2 + #2^2)^2 + 7.51767 × 10^-6 (#1^2 + #2^2)^3) +
    0.00223709 Cos[5 ArcTan[#1, #2]] (-0.0000265478 (#1^2 + #2^2)^(5/2) +
       2.23525 × 10^-7 (#1^2 + #2^2)^(7/2)) - 0.015043 Cos[3 ArcTan[#1, #2]]
     (0.00613093 (#1^2 + #2^2)^(3/2) - 0.000132739 (#1^2 + #2^2)^(5/2) + 6.70576 × 10^-7 (#1^2 + #2^2)^(7/2)) -
    0.10943 Cos[ArcTan[#1, #2]] (-0.339809 Sqrt[#1^2 + #2^2] + 0.0183928 (#1^2 + #2^2)^(3/2) -
       0.000265478 (#1^2 + #2^2)^(5/2) + 1.11763 × 10^-6 (#1^2 + #2^2)^(7/2)) + 0.000850661
     Cos[6 ArcTan[#1, #2]] (-2.63118 × 10^-6 (#1^2 + #2^2)^3 + 2.17017 × 10^-8 (#1^2 + #2^2)^4) +
    0.00324143 Cos[4 ArcTan[#1, #2]] (0.000781256 (#1^2 + #2^2)^2 - 0.0000157871 (#1^2 + #2^2)^3 +
       7.5956 × 10^-8 (#1^2 + #2^2)^4) - 0.0148225 Cos[2 ArcTan[#1, #2]] (-0.0721691 (#1^2 + #2^2) +
       0.00312502 (#1^2 + #2^2)^2 - 0.0000394678 (#1^2 + #2^2)^3 + 1.51912 × 10^-7 (#1^2 + #2^2)^4) -
    0.0604679 (1 - 0.144338 (#1^2 + #2^2) + 0.00468754 (#1^2 + #2^2)^2 -
       0.0000526237 (#1^2 + #2^2)^3 + 1.8989 × 10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OpticalPathRange → {126.144, 126.216}, OutputGraphics →
 {WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -),
  PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
PerfectModulationTransferFunction →
 InterpolatingFunction[{{-215.218, 214.798}, {-218.021, 217.595}}, <>],
PerfectPhaseTransferFunction →
 InterpolatingFunction[{{-215.218, 214.798}, {-218.021, 217.595}}, <>],
PerfectPointSpreadFunction →
 InterpolatingFunction[{{-0.302548, 0.303735}, {-0.298806, 0.299978}}, <>],
PhaseTransferFunction → InterpolatingFunction[
  {{-215.218, 214.798}, {-218.021, 217.595}}, <>],
Plot2D → True, PlotPoints → 64, PointSpreadFunction →
 InterpolatingFunction[{{-0.302548, 0.303735}, {-0.298806, 0.299978}}, <>],
PrinciplePoints → {{7.15509, -2.37185, 0}, {13.7372, -2.49811, 0}},
PrinciplePointSeparation → 6.58337,
RayBoundary → {{-8.26882, 8.26882}, {-8.37796, 8.37796}},
```

```
RefractiveIndex → 1.00027,
RenderedParameters →
 {ModulationTransferFunction, PointSpreadFunction, OpticalPathDifference},
ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
ResidualFitError → 0.00161023,
SampleFactor → 2,
SeidelAberrations → {SphericalAberration → 591.972, Astigmatism → 220.653,
  FieldCurvature → -661.599, Distortion → -61.0418, Coma → 81.7115},
ShowPerfectCase → True, SignalPlotCutoff → 0.01,
SmoothKernelRange → 3,
SmoothKernelSize → 1.25,
SourceID → 1351,
SpatialScale → 1,
StrehlRatio → 0.000390164,
SurfaceNumber → 1,
SystemABCDMatrix → {{-1.137, -19.1063}, {-0.0218353, -1.24643}},
TransverseMagnification → -1.137,
WaveFrontID → 1,
WaveFrontIntensity →
 InterpolatingFunction[{{-11.152, 11.152}, {-11.2611, 11.2611}}, <>],
WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}
```




```
Warning: too few sample points.
Need to increase the NumberOfPoints option.
Suggestion: NumberOfPoints -> 128.
```





Out[349]=

```
{AngularMagnification → -1.24806, BackFocalLength → 39.4526,
 BackFocalPoint → {57.8163, -14.8046, 0}, DiffractionSpotSize → {0.0753291, 0.0797306},
 EntrancePupilBoundary → {-57.4299, -57.4299},
 EntrancePupilDistance → -14.8563, EntrancePupilOffset → {0, 0},
 EntrancePupilPosition → {14.5343, -3.07646, 0}, EntrancePupilRotationMatrix →
  {{-0.978324, 0.207081, 0}, {-0.207081, -0.978324, 0}, {0, 0, 1}},
 ExitPupilBoundary → {-50., -50.}, ExitPupilDistance → 0, ExitPupilOffset → {0, 0},
 ExitPupilPosition → {19.8171, -4.19468, 0}, ExitPupilRotationMatrix →
  {{0.96316, -0.268929, 0}, {0.268929, 0.96316, 0}, {0, 0, 1}}, FieldStopPosition → 2,
 FieldStopSurface → {IntersectionNumber → 2, SurfaceNumber → 2, ComponentNumber → 1},
 FindImagePoint → True, FocalLength → 45.7634, FrequencyCutoff → {426.149, 401.793},
 FrontFocalLength → 38.2274, FrontFocalPoint → {-36.2857, 12.0284, 0},
 ImagePlaneTilt → {0.96316, -0.268929, 0}, ImagePoint → {89.2906, -23.4088, 0},
 ImageSampleSize → {0.0011957, 0.00126556}, ImagingOptics → {BiConvexLens[50, 50, 20],
   Boundary[{0, -54, -54}, {108, 54, 54}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{0.862099, 10.2862}, {-0.0218515, 0.899236}},
 ModulationTransferFunction →
  InterpolatingFunction[{{-432.913, 426.149}, {-408.17, 401.793}}, <>],
 NumberOfRays → 32, ObjectSource → Move[PointOfRays[{10, 10}], {-90.5, 30.}, -18.3399],
 OpticalMedium → Vacuum, OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0, 0,
```

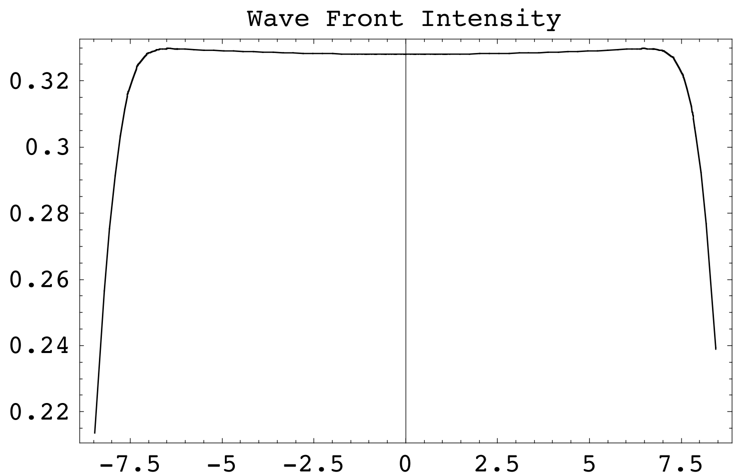
$$-53.7272 + 0.684137 \,\mathrm{Cos}[\mathrm{ArcTan}[\#1, \#2]] \sqrt{\#1^2 + \#2^2} + 0.826809 \,\mathrm{Cos}[2\, \mathrm{ArcTan}[\#1, \#2]] (\#1^2 + \#2^2) + 0.000523731 \,\mathrm{Cos}[3\, \mathrm{ArcTan}[\#1, \#2]] (\#1^2 + \#2^2)^{3/2} + 2.38038 \times 10^{-6}$$

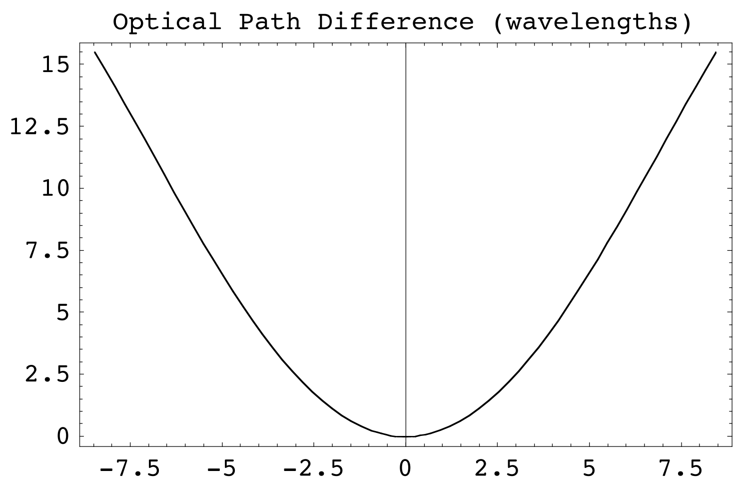
```
   Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 + 6.24216×10^-8 Cos[5 ArcTan[#1, #2]] (#1^2 + #2^2)^(5/2) -
  7.52762×10^-10 Cos[6 ArcTan[#1, #2]] (#1^2 + #2^2)^3 - 33.3179 (-1 + 0.0150516 (#1^2 + #2^2)) -
  25.7545 Cos[ArcTan[#1, #2]] (-0.173503 Sqrt[#1^2 + #2^2] + 0.00195862 (#1^2 + #2^2)^(3/2)) +
  3.28743 Cos[2 ArcTan[#1, #2]] (-0.0225774 (#1^2 + #2^2) + 0.00022655 (#1^2 + #2^2)^2) + 21.2981
   (1 - 0.0451547 (#1^2 + #2^2) + 0.000339825 (#1^2 + #2^2)^2) + 0.19316 Cos[3 ArcTan[#1, #2]]
   (-0.00261149 (#1^2 + #2^2)^(3/2) + 0.0000245669 (#1^2 + #2^2)^(5/2)) + 0.470401 Cos[ArcTan[#1, #2]]
   (0.260254 Sqrt[#1^2 + #2^2] - 0.00783446 (#1^2 + #2^2)^(3/2) + 0.0000491337 (#1^2 + #2^2)^(5/2)) +
  0.00165475 Cos[4 ArcTan[#1, #2]] (-0.000283187 (#1^2 + #2^2)^2 + 2.55745×10^-6 (#1^2 + #2^2)^3) +
  0.171863 Cos[2 ArcTan[#1, #2]]
   (0.0451547 (#1^2 + #2^2) - 0.00113275 (#1^2 + #2^2)^2 + 6.39362×10^-6 (#1^2 + #2^2)^3) +
  0.938078 (-1 + 0.0903094 (#1^2 + #2^2) - 0.00169912 (#1^2 + #2^2)^2 + 8.52483×10^-6 (#1^2 + #2^2)^3) -
  0.00160084 Cos[5 ArcTan[#1, #2]] (-0.0000294802 (#1^2 + #2^2)^(5/2) +
     2.58839×10^-7 (#1^2 + #2^2)^(7/2)) + 0.0127895 Cos[3 ArcTan[#1, #2]]
   (0.00652872 (#1^2 + #2^2)^(3/2) - 0.000147401 (#1^2 + #2^2)^(5/2) + 7.76517×10^-7 (#1^2 + #2^2)^(7/2)) +
  0.0884121 Cos[ArcTan[#1, #2]] (-0.347005 Sqrt[#1^2 + #2^2] + 0.0195862 (#1^2 + #2^2)^(3/2) -
     0.000294802 (#1^2 + #2^2)^(5/2) + 1.2942×10^-6 (#1^2 + #2^2)^(7/2)) - 0.000858399
   Cos[6 ArcTan[#1, #2]] (-2.98369×10^-6 (#1^2 + #2^2)^3 + 2.56624×10^-8 (#1^2 + #2^2)^4) -
  0.00274833 Cos[4 ArcTan[#1, #2]] (0.000849562 (#1^2 + #2^2)^2 - 0.0000179021 (#1^2 + #2^2)^3 +
     8.98185×10^-8 (#1^2 + #2^2)^4) + 0.0114937 Cos[2 ArcTan[#1, #2]] (-0.0752579 (#1^2 + #2^2) +
     0.00339825 (#1^2 + #2^2)^2 - 0.0000447554 (#1^2 + #2^2)^3 + 1.79637×10^-7 (#1^2 + #2^2)^4) +
  0.0492482 (1 - 0.150516 (#1^2 + #2^2) + 0.00509737 (#1^2 + #2^2)^2 -
     0.0000596738 (#1^2 + #2^2)^3 + 2.24546×10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OutputGraphics → {OpticalPathDifference → (- Graphics -),
  PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
PerfectModulationTransferFunction → InterpolatingFunction[
  {{-432.913, 426.149}, {-408.17, 401.793}}, <>],
PerfectPhaseTransferFunction → PerfectPhaseTransferFunction,
PerfectPointSpreadFunction →
 InterpolatingFunction[{{-0.0370667, 0.0382624}, {-0.0392325, 0.0404981}}, <>],
PhaseTransferFunction → InterpolatingFunction[
  {{-432.913, 426.149}, {-408.17, 401.793}}, <>],
PointSpreadFunction → InterpolatingFunction[
  {{-0.0370667, 0.0382624}, {-0.0392325, 0.0404981}}, <>],
PrinciplePoints → {{7.15322, -2.37123, 0}, {13.7388, -2.49751, 0}},
PrinciplePointSeparation → 6.58677,
RayBoundary → {{-8.23116, 8.21654}, {-7.74823, 7.74823}},
ResidualFitError → 0.00121112,
SeidelAberrations →
 {SphericalAberration → -486.444, Asigmatism → -211.532,
  FieldCurvature → 589.235, Distortion → 60.3225, Coma → -79.5401},
SourceID → 90485, StrehlRatio → 513.733,
SystemABCDMatrix → {{-1.13853, -19.2644}, {-0.0218515, -1.24806}},
TransverseMagnification → -1.13853,
WaveFrontID → 1,
WaveFrontIntensity →
 InterpolatingFunction[{{-8.23116, 8.21654}, {-7.74823, 7.74823}}, <>],
WaveLength → 0.532]
```

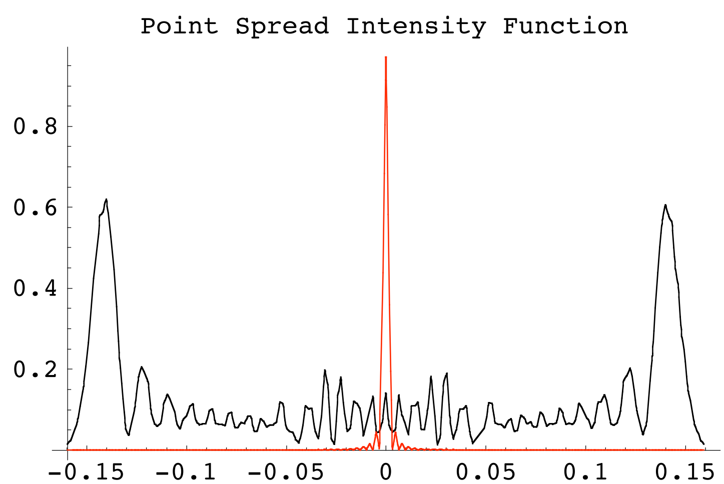
In[350]:=

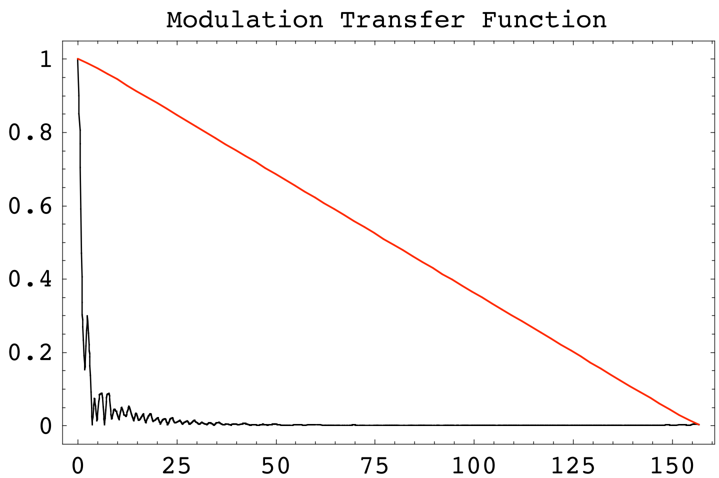
```
(*Note NumberOfPoints->256 can run out of Kernal memory sometimes*)
```

In[78]:=
```
MTF[sys3D, FindImagePoint->True, FocalFraction->1, NumberOfPoints->256]//Timing
```







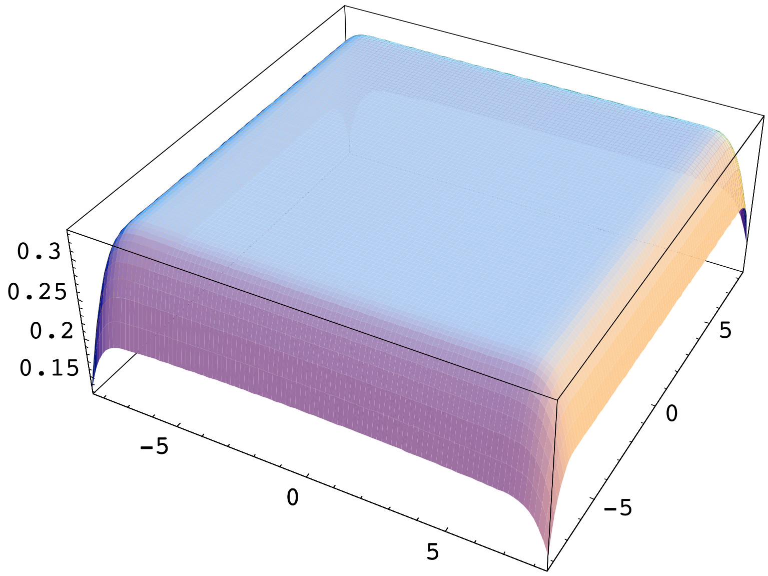



```
Out[78]= {56.07 Second, {AngularMagnification → -0.93012,
    BackFocalLength → 43.791, BackFocalPoint → {153.291, 0, 0},
    ColorFunction → (Hue[0.65 - #1 0.65, 1, #1 0.9 + 0.1] &), ComponentNumber → 5,
    CosineCompensation → True, DiffractionSpotSize → {0.318928, 0.319025}, Energy → 100.,
    EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
    EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
    EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
    ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
    ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
    ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
    FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
    FilterTrace → True, FindImagePoint → True, FindPupils → Automatic, FocalFraction → 1,
    FocalLength → 50.2182, FrequencyCutoff → {313.639, 313.541}, FrontFocalLength → 43.791,
    FrontFocalPoint → {46.709, 0, 0}, GeometricPointSpreadFunction → False,
    ImagePlaneTilt → {1., 0, 0}, ImagePoint → {204.057, 0, -0.0000575337},
    ImageSampleSize → {0.00161075, 0.00161124}, ImagingOptics →
     {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
      Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
      Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
    IntensityScale → 1, IntensitySetting → Automatic, IntensityTransform → True,
    InterceptHole → True, InterpolationOrder → 1, KernelScale → Relative,
    LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
    ModulationTransferFunction → InterpolatingFunction[
      {{-157.435, 156.82}, {-157.385, 156.771}}, <>], NormalizePlot → True,
    NumberOfPoints → 256, ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5],
    Offset → 112.446, OpticalLengthFunction → OpticalLengthFunction,
    OpticalMedium → Air, OpticalPathDifference →
     CompiledFunction[If[#1 == 0 && #2 == 0, -0.0425858, 13.7408 + 0.00107121
         Sin[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] - 3.72589×10^-9 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
        2.07395×10^-10 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 + 10.3498 (-1 + 0.0139949 (#1^2 + #2^2)) +
        2.65933×10^-7 Sin[ArcTan[#1, #2]] (-0.167302 Sqrt[#1^2 + #2^2] + 0.00175603 (#1^2 + #2^2)^(3/2)) -
        5.53568×10^-7 Cos[2 ArcTan[#1, #2]] (-0.0209924 (#1^2 + #2^2) + 0.000195858 (#1^2 + #2^2)^2) -
        3.45532 (1 - 0.0419848 (#1^2 + #2^2) + 0.000293788 (#1^2 + #2^2)^2) - 1.68437×10^-10
         Sin[3 ArcTan[#1, #2]] (-0.00234138 (#1^2 + #2^2)^(3/2) + 0.0000204797 (#1^2 + #2^2)^(5/2)) -
        2.78406×10^-7 Sin[ArcTan[#1, #2]] (0.250953 Sqrt[#1^2 + #2^2] - 0.00702414 (#1^2 + #2^2)^(3/2) +
```

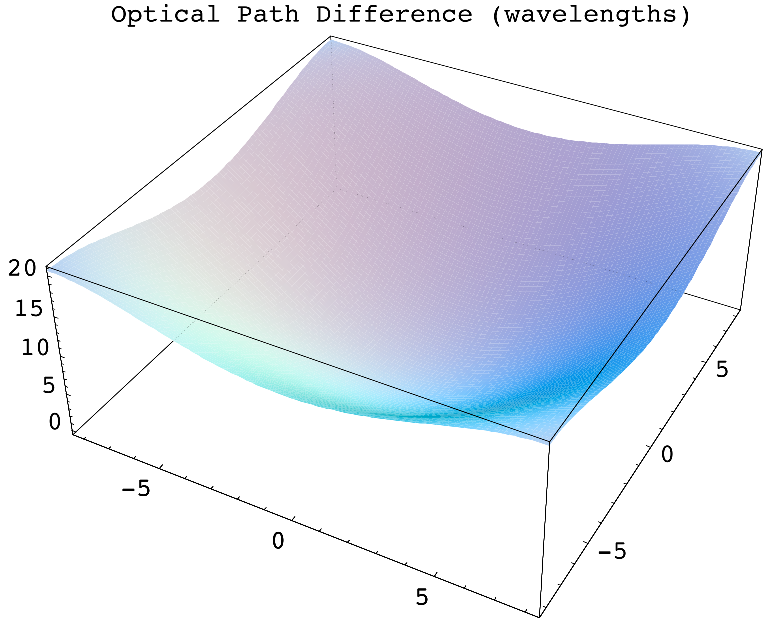
```
        0.0000409593 (#1^2 + #2^2)^(5/2)) + 4.5239×10^-6 Cos[4 ArcTan[#1, #2]] (-0.000244823
         (#1^2 + #2^2)^2 + 2.05577×10^-6 (#1^2 + #2^2)^3) - 3.58079×10^-7 Cos[2 ArcTan[#1, #2]]
      (0.0419848 (#1^2 + #2^2) - 0.000979292 (#1^2 + #2^2)^2 + 5.13943×10^-6 (#1^2 + #2^2)^3) -
      0.0219478 (-1 + 0.0839697 (#1^2 + #2^2) - 0.00146894 (#1^2 + #2^2)^2 +
        6.85257×10^-6 (#1^2 + #2^2)^3) - 1.00287×10^-10 Sin[3 ArcTan[#1, #2]]
      (0.00585345 (#1^2 + #2^2)^(3/2) - 0.000122878 (#1^2 + #2^2)^(5/2) + 6.01885×10^-7 (#1^2 + #2^2)^(7/2)) -
      4.7118×10^-9 Sin[ArcTan[#1, #2]] (-0.334604 Sqrt[#1^2 + #2^2] + 0.0175603 (#1^2 + #2^2)^(3/2) -
        0.000245756 (#1^2 + #2^2)^(5/2) + 1.00314×10^-6 (#1^2 + #2^2)^(7/2)) - 7.76697×10^-8
       Cos[6 ArcTan[#1, #2]] (-2.3984×10^-6 (#1^2 + #2^2)^3 + 1.91803×10^-8 (#1^2 + #2^2)^4) +
      3.36092×10^-6 Cos[4 ArcTan[#1, #2]]
       (0.000734469 (#1^2 + #2^2)^2 - 0.0000143904 (#1^2 + #2^2)^3 + 6.7131×10^-8 (#1^2 + #2^2)^4) -
      1.36179×10^-7 Cos[2 ArcTan[#1, #2]] (-0.0699747 (#1^2 + #2^2) +
        0.00293788 (#1^2 + #2^2)^2 - 0.000035976 (#1^2 + #2^2)^3 + 1.34262×10^-7 (#1^2 + #2^2)^4) -
      0.000220213 (1 - 0.139949 (#1^2 + #2^2) + 0.00440682 (#1^2 + #2^2)^2 -
        0.000047968 (#1^2 + #2^2)^3 + 1.67827×10^-7 (#1^2 + #2^2)^4)], -CompiledCode-],
OpticalPathRange → {112.446, 112.457}, OutputGraphics →
 {WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -),
  PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
PaddingFactor → Automatic, ParaxialReductionRatio → 0.001,
PerfectModulationTransferFunction →
 InterpolatingFunction[{{-157.435, 156.82}, {-157.385, 156.771}}, <>],
PerfectPhaseTransferFunction →
 InterpolatingFunction[{{-157.435, 156.82}, {-157.385, 156.771}}, <>],
PerfectPointSpreadFunction → InterpolatingFunction[
  {{-0.204565, 0.206176}, {-0.204627, 0.206239}}, <>], PhaseTransferFunction →
 InterpolatingFunction[{{-157.435, 156.82}, {-157.385, 156.771}}, <>],
Plot2D → True, PlotPoints → 64, PointSpreadFunction →
 InterpolatingFunction[{{-0.204565, 0.206176}, {-0.204627, 0.206239}}, <>],
PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
PrinciplePointSeparation → 6.14556,
RayBoundary → {{-8.4544, 8.4544}, {-8.45174, 8.45174}},
RefractiveIndex → 1.00027,
RenderedParameters →
 {ModulationTransferFunction, PointSpreadFunction, OpticalPathDifference},
ReportedParameters → {OpticalPathDifference, WaveFrontIntensity},
ResidualFitError → 7.71723×10^-7,
SampleFactor → 2,
SeidelAberrations → {SphericalAberration → 81.5701, Astigmatism → 0.000112137,
  FieldCurvature → -82.2047, Distortion → 9.02326×10^-13, Coma → -4.77489×10^-13},
ShowPerfectCase → True, SignalPlotCutoff → 0.01, SmoothKernelRange → 3,
SmoothKernelSize → 1.25,
SourceID → 584, SpatialScale → 1,
StrehlRatio → 0.00066111, SurfaceNumber → 1,
SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
TransverseMagnification → -1.08942,
WaveFrontID → 1, WaveFrontIntensity →
 InterpolatingFunction[{{-11.345, 11.345}, {-11.3424, 11.3424}}, <>],
WaveLength → 0.532, ZernikeFit → True, ZernikeOrder → 8}}
```

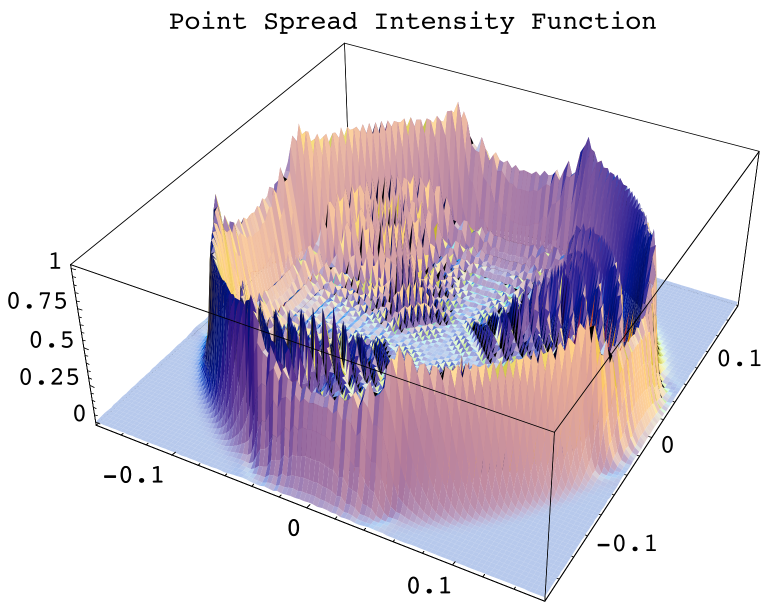
```
In[80]:= MTF[sys3D, FindImagePoint->True, FocalFraction->1, NumberOfPoints->256,
         Plot2D->False, PlotPoints->100, ColorFunction->Automatic, Mesh->False];
```

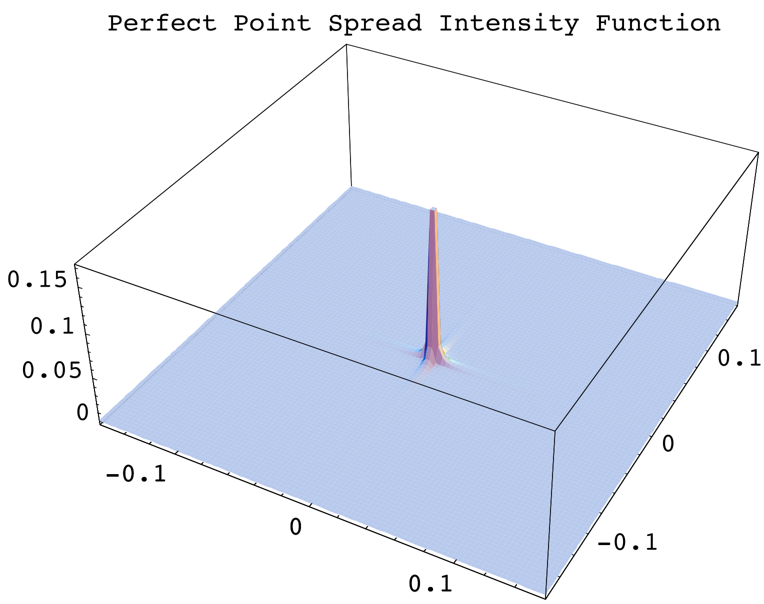
Wave Front Intensity

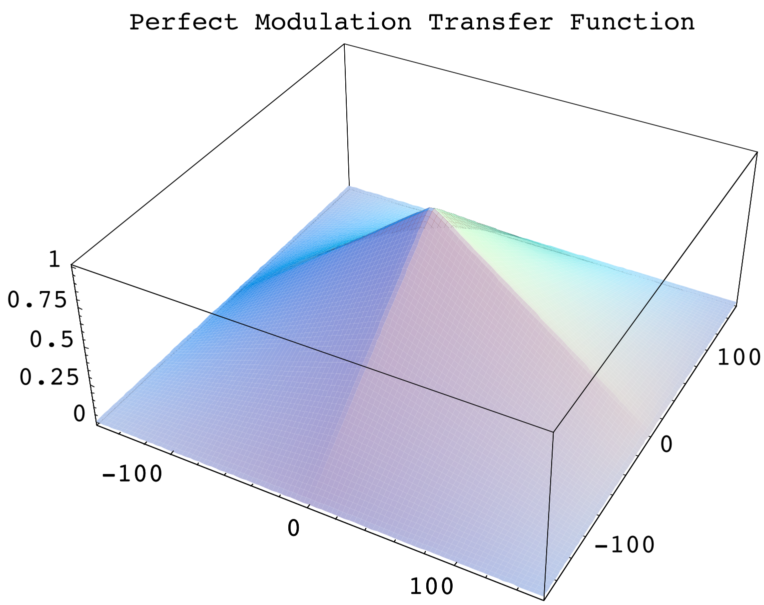


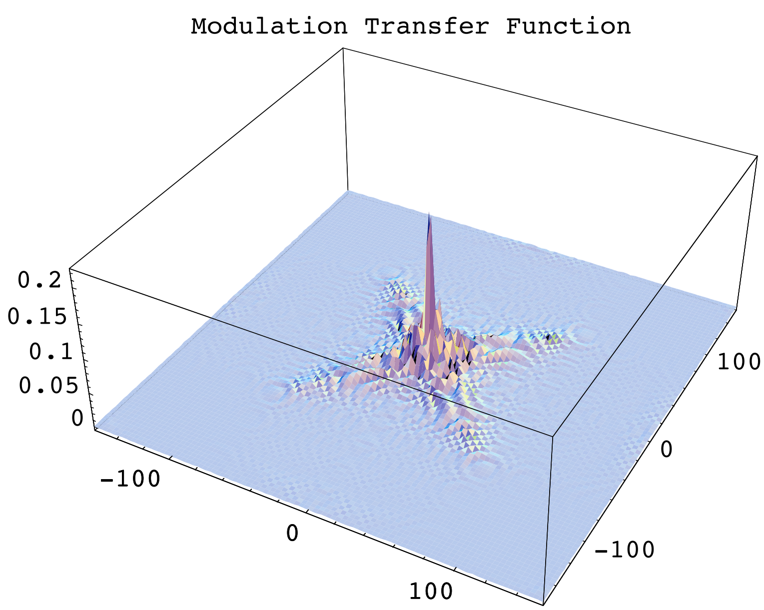
Optical Path Difference (wavelengths)




Point Spread Intensity Function
1
0.75
0.5
0.25
0
-0.1
0
0.1
0.1
0
-0.1


Perfect Point Spread Intensity Function
0.15
0.1
0.05
0
-0.1
0
0.1
0.1
0
-0.1





Go to list of topics

5.7 GeometricPointSpreadFunction

In[12]:= **?GeometricPointSpreadFunction**

```
GeometricPointSpreadFunction[system, options] calculates a histogram
  to show the geometric point spread function of an imaging system.

GeometricPointSpreadFunction works together with GeometricFocalData
  to show the geometric point spread function of an imaging system.
  GeometricPointSpreadFunction can work directly with multiple wavelength (
  white light) sources, since it works with intensity (not amplitude and
  phase). GeometricPointSpreadFunction first calls GeometricFocalData to
  find the focal plane of an imaging system and trace a quantity of rays
  to the focal plane of the system. Finally, GeometricPointSpreadFunction
  calculates a histogram of the ray intercept points and intensities to
  determine the geometric point spread function of the optical system. As
  input, GeometricPointSpreadFunction takes either an optical system or the
  returned output from GeometricFocalData. The optical system must contain
  a light source followed by the imaging optics with the approximate focal
  surface as its last element. For FindImagePoint->True, the last system
  element need not be the exact focal surface, since the exact focal surface
  position is automatically determined. When the default FindImagePoint->
  False is given, the last system element is assumed to be the exact focal
  surface. In addition, the light source need only contain a small number
  of rays (since the actual number of rays is internally specified).
  The user can also give as input a specific focal point or exit pupil
  position with the FocalPoint and ExitPupilPosition options, although
  GeometricPointSpreadFunction does not make use of the exit pupil information.

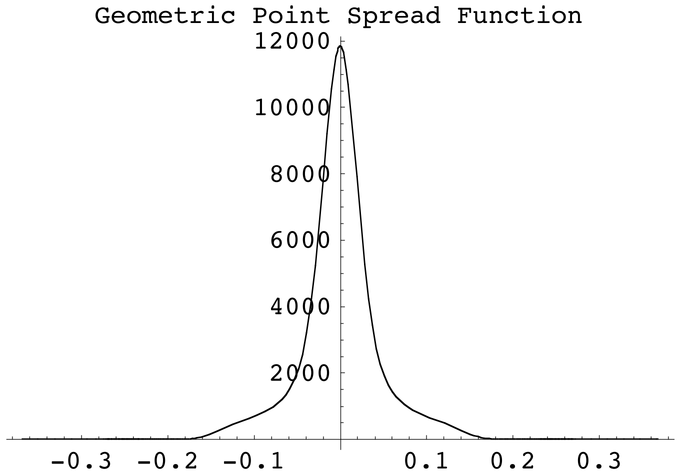
Note: GeometricPointSpreadFunction works equally well for both point sources and
  planar sources, as long as the described imaging system contains a focus.
  If a one-dimensional light source is used (ie. WedgeOfRays or LineOfRays),
  then a one-dimensional geometric point spread function is calculated. If a
  two-dimensional light source is used (ie. PointOfRays or GridOfRays), then
  the geometric point spread calculations are carried out in two-dimensions.

GeometricPointSpreadFunction does not utilize PupilFunction.
  GeometricPointSpreadFunction is also an option of OpticalTransferFunction
  and PointSpreadFunction that indicates whether the diffraction
  point spread function or the geometric point spread function is to
  be used. GeometricPointSpreadFunction->True invokes the direct use
  of PointSpreadFunction, while GeometricPointSpreadFunction->False
  invokes the initial use of GeometricPointSpreadFunction. See also:
  FindLensParameters, GeometricFocalData, GPSF and OpticalTransferFunction.
```

In[13]:= **Options[GeometricPointSpreadFunction]**

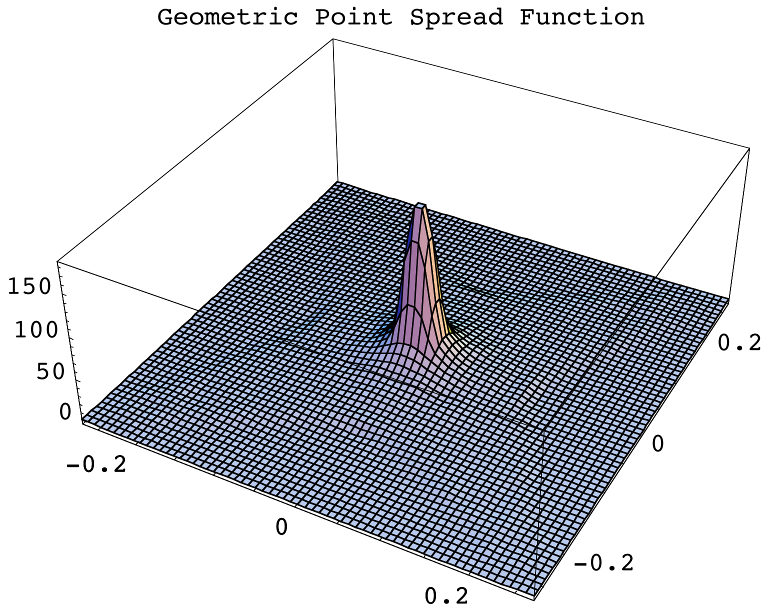
Out[13]=
```
{FindImagePoint → False, NumberOfRays → 64, GeometricCellCapacity → 2,
 FocalFraction → Automatic, FocalPoint → Automatic, FindPupils → False,
 FieldStopPosition → -2, GeometricScaleFactor → Automatic,
 GeometricMinimumCellWidth → 0.001, PlotRange → All, PlotPoints → 64, Plot2D → True,
 InterpolationOrder → 1, RenderedParameters → {PointSpreadFunction}}
```

```
In[14]:= GPSF[sys3D, FindImagePoint->True, FocalFraction->0]
```



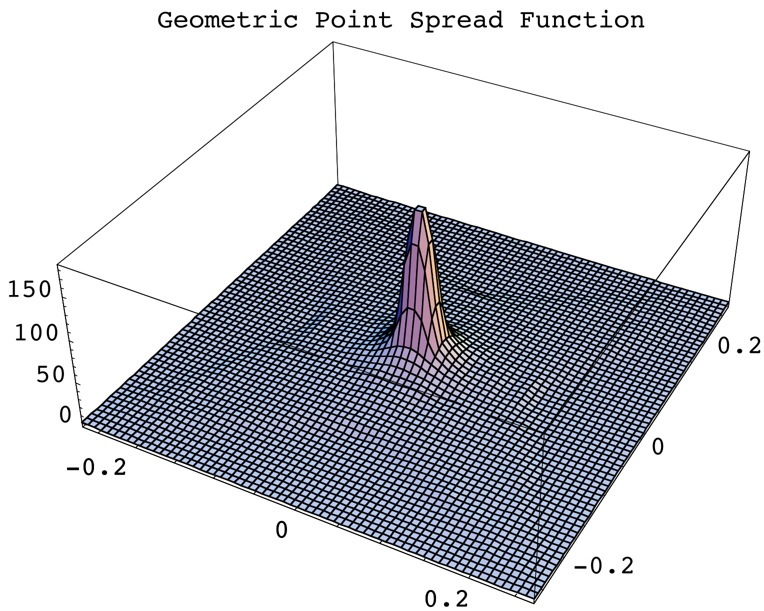

```
Out[184]=
  {AngularMagnification → -0.93012, BackFocalLength → 43.791,
   BackFocalPoint → {153.291, 0, 0}, ComponentNumber → 4., Energy → 100.,
   EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
   EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
   EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
   ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
   ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
   ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
   FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
   FindImagePoint → True, FocalFraction → 0, FocalLength → 50.2182,
   FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0}, Full3D → True,
   ImagePlaneTilt → {1., 0, 0}, ImagePoint → {207.282, 0, 0}, ImagingOptics →
    {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
     Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
     Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
   LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}}, NumberOfRays → 4096,
   ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5], OpticalMedium → Air,
   -Options-, OutputGraphics → {PointSpreadFunction → (- Graphics -)},
   PointSpreadFunction → CompiledFunction[-intensity data-],
   PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}}, PrinciplePointSeparation → 6.14556,
   RayBoundary → {{-0.271851, 0.271851}, {-0.269793, 0.269793}},
   RayTraceFunction → RayTraceFunction[{f1, f2}, -raytrace code: 7545428 Bytes- ],
   RotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, SmoothKernelSize → 0.0241913,
   SourceID → 19474, SurfaceNormalFunction → ({{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} &),
   SurfaceNumber → 1., SymbolicValues → {f1 → 100., f2 → 100.},
   SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
   TransverseMagnification → -1.08942, WaveFrontID → 1., WaveLength → 0.532}
```

```
In[15]:= GPSF[sys3D, FindImagePoint->True, FocalFraction->0, Plot2D->False]
```



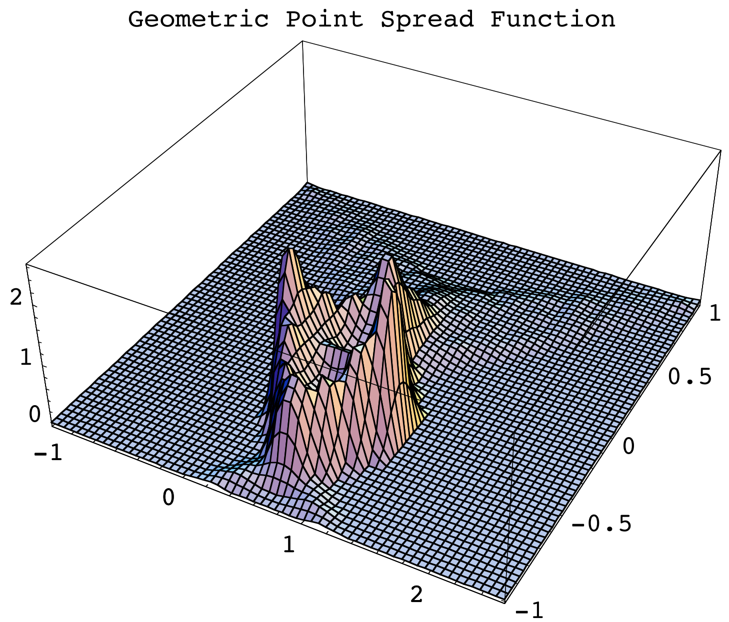

```
Out[15]= {AngularMagnification → -0.931602, BackFocalLength → 43.7532,
  BackFocalPoint → {153.253, 0, 0}, ExitPupilBoundary → {-50., -50.},
  ExitPupilDistance → 0, ExitPupilOffset → {0, 0}, ExitPupilPosition → {109.5, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 5,
  FieldStopSurface → {IntersectionNumber → 5, SurfaceNumber → 2, ComponentNumber → 3},
  FindImagePoint → True, FocalFraction → 0, FocalLength → 50.1789,
  ImagePlaneTilt → {1., 0, 0}, ImagePoint → {207.115, 0, 0}, ImagingOptics →
   {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
    Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
    Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
  LensABCDMatrix → {{0.871946, 12.0284}, {-0.0199287, 0.871946}},
  ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5], OpticalMedium → Vacuum,
  -Options-, OutputGraphics → {PointSpreadFunction → (- SurfaceGraphics -)},
  Plot2D → False, PointSpreadFunction →
   InterpolatingFunction[{{-0.270512, 0.270512}, {-0.270512, 0.270512}}, <>],
  SourceID → 518, SystemABCDMatrix → {{-1.09103, -0.823348}, {-0.0199287, -0.931602}},
  TransverseMagnification → -1.09103, WaveFrontID → 1, WaveLength → 0.532}
```

```
In[16]:= GPSF[sys3D, FindImagePoint->True, FocalFraction->0, Plot2D->False]
```



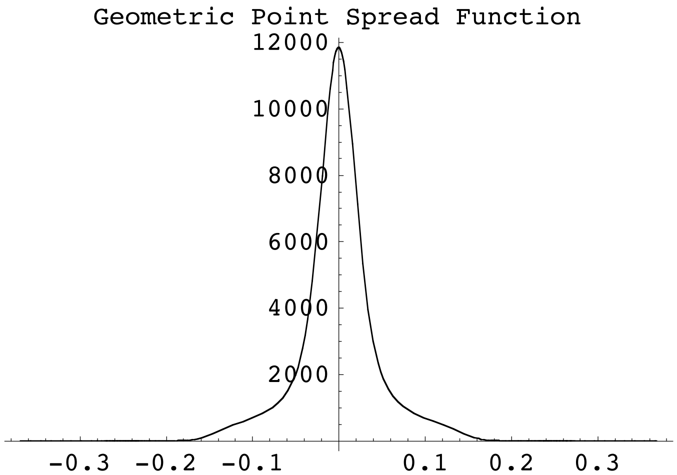

```
Out[16]= {AngularMagnification → -0.931602, BackFocalLength → 43.7532,
  BackFocalPoint → {153.253, 0, 0}, ExitPupilBoundary → {-50., -50.},
  ExitPupilDistance → 0, ExitPupilOffset → {0, 0}, ExitPupilPosition → {109.5, 0, 0},
  ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 5,
  FieldStopSurface → {IntersectionNumber → 5, SurfaceNumber → 2, ComponentNumber → 3},
  FindImagePoint → True, FocalFraction → 0, FocalLength → 50.1789,
  ImagePlaneTilt → {1., 0, 0}, ImagePoint → {207.115, 0, 0}, ImagingOptics →
   {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
    Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
    Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
  LensABCDMatrix → {{0.871946, 12.0284}, {-0.0199287, 0.871946}},
  ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5], OpticalMedium → Vacuum,
  -Options-, OutputGraphics → {PointSpreadFunction → (- SurfaceGraphics -)},
  Plot2D → False, PointSpreadFunction →
   InterpolatingFunction[{{-0.270512, 0.270512}, {-0.270512, 0.270512}}, <>],
  SourceID → 518, SystemABCDMatrix → {{-1.09103, -0.823348}, {-0.0199287, -0.931602}},
  TransverseMagnification → -1.09103, WaveFrontID → 1, WaveLength → 0.532}
```

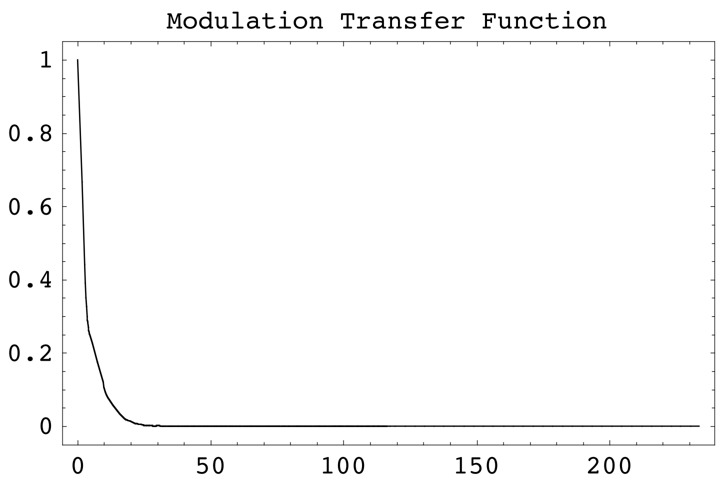
```
In[17]:= GeometricPointSpreadFunction[offaxis3D, FindImagePoint->True, FocalFraction->0,
         Plot2D->False]
```




```
Out[17]= {AngularMagnification → -1.24806, BackFocalLength → 39.4526,
          BackFocalPoint → {57.8163, -14.8046, 0}, ExitPupilBoundary → {-50., -50.},
          ExitPupilDistance → 0, ExitPupilOffset → {0, 0},
          ExitPupilPosition → {19.8171, -4.19468, 0}, ExitPupilRotationMatrix →
           {{0.96316, -0.268929, 0}, {0.268929, 0.96316, 0}, {0, 0, 1}}, FieldStopPosition → 2,
          FieldStopSurface → {IntersectionNumber → 2, SurfaceNumber → 2, ComponentNumber → 1},
          FindImagePoint → True, FocalFraction → 0, FocalLength → 45.7634,
          ImagePlaneTilt → {0.96316, -0.268929, 0}, ImagePoint → {97.2007, -25.8014, 0},
          ImagingOptics → {BiConvexLens[50, 50, 20],
            Boundary[{0, -54, -54}, {108, 54, 54}, {GraphicDesign → Off}]},
          LensABCDMatrix → {{0.862099, 10.2862}, {-0.0218515, 0.899236}},
          ObjectSource → Move[PointOfRays[{10, 10}], {-90.5, 30.}, -18.3399],
          OpticalMedium → Vacuum, -Options-,
          OutputGraphics → {PointSpreadFunction → (- SurfaceGraphics -)},
          Plot2D → False, PointSpreadFunction →
           InterpolatingFunction[{{-1.17876, 2.57075}, {-1.00931, 1.00931}}, <>],
          SourceID → 1212, SystemABCDMatrix → {{-1.13853, -19.2644}, {-0.0218515, -1.24806}},
          TransverseMagnification → -1.13853, WaveFrontID → 1, WaveLength → 0.532}
```

```
In[187]:=
    MTF[sys3D, FindImagePoint->True, FocalFraction->0, NumberOfPoints->128,
    GeometricPointSpreadFunction->True]
```

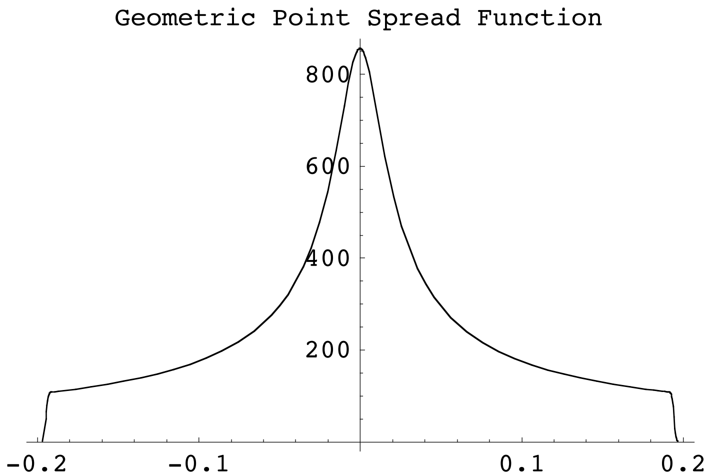

Geometric Point Spread Function
12000
10000
8000
6000
4000
2000
-0.3 -0.2 -0.1 0.1 0.2 0.3


Modulation Transfer Function
1
0.8
0.6
0.4
0.2
0
0 50 100 150 200

```
Out[187]=
   {AngularMagnification → -0.93012, BackFocalLength → 43.791,
    BackFocalPoint → {153.291, 0, 0}, ComponentNumber → 4., Energy → 100.,
    EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
    EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
    EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
    ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
    ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
    ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
    FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
    FindImagePoint → True, FocalFraction → 0, FocalLength → 50.2182,
    FrequencyCutoff → {233.584, 235.365}, FrontFocalLength → 43.791,
    FrontFocalPoint → {46.709, 0, 0}, Full3D → True, GeometricPointSpreadFunction → True,
    ImagePlaneTilt → {1., 0, 0}, ImagePoint → {207.282, 0, 0},
    ImageSampleSize → {0.00142704, 0.00141624}, ImagingOptics →
     {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
      Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
      Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
    LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}},
    ModulationTransferFunction → InterpolatingFunction[
      {{-117.252, 116.641}, {-118.146, 117.531}}, <>], NumberOfPoints → 128,
    NumberOfRays → 4096, ObjectSource → PointOfRays[{10, 10}, NumberOfRays → 5],
    OpticalMedium → Air, OutputGraphics →
     {PointSpreadFunction → (- Graphics -), ModulationTransferFunction → (- Graphics -)},
    PerfectModulationTransferFunction → PerfectModulationTransferFunction,
    PerfectPhaseTransferFunction → PerfectPhaseTransferFunction, PhaseTransferFunction →
     InterpolatingFunction[{{-117.252, 116.641}, {-118.146, 117.531}}, <>],
    PointSpreadFunction → CompiledFunction[-intensity data-],
    PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}}, PrinciplePointSeparation → 6.14556,
    RayBoundary → {{-0.271851, 0.271851}, {-0.269793, 0.269793}},
    RayTraceFunction → RayTraceFunction[{f1, f2}, -raytrace code: 7545428 Bytes- ],
    RotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, SmoothKernelSize → 0.0241913,
    SourceID → 19474, SurfaceNormalFunction → ({{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} &),
    SurfaceNumber → 1., SymbolicValues → {f1 → 100., f2 → 100.},
    SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
    TransverseMagnification → -1.08942, WaveFrontID → 1., WaveLength → 0.532}
```

In[185]:=

```
GPSF[sys, NumberOfPoints->512]
```

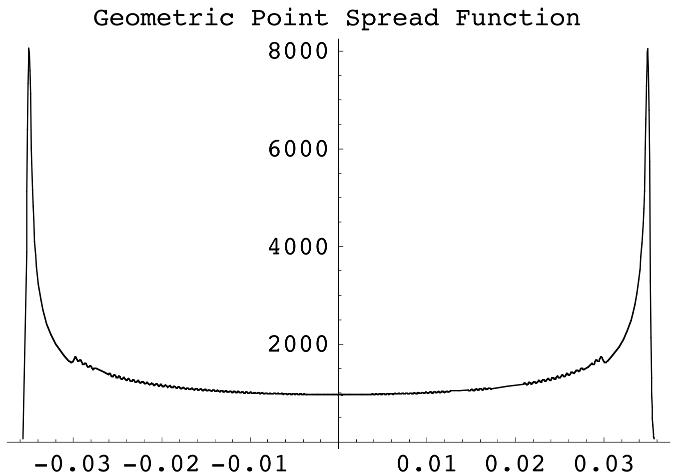


Out[185]=

```
{AngularMagnification → -0.93012, BackFocalLength → 43.791,
 BackFocalPoint → {153.291, 0, 0}, ComponentNumber → 4., Energy → 100.,
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FocalLength → 50.2182, FrontFocalLength → 43.791, FrontFocalPoint → {46.709, 0, 0},
 Full3D → False, ImagePlaneTilt → {1., 0, 0}, ImagePoint → {208., 0., 0.}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}}, NumberOfPoints → 512,
 NumberOfRays → 1064, ObjectSource → WedgeOfRays[10, NumberOfRays → 5],
 OpticalMedium → Air, -Options-, OutputGraphics → {PointSpreadFunction → (- Graphics -)},
 PointSpreadFunction → CompiledFunction[-intensity data-],
 PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
 PrinciplePointSeparation → 6.14556, RayBoundary → {-0.194042, 0.194042},
 RayTraceFunction → RayTraceFunction[{f1, f2}, -raytrace code: 2672860 Bytes- ],
 RotationMatrix → {{-1, 0, 0}, {0, -1, 0}, {0, 0, 1}}, SmoothKernelSize → 0.00139917,
 SourceID → 19920, SurfaceNormalFunction → ({{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} &),
 SurfaceNumber → 1., SymbolicValues → {f1 → 100., f2 → 100.},
 SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
 TransverseMagnification → -1.08942, WaveFrontID → 1., WaveLength → 0.532}
```

In[186]:=

```
GPSF[sys, NumberOfPoints->512, FindImagePoint->True]
```



Out[186]=

```
{AngularMagnification → -0.93012, BackFocalLength → 43.791,
 BackFocalPoint → {153.291, 0, 0}, ComponentNumber → 4., Energy → 100.,
 EntrancePupilBoundary → {-30.1509, -30.1509}, EntrancePupilDistance → -6.42722,
 EntrancePupilOffset → {0, 0}, EntrancePupilPosition → {96.9272, 0, 0},
 EntrancePupilRotationMatrix → {{-1., 0, 0}, {0, -1., 0}, {0, 0, 1}},
 ExitPupilBoundary → {-30.1509, -30.1509}, ExitPupilDistance → -6.42722,
 ExitPupilOffset → {0, 0}, ExitPupilPosition → {103.073, 0, 0},
 ExitPupilRotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, FieldStopPosition → 3,
 FieldStopSurface → {IntersectionNumber → 3, SurfaceNumber → 1, ComponentNumber → 2},
 FindImagePoint → True, FocalLength → 50.2182, FrontFocalLength → 43.791,
 FrontFocalPoint → {46.709, 0, 0}, Full3D → False,
 ImagePlaneTilt → {1., 0, 0}, ImagePoint → {206.012, 0, 0}, ImagingOptics →
  {Move[PlanoConvexLens[{f1, 100}, 50, 9, {CurvatureDirection → Back}], 90.5],
   Move[ApertureStop[50, 30], 100.], Move[PlanoConvexLens[{f2, 100}, 50, 9], 100.5],
   Boundary[{0, -104, -104}, {208, 104, 104}, {GraphicDesign → Off}]},
 LensABCDMatrix → {{0.872014, 12.0318}, {-0.0199131, 0.872014}}, NumberOfPoints → 512,
 NumberOfRays → 1064, ObjectSource → WedgeOfRays[10, NumberOfRays → 5],
 OpticalMedium → Air, -Options-, OutputGraphics → {PointSpreadFunction → (- Graphics -)},
 PointSpreadFunction → CompiledFunction[-intensity data-],
 PrinciplePoints → {{96.9272, 0, 0}, {103.073, 0, 0}},
 PrinciplePointSeparation → 6.14556, RayBoundary → {-0.0351218, 0.0351218},
 RayTraceFunction → RayTraceFunction[{f1, f2}, -raytrace code: 2669972 Bytes- ],
 RotationMatrix → {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, SmoothKernelSize → 0.000279382,
 SourceID → 19920, SurfaceNormalFunction → ({{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} &),
 SurfaceNumber → 1., SymbolicValues → {f1 → 100., f2 → 100.},
 SystemABCDMatrix → {{-1.08942, -0.667674}, {-0.0199131, -0.93012}},
 TransverseMagnification → -1.08942, WaveFrontID → 1., WaveLength → 0.532}
```

Go to list of topics

6. Interference and Wavefront Calculations

6.1 Overview

In[3]:=

Wavica contains two tools used for interference and wavefront calculations. These functions are **FindWaveFronts** and **FindInterference**. Finally, at the end of this chapter, we introduce a method to calculate propagated optical wavefronts that uses Gaussian wavelets.

In[21]:= **?FindWaveFronts**

```
FindWaveFronts[system, options] is a function
  designed to find the optical path length function and wave
  intensity function on a last surface in the optical system.

FindWaveFronts calculates the wavefront function and the intensity mapping
  function at the final surface. This information is used together by
  FindInterference to calculate the complex field as well as the interference
  at the surface. As input, FindWaveFronts uses an optical system containing
  one or more light sources. FindWaveFronts works equally well for both point
  sources and planar sources. In addition, the light source need only contain a
  small number of rays (since the actual number of rays is internally specified).

If a one-dimensional light source is used (ie. WedgeOfRays or
  LineOfRays), then one-dimensional wavefront and intensity functions
  are calculated. If a two-dimensional light source is used (ie.
  PointOfRays or GridOfRays), then two-dimensional wavefront and
  intensity functions are determined. FindWaveFronts is also a label
  used with the RenderedParameters option. See also: FindInterference,
  OpticalPathDifference, WaveFrontIntensity, and OpticalLengthFunction.
```

In[22]:= **Options[FindWaveFronts]**

```
Out[22]= {NumberOfRays → 32, IntensitySamplePoints → 32,
  InterpolationOrder → 1, ZernikeFit → False, ZernikeOrder → 10,
  RenderedParameters → {All}, PlotDomain → RayBoundary,
  Plot2D → False, PlotPoints → 32, IntensityScaleFactor → Automatic}
```

```
In[23]:= ?FindInterference
```

```
FindInterference[system, options] is a function
  that determines the interference function between multiple
  wavefronts hitting the last optical surface in an optical system.

As input, FindInterference takes either an optical system containing multiple
  light sources or the output from one or more FindWaveFronts calculations.
  FindInterference calculates the interference function by adding together the
  complex fields from the individual wavefronts hitting the optical surface
  and multiplying by the complex conjugate to get the intensity distribution
  on the surface. FindInterference works equally well for both point sources
  and planar sources. In addition, the light source need only contain a small
  number of rays (since the actual number of rays is internally specified).

If one-dimensional light sources are used (ie. WedgeOfRays or LineOfRays), then a
  one-dimensional interference function is calculated. If two-dimensional light
  sources are used (ie. PointOfRays or GridOfRays), then a two-dimensional
  interference functions is determined. FindInterference is also a label used with
  the RenderedParameters option. See also: FindWaveFronts, InterferenceFunction,
  OpticalPathDifference, WaveFrontIntensity, and OpticalLengthFunction.
```

```
In[24]:= Options[FindInterference]

Out[24]= {NumberOfRays → 32, NumberOfPoints → 128, InterpolationOrder → 1, ZernikeFit → False,
          ZernikeOrder → 10, RenderedParameters → {OpticalPathDifference, InterferenceFunction},
          PlotDomain → RayBoundary, PlotPoints → 64, Mesh → False, Plot2D → False}
```

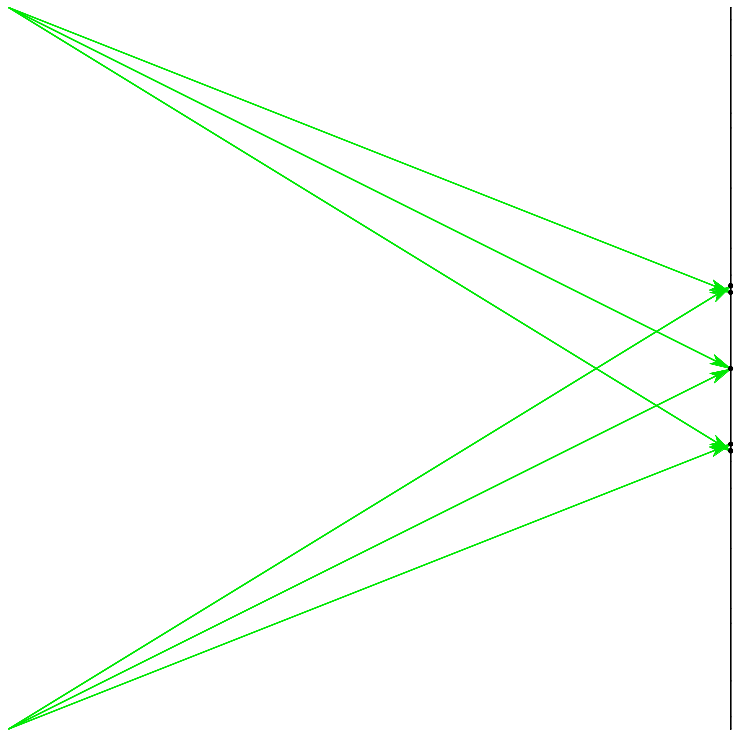
Go to list of topics

6.2 One-Dimensional Wavefronts

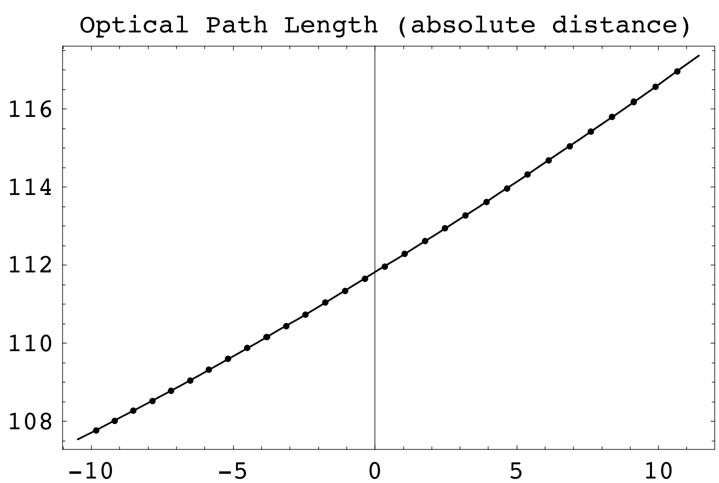
Example 1

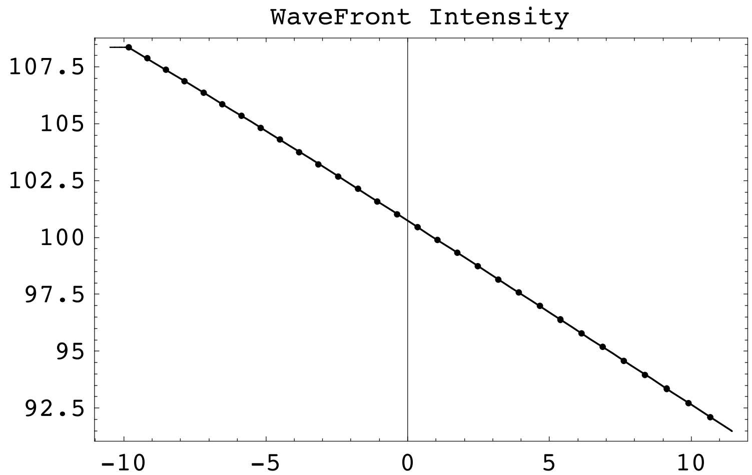
```
In[25]:= sys = {
         MoveDirected[WedgeOfRays[10],{0,-50},{100,0},SideOfObject->After],
         MoveDirected[WedgeOfRays[10],{0,50},{100,0},SideOfObject->After],
         Move[Screen[100],100]};
```

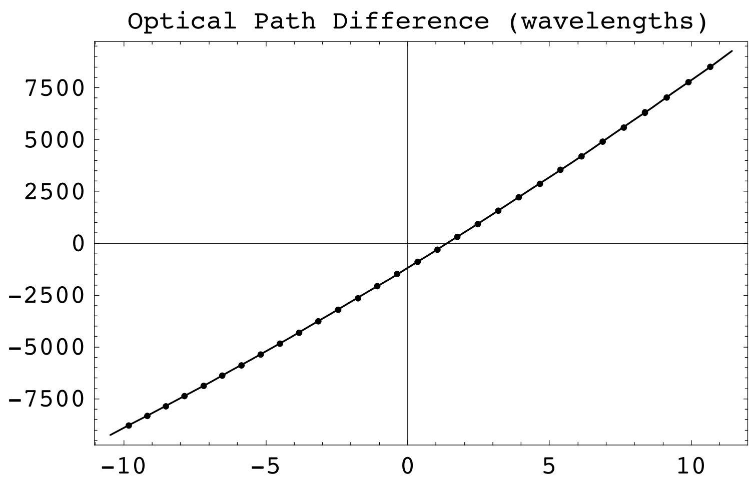
In[26]:= **DrawSystem[sys,PlotType->TopView];**

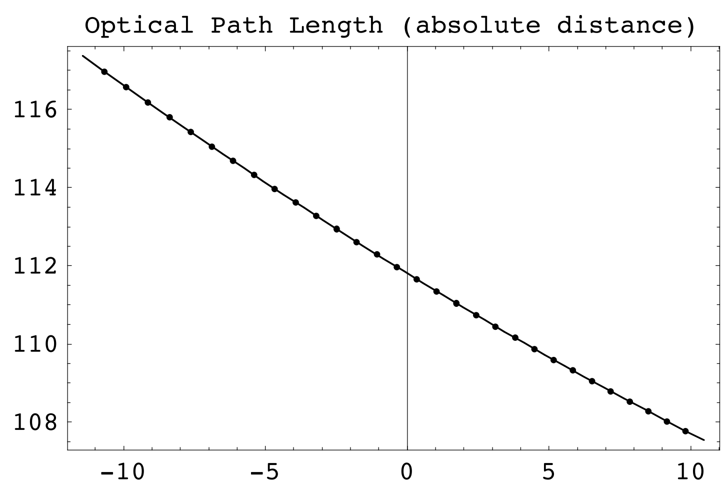


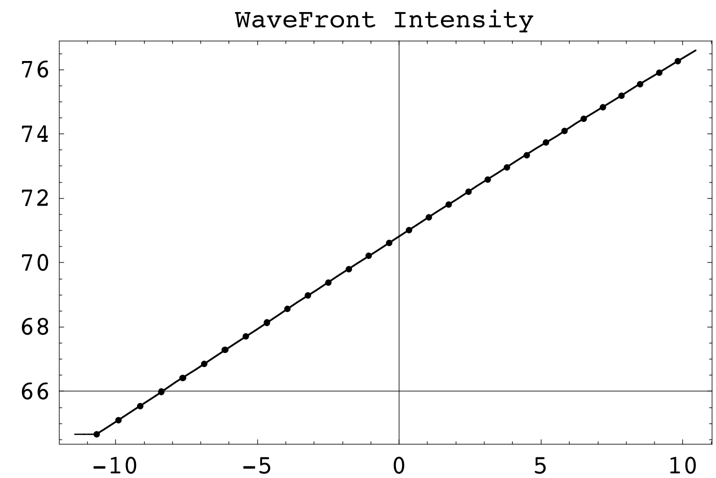
In[27]:= **wavefront = FindWaveFronts[sys]**

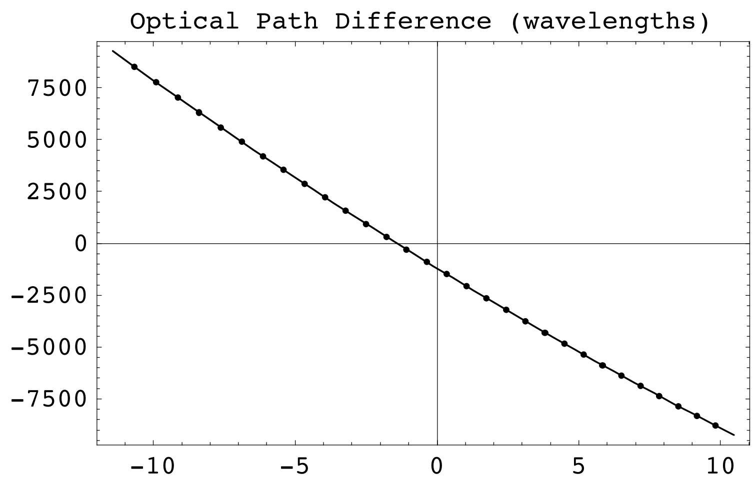



WaveFront Intensity
107.5
105
102.5
100
97.5
95
92.5
-10
-5
0
5
10


Optical Path Difference (wavelengths)
7500
5000
2500
0
-2500
-5000
-7500
-10
-5
0
5
10

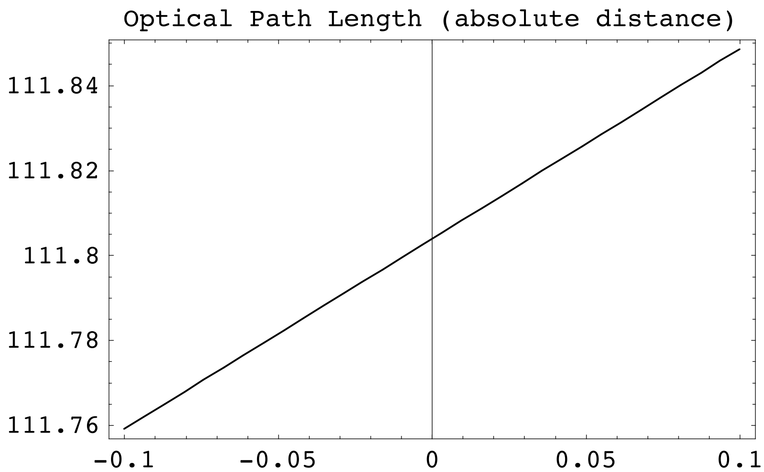

Optical Path Length (absolute distance)
116
114
112
110
108
-10
-5
0
5
10

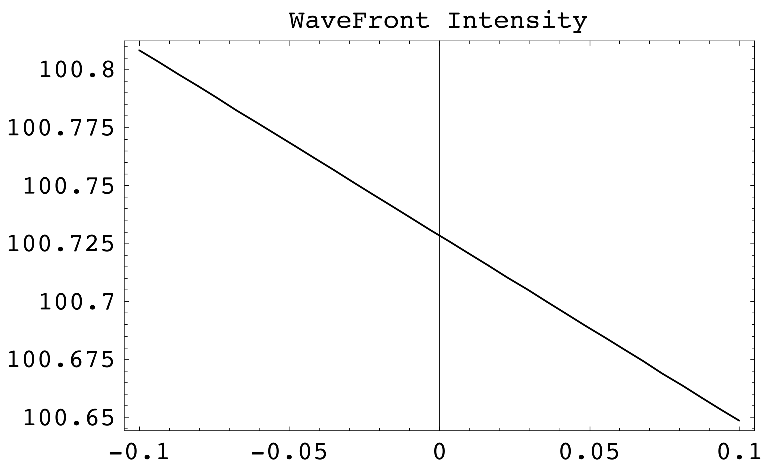


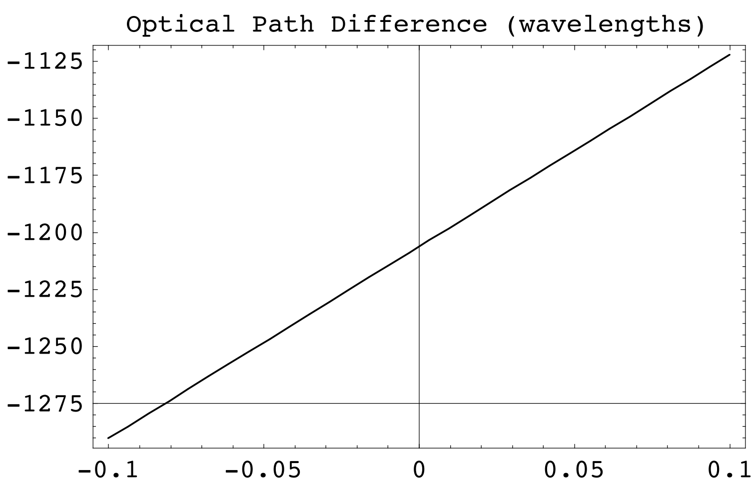


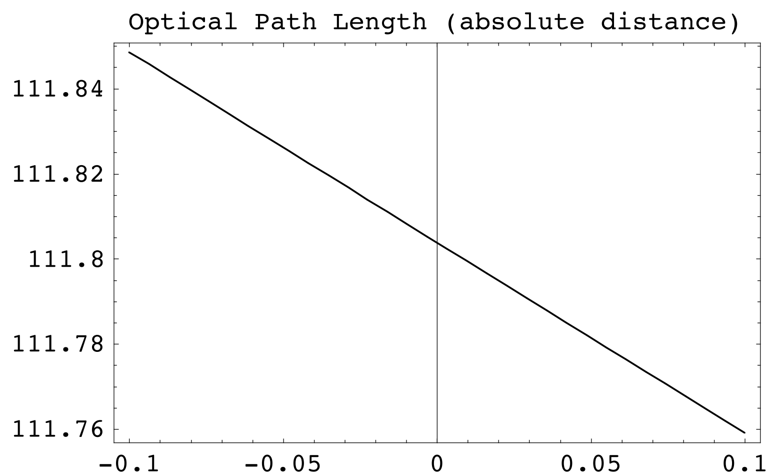

```
Out[27]= {{OpticalLengthFunction → InterpolatingFunction[{{-10.4777, 11.4364}}, <>],
  OpticalPathDifference → InterpolatingFunction[{{-10.4777, 11.4364}}, <>],
  OpticalPathRange → {107.527, 117.365},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
  RayBoundary → {-10.4777, 11.4364}, WaveFrontIntensity →
   InterpolatingFunction[{{-10.4777, 11.4364}}, <>], WaveLength → 0.532},
 {OpticalLengthFunction → InterpolatingFunction[{{-11.4364, 10.4777}}, <>],
  OpticalPathDifference → InterpolatingFunction[{{-11.4364, 10.4777}}, <>],
  OpticalPathRange → {107.527, 117.365},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
  RayBoundary → {-11.4364, 10.4777}, WaveFrontIntensity →
   InterpolatingFunction[{{-11.4364, 10.4777}}, <>], WaveLength → 0.532}}
```

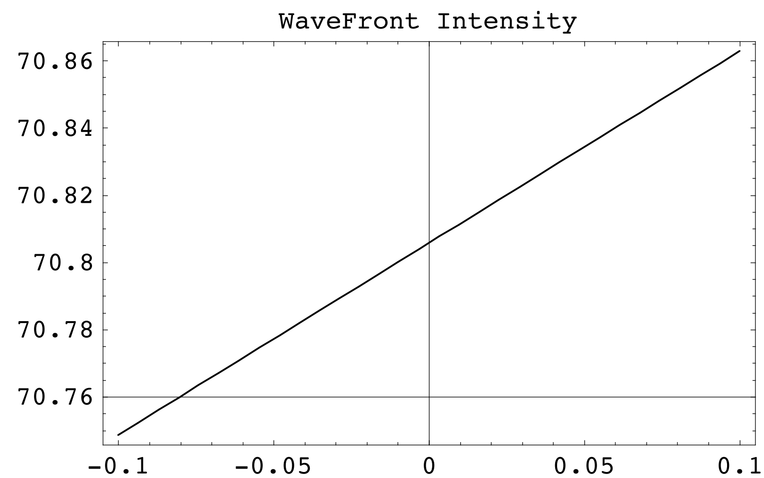
```
In[28]:= wavefront = FindWaveFronts[sys,PlotDomain->{-.1,.1}]
```

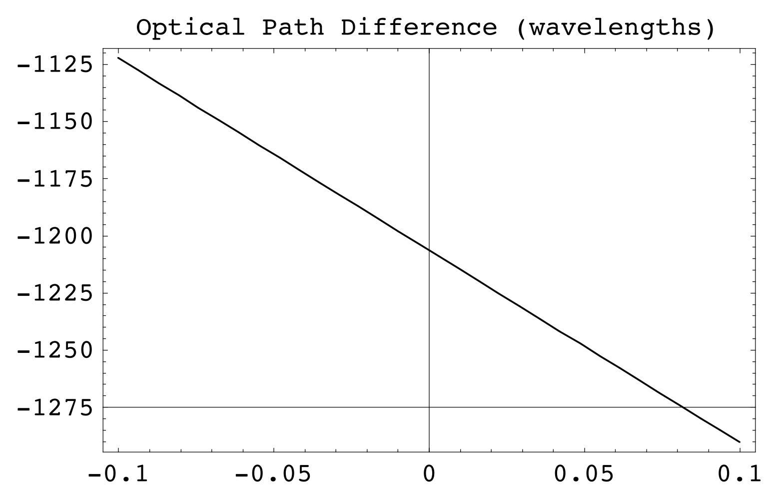

Optical Path Length (absolute distance)
111.84
111.82
111.8
111.78
111.76
-0.1
-0.05
0
0.05
0.1


WaveFront Intensity
100.8
100.775
100.75
100.725
100.7
100.675
100.65
-0.1
-0.05
0
0.05
0.1


Optical Path Difference (wavelengths)
-1125
-1150
-1175
-1200
-1225
-1250
-1275
-0.1
-0.05
0
0.05
0.1

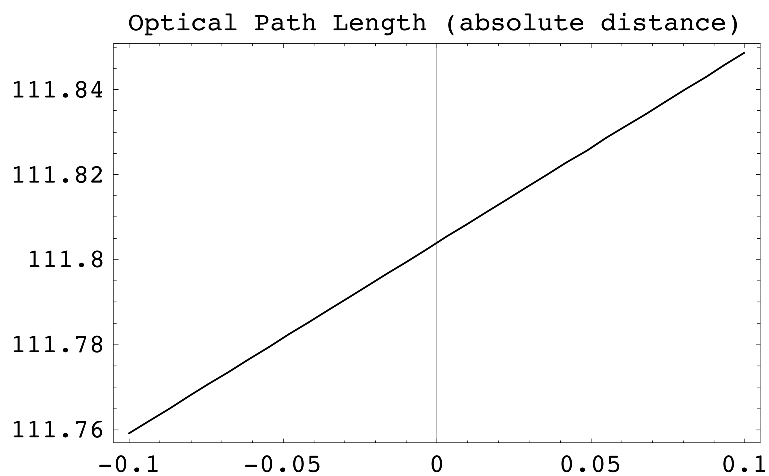

Optical Path Length (absolute distance)
111.84
111.82
111.8
111.78
111.76
-0.1
-0.05
0
0.05
0.1

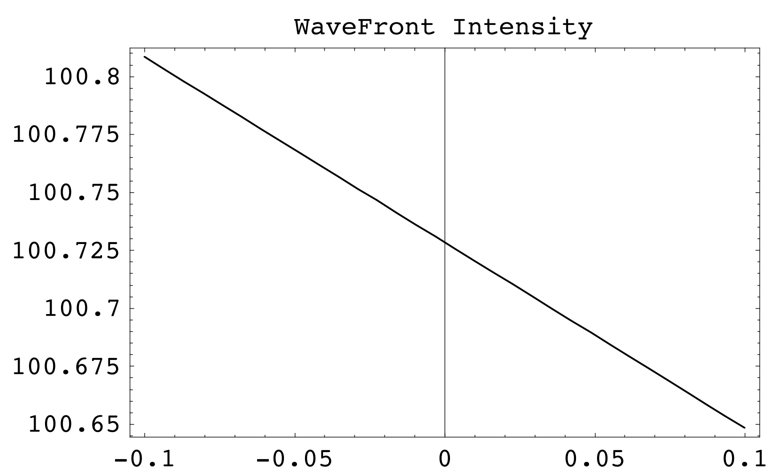

WaveFront Intensity
70.86
70.84
70.82
70.8
70.78
70.76
-0.1
-0.05
0
0.05
0.1

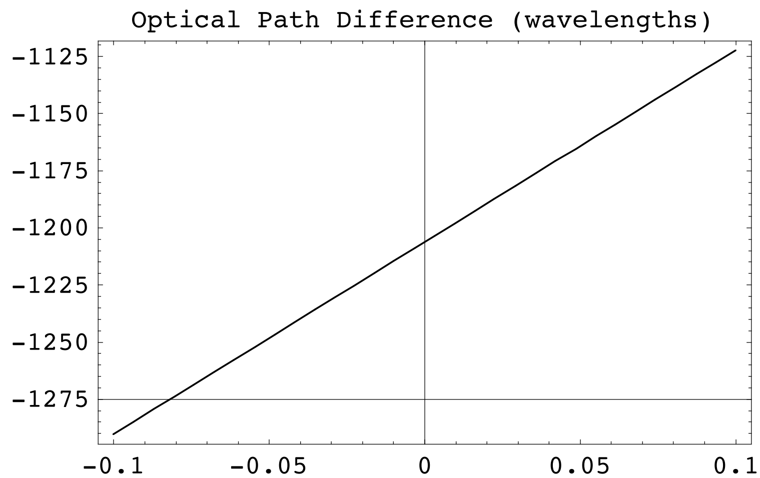

Optical Path Difference (wavelengths)
-1125
-1150
-1175
-1200
-1225
-1250
-1275
-0.1
-0.05
0
0.05
0.1

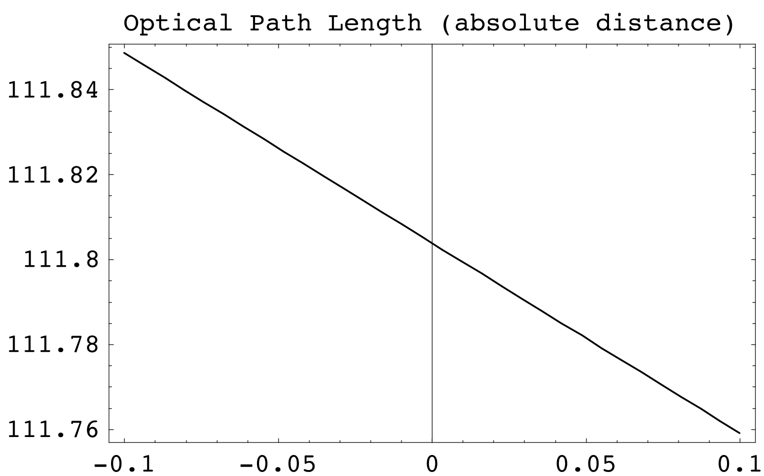
```
Out[28]=  {{OpticalLengthFunction → InterpolatingFunction[{{-10.4777, 11.4364}}, <>],
            OpticalPathDifference → InterpolatingFunction[{{-10.4777, 11.4364}}, <>],
            OpticalPathRange → {107.527, 117.365},
            OutputGraphics → {OpticalLengthFunction → (- Graphics -),
              WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
            PlotDomain → {-0.1, 0.1}, RayBoundary → {-10.4777, 11.4364}, WaveFrontIntensity →
             InterpolatingFunction[{{-10.4777, 11.4364}}, <>], WaveLength → 0.532},
          {OpticalLengthFunction → InterpolatingFunction[{{-11.4364, 10.4777}}, <>],
            OpticalPathDifference → InterpolatingFunction[{{-11.4364, 10.4777}}, <>],
            OpticalPathRange → {107.527, 117.365},
            OutputGraphics → {OpticalLengthFunction → (- Graphics -),
              WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
            PlotDomain → {-0.1, 0.1}, RayBoundary → {-11.4364, 10.4777}, WaveFrontIntensity →
             InterpolatingFunction[{{-11.4364, 10.4777}}, <>], WaveLength → 0.532}}
```

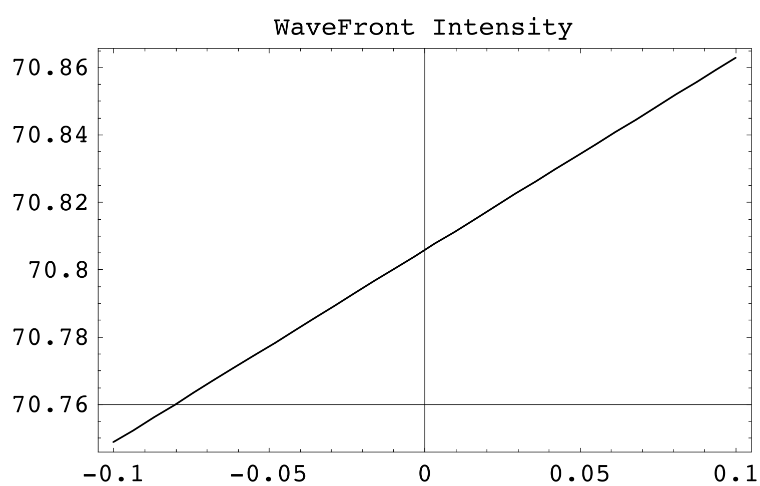
```
In[29]:= wavefront = FindWaveFronts[sys,PlotDomain->{-.1,.1}]
```

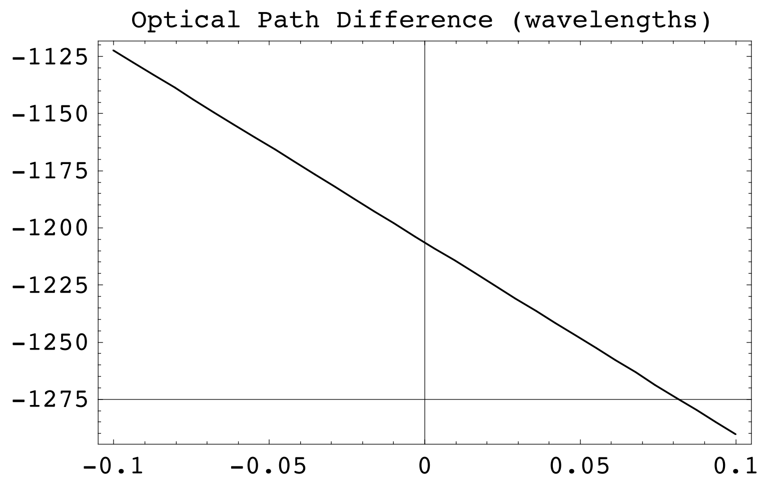





Optical Path Difference (wavelengths)
-1125
-1150
-1175
-1200
-1225
-1250
-1275
-0.1
-0.05
0
0.05
0.1

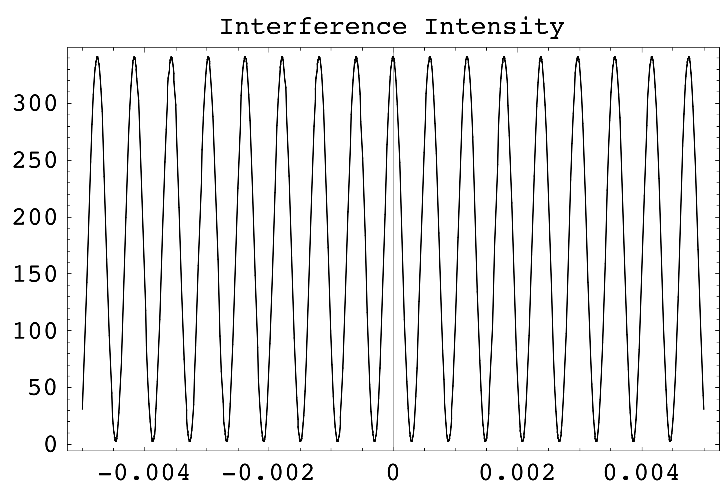

Optical Path Length (absolute distance)
111.84
111.82
111.8
111.78
111.76
-0.1
-0.05
0
0.05
0.1


WaveFront Intensity
70.86
70.84
70.82
70.8
70.78
70.76
-0.1
-0.05
0
0.05
0.1

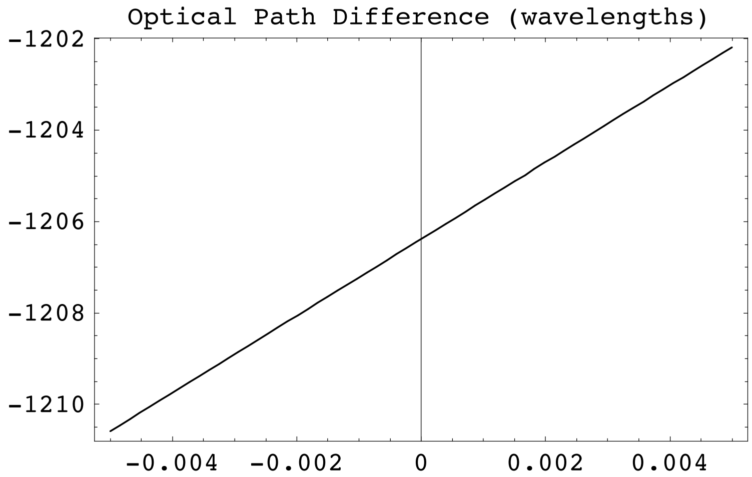


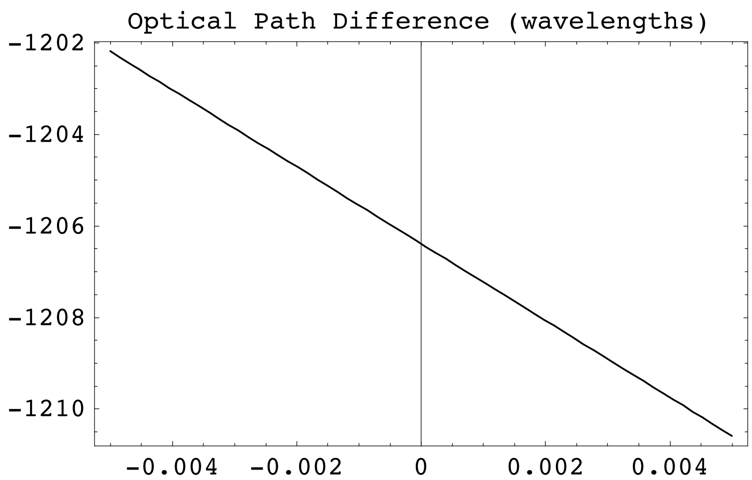

```
Out[29]= {{OpticalLengthFunction → InterpolatingFunction[{{-10.4777, 11.4364}}, <>],
  OpticalPathDifference → InterpolatingFunction[{{-10.4777, 11.4364}}, <>],
  OpticalPathRange → {107.527, 117.365},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
  PlotDomain → {-0.1, 0.1}, RayBoundary → {-10.4777, 11.4364}, WaveFrontIntensity →
   InterpolatingFunction[{{-10.4777, 11.4364}}, <>], WaveLength → 0.532},
 {OpticalLengthFunction → InterpolatingFunction[{{-11.4364, 10.4777}}, <>],
  OpticalPathDifference → InterpolatingFunction[{{-11.4364, 10.4777}}, <>],
  OpticalPathRange → {107.527, 117.365},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
  PlotDomain → {-0.1, 0.1}, RayBoundary → {-11.4364, 10.4777}, WaveFrontIntensity →
   InterpolatingFunction[{{-11.4364, 10.4777}}, <>], WaveLength → 0.532}}
```

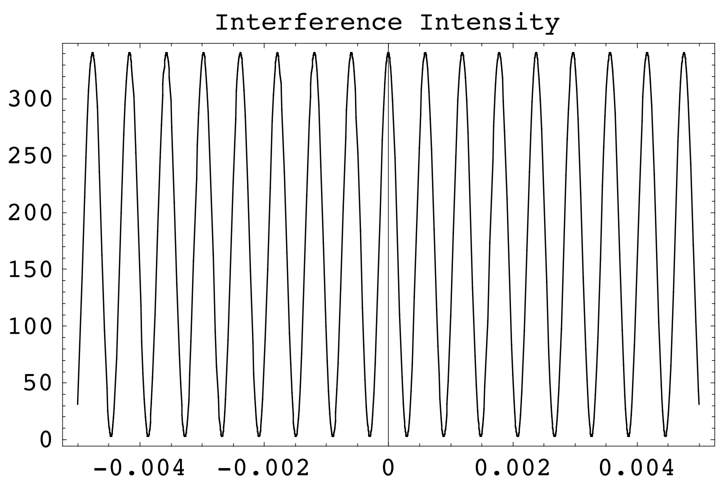
```
In[30]:= FindInterference[wavefront,PlotDomain->{-.005,.005}];
```




```
In[31]:= FindInterference[sys,PlotDomain->{-.005,.005}];
```







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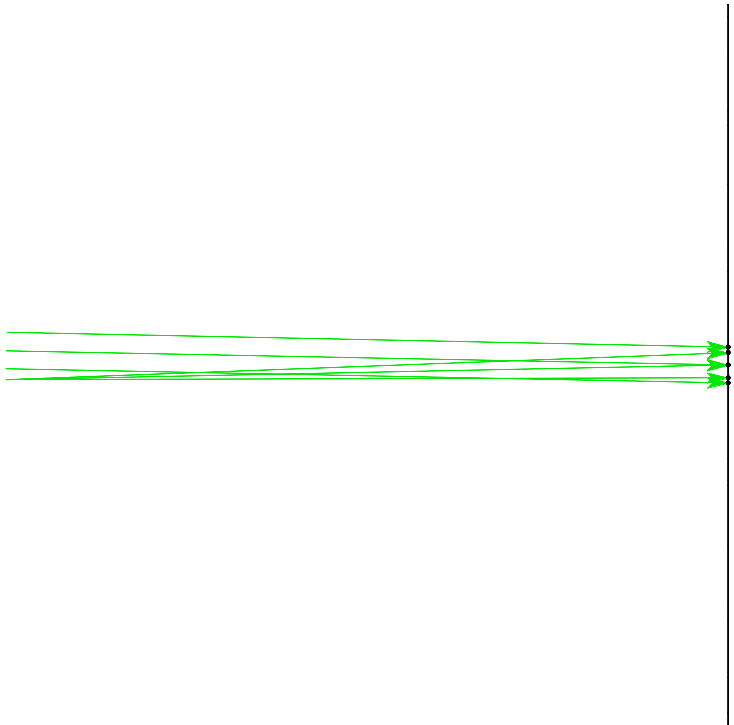
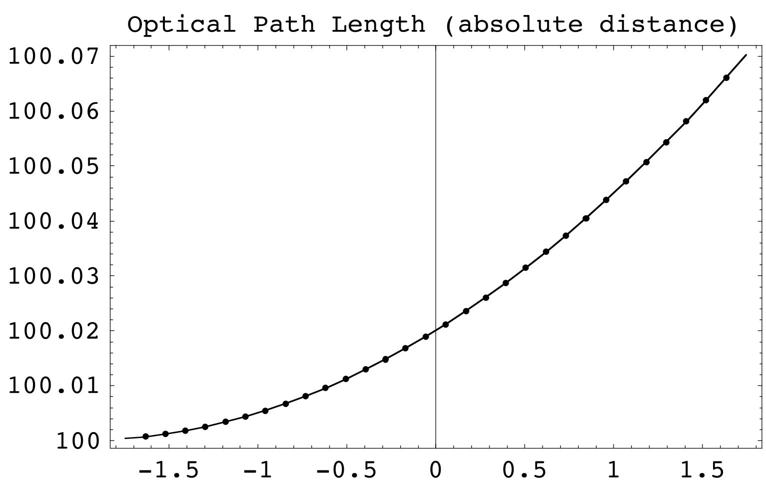
Example 2

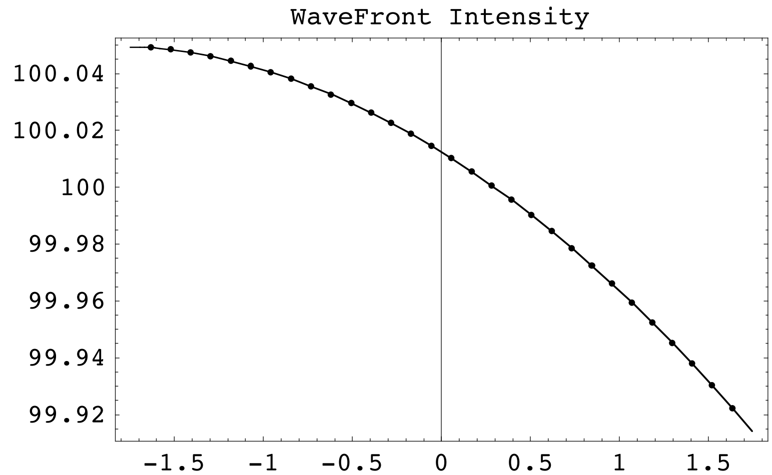
```
In[32]:= sys = {
         MoveDirected[WedgeOfRays[2],{0,-2},{100,0},SideOfObject->After],
         MoveDirected[LineOfRays[5],{0,2},{100,0},SideOfObject->After],
         Move[Screen[100],100]};
```

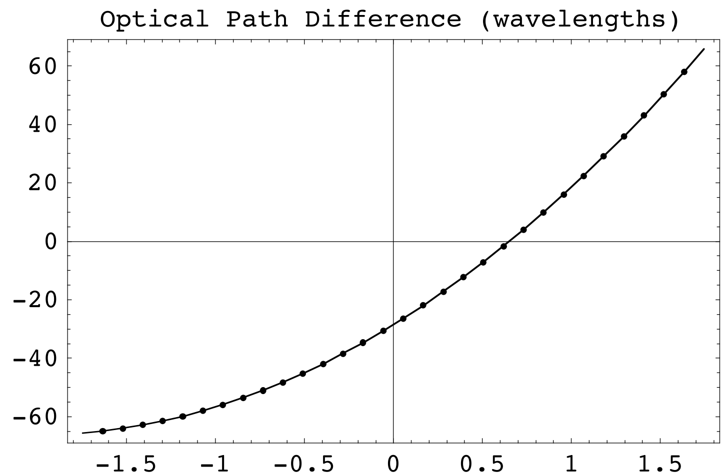
```
In[33]:= DrawSystem[sys,PlotType->TopView];
```

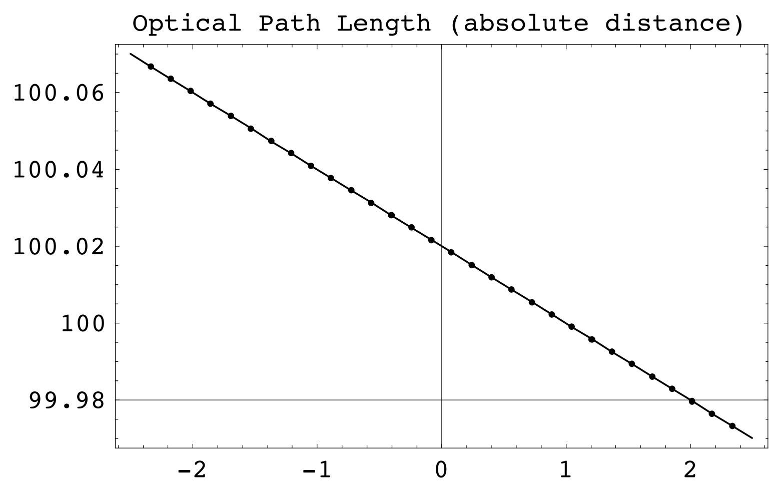


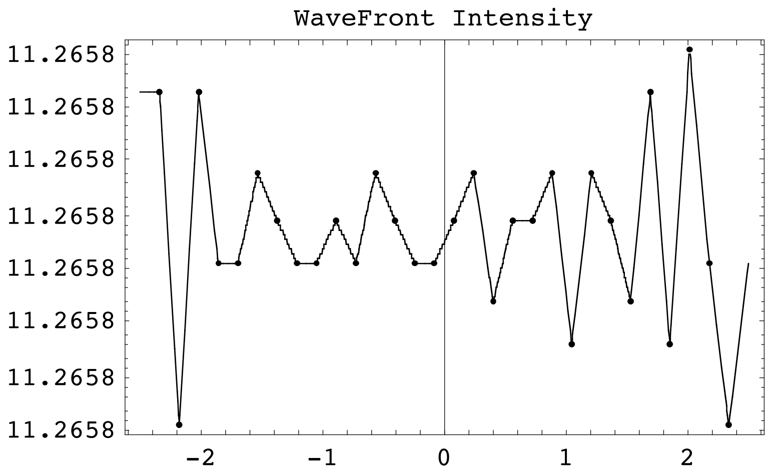
```
In[34]:= wavefront = FindWaveFronts[sys]
```

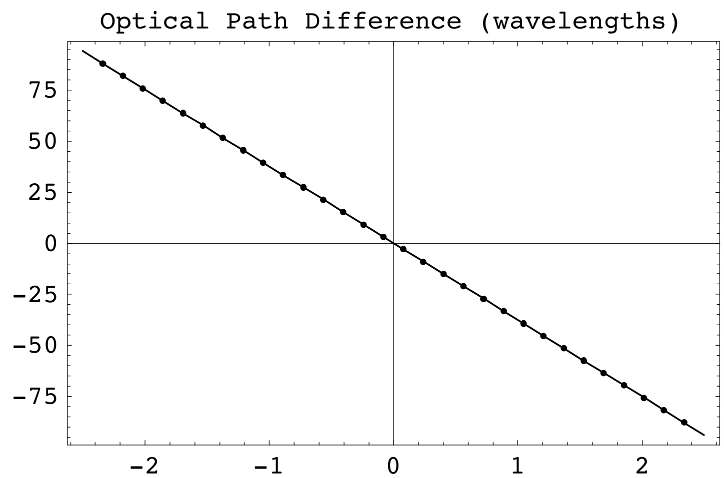



WaveFront Intensity
100.04
100.02
100
99.98
99.96
99.94
99.92
-1.5 -1 -0.5 0 0.5 1 1.5


Optical Path Difference (wavelengths)
60
40
20
0
-20
-40
-60
-1.5 -1 -0.5 0 0.5 1 1.5

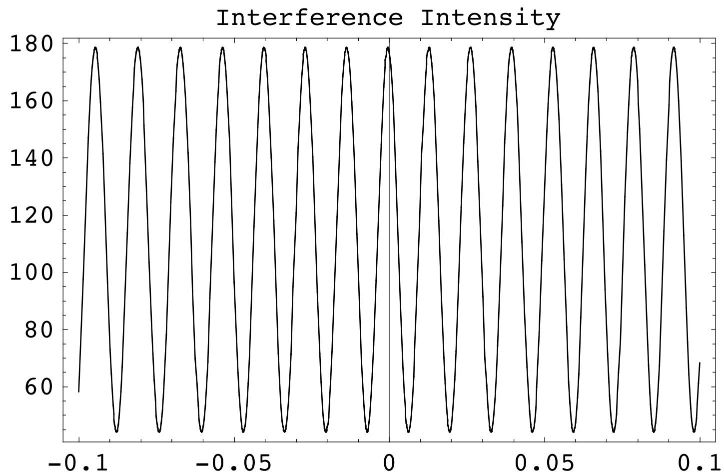

Optical Path Length (absolute distance)
100.06
100.04
100.02
100
99.98
-2 -1 0 1 2






```
Out[34]= {{OpticalLengthFunction → InterpolatingFunction[{{-1.7456, 1.74681}}, <>],
   OpticalPathDifference → InterpolatingFunction[{{-1.7456, 1.74681}}, <>],
   OpticalPathRange → {100., 100.07},
   OutputGraphics → {OpticalLengthFunction → (- Graphics -),
     WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
   RayBoundary → {-1.7456, 1.74681}, WaveFrontIntensity →
    InterpolatingFunction[{{-1.7456, 1.74681}}, <>], WaveLength → 0.532},
  {OpticalLengthFunction → InterpolatingFunction[{{-2.5005, 2.5005}}, <>],
   OpticalPathDifference → InterpolatingFunction[{{-2.5005, 2.5005}}, <>],
   OpticalPathRange → {99.97, 100.07},
   OutputGraphics → {OpticalLengthFunction → (- Graphics -),
     WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
   RayBoundary → {-2.5005, 2.5005}, WaveFrontIntensity →
    InterpolatingFunction[{{-2.5005, 2.5005}}, <>], WaveLength → 0.532}}
```

```
In[35]:= FindInterference[wavefront,PlotDomain->{-.1,.1}];
```



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Example 3

This example shows how to create each wave-front separately and interfere them together afterwards.

```
In[36]:= outerradius = .5 106

Out[36]= 53.

In[37]:= Piston = ComponentRendering[{Move[CylindricalLensSurface[40, {1.9999 40, 120},
              ComponentMedium → FusedSilica, PlotPoints → 50], outerradius - 40],
           CylindricalLensSurface[outerradius, {1.999 outerradius, 120},
            ComponentMedium → FusedSilica, PlotPoints → 50]}, Labels → {""}, PlotPoints → 50];

In[38]:= Clear[H, L1, L2, CL];
         H = Window[{5. 25.4, 4 25.4}, 1.5, "H"];
         L1 = PlanoConvexLens[450, 145, 15.5, "L1", CurvatureDirection → Back];
         L2 = PlanoConvexLens[450, 145, 15.5, "L2", CurvatureDirection → Back];
         CL = PlanoConvexCylindricalLens[100, {50, 60}, 8.89, "CL", CurvatureDirection → Back];

In[61]:= reference = Move[WedgeOfRays[7., WaveLength → .532], {422, 0}, 180];

In[65]:= HOE = Move[{Window[60, 1.5, "HOE"], Move[CircleGraphic[{57, 85}], {-1.5, 5.5}],
           CircleGraphic[4]}, {0, 0}, -(ArcTan[-30., 48.] - ArcTan[-45., 28.]) / Degree];

In[45]:= in = 24.5;
         mm = 1;
         L = PlanoConvexLens[100, 25, 5, CurvatureDirection -> Back];
         M = Mirror[1 in, 6, GraphicDesign → Solid];

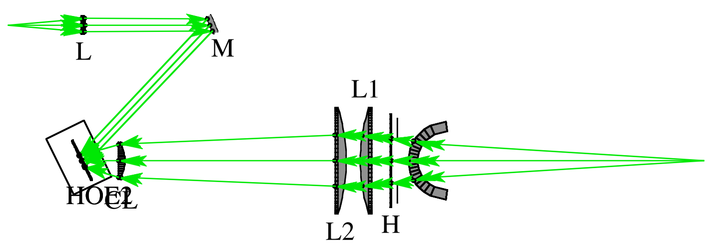
In[49]:= planesource = {Move[WedgeOfRays[10],{-640+115,185}],
                        Move[L,{-640+115+100,185}],
                        MoveReflected[M,{-640+115+100,185},{-640+115+100+100,185},{-425,0}],
                        MoveReflected[M,{-640+115+100,185},{-640+115+100+100+75,185},{-425,0}]};

In[50]:= planesource = {Move[WedgeOfRays[10],{-640+115,185}],
                        Move[L,{-640+115+100,185}],
                        MoveReflected[M,{-640+115+100,185},{-640+115+100+100+75,185},{-425,0}]};
```

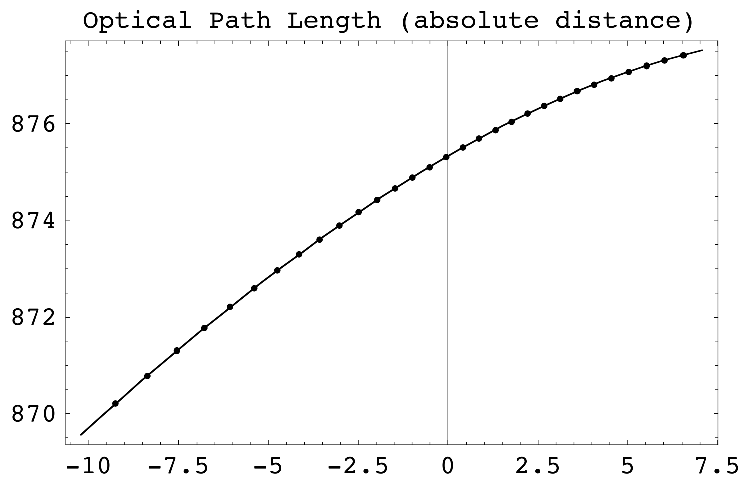
```
In[66]:= sys1 = Flatten[{planesource,
                 Move[HOE,-425,MoveRelative->False]}];

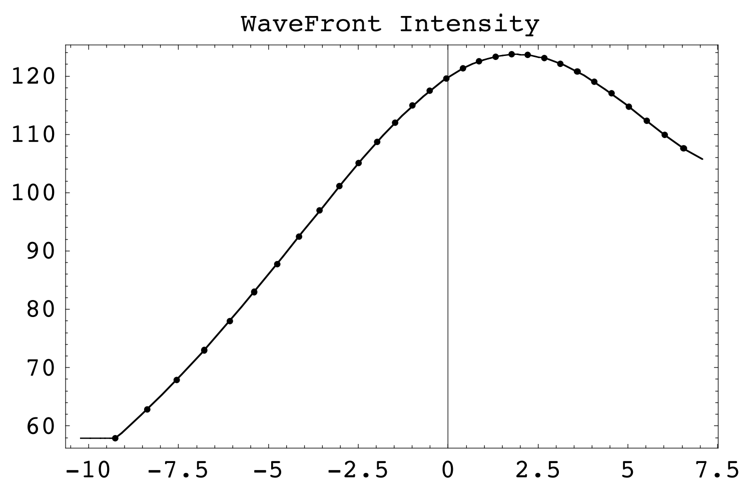
In[69]:= sys2 = {    reference,
                 Move[Piston,20],
                 Move[{Move[Baffle[{25,120},Labels→""],{0,.5 65+12.5}],
                 Move[Baffle[{25,120},Labels→""],{0,-.5 65-12.5}]},5],
                 Move[H,-4.5],
                 Move[L1,-30,180],
                 Move[L2,-80],
                 Move[CL,-375],
                 Move[HOE,-425,MoveRelative->False]};

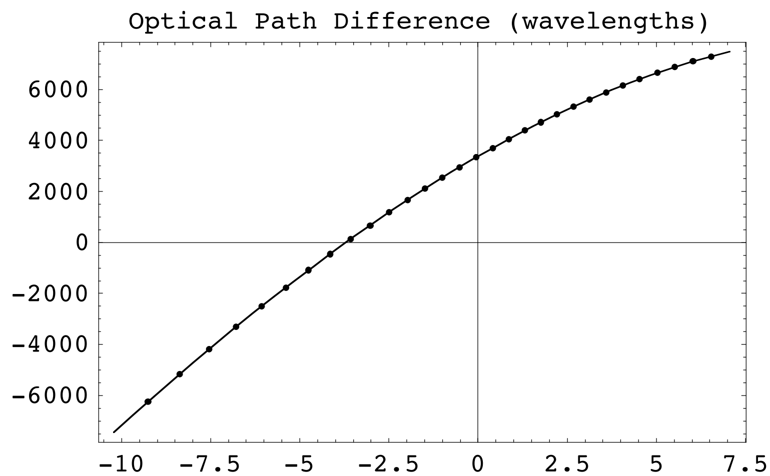
In[70]:=  imagingoptics = DrawSystem[{sys1, sys2}, PlotType → TopView, PlotPoints → 50];
```



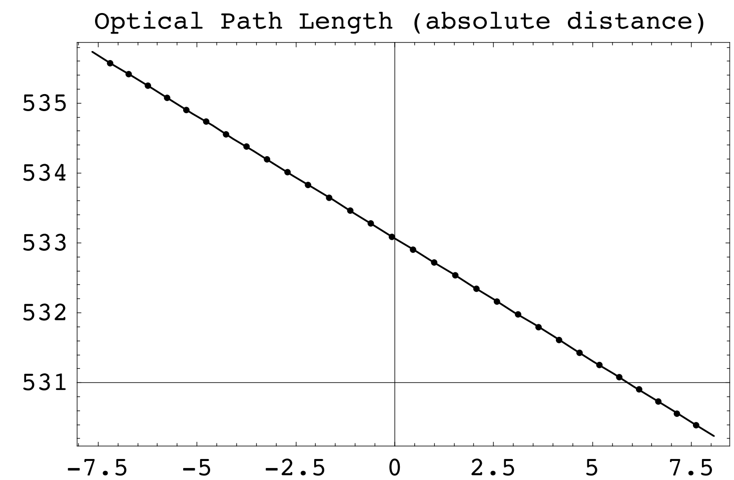

```
In[71]:= wave2 = FindWaveFronts[sys2];
```

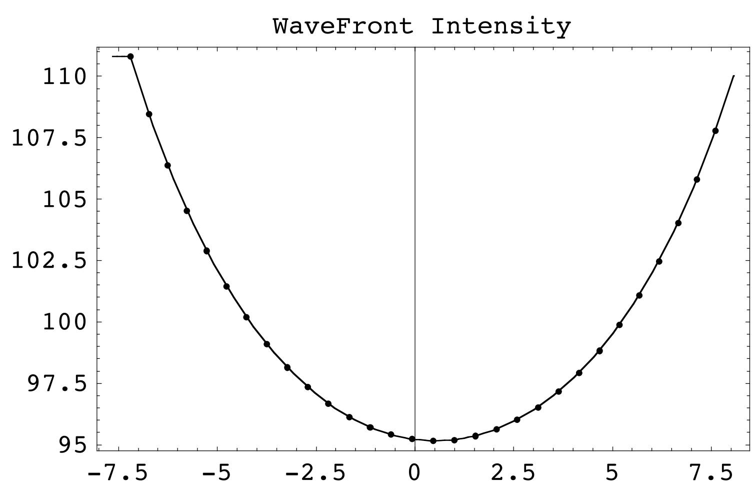


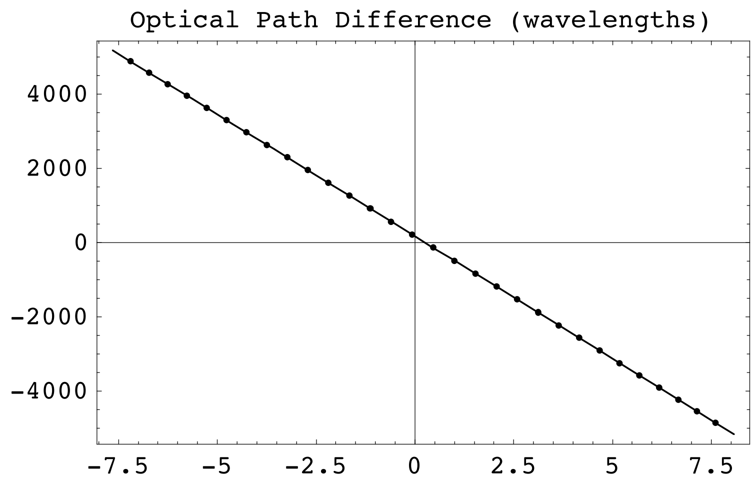




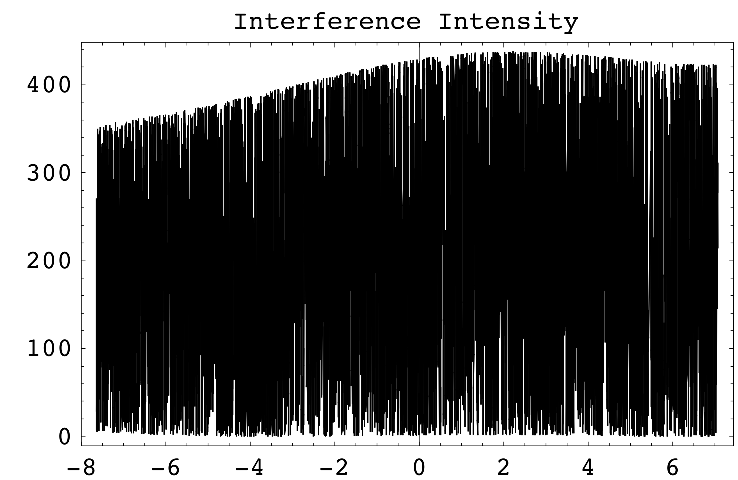
In[72]:= **wave1 = FindWaveFronts[sys1];**



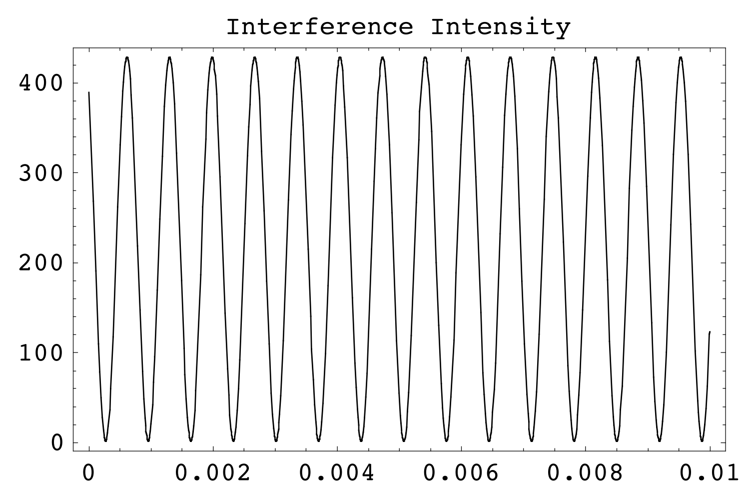





```
In[73]:= FindInterference[Join[wave1,wave2]];
```




```
In[74]:= FindInterference[Join[wave1,wave2],PlotDomain->{0,.01}];
```



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Example 4

This example shows how to create and interfere two wave-fronts simultaneously.

```
In[3]:= outerradius = .5 106

Out[3]= 53.

In[4]:= Piston = ComponentRendering[{Move[CylindricalLensSurface[40, {1.9999 40, 120},
              ComponentMedium → FusedSilica, PlotPoints → 50], outerradius - 40],
           CylindricalLensSurface[outerradius, {1.999 outerradius, 120},
            ComponentMedium → FusedSilica, PlotPoints → 50]}, Labels → {""}, PlotPoints → 50];

In[5]:= Clear[H, L1, L2, CL];
        H = Window[{5. 25.4, 4 25.4}, 1.5, "H"];
        L1 = PlanoConvexLens[450, 145, 15.5, "L1", CurvatureDirection → Back];
        L2 = PlanoConvexLens[450, 145, 15.5, "L2", CurvatureDirection → Back];
        CL = PlanoConvexCylindricalLens[100, {50, 60}, 8.89, "CL", CurvatureDirection → Back];

In[10]:= reference  = Move[WedgeOfRays[7., WaveLength → .532], {422, 0}, 180];

In[11]:= HOE = Move[{Window[60, 1.5, "HOE"], Move[CircleGraphic[{57, 85}], {-1.5, 5.5}],
             CircleGraphic[4]}, {0, 0}, -(ArcTan[-30., 48.] - ArcTan[-45., 28.]) / Degree];

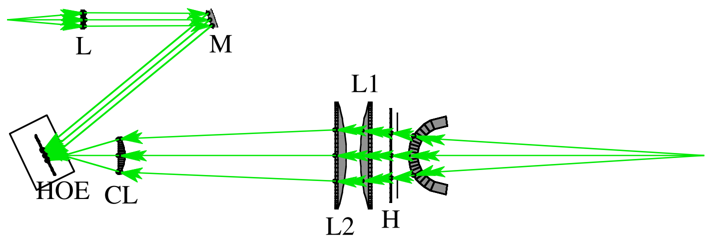
In[12]:= in = 24.5;
         mm = 1;
         L = PlanoConvexLens[100, 25, 5, CurvatureDirection -> Back];
         M = Mirror[1 in, 6, GraphicDesign → Solid];

In[16]:= planesource  = {Move[WedgeOfRays[10],{-640+115,185}],
                        Move[L,{-640+115+100,185}],
                        MoveReflected[M,{-640+115+100,185},{-640+115+100+100+75,185},{-475,0}]};

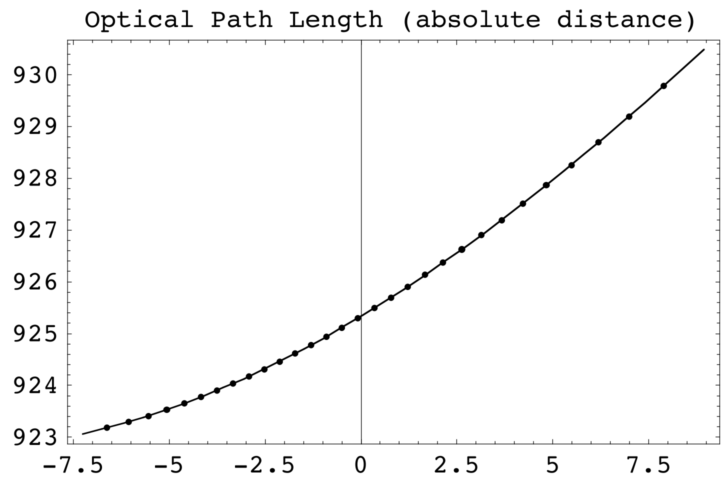
In[17]:= combinedsys  =
                  {    reference,
                       Move[Piston,20],
                       Move[{Move[Baffle[{25,120},Labels→""],{0,.5  65+12.5}],
                       Move[Baffle[{25,120},Labels→""],{0,-.5  65-12.5}]},5],
                       Move[H,-4.5],
                       Move[L1,{-30,0},180],
                       Move[L2,{-80,0},0],
                       Move[CL,-375],
                       planesource,
                       Move[HOE,-475,MoveRelative->False]};

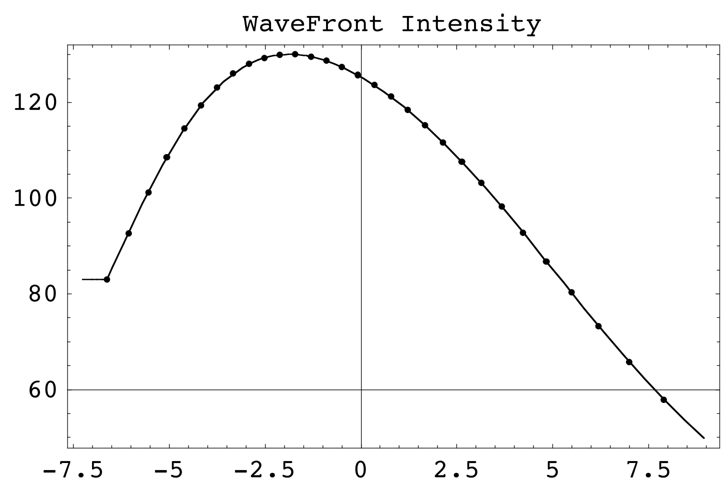
In[18]:= combinedsys  =
                  {    reference,
                       planesource,
                       Move[Piston,20],
                       Move[{Move[Baffle[{25,120},Labels→""],{0,.5  65+12.5}],
                       Move[Baffle[{25,120},Labels→""],{0,-.5  65-12.5}]},5],
                       Move[H,-4.5],
                       Move[L1,{-30,0},180],
                       Move[L2,{-80,0},0],
                       Move[CL,-375],
                       Move[HOE,-475,MoveRelative->False]};
```

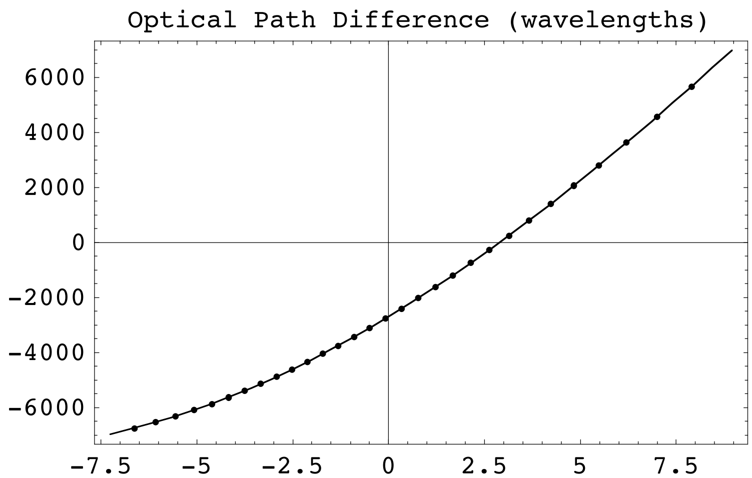
```
In[19]:= DrawSystem[combinedsys, PlotType → TopView, PlotPoints → 50];
```

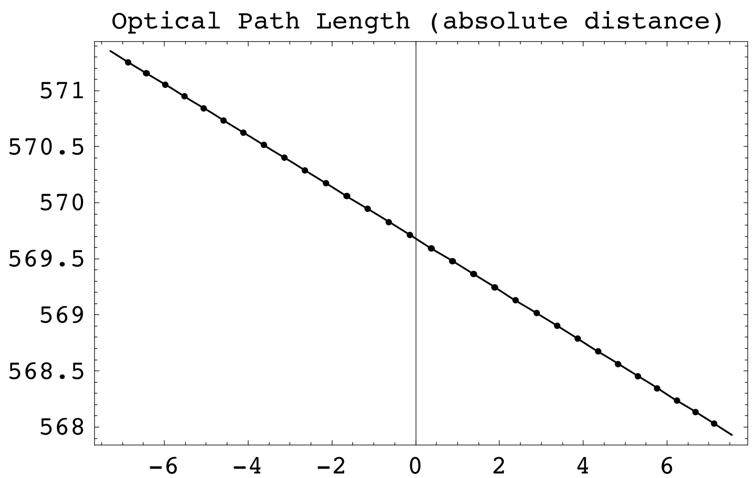


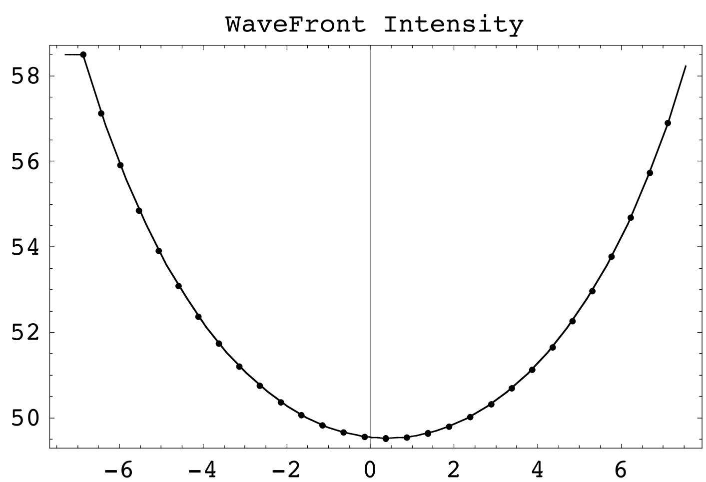

```
In[20]:= FindWaveFronts[combinedsys]
```

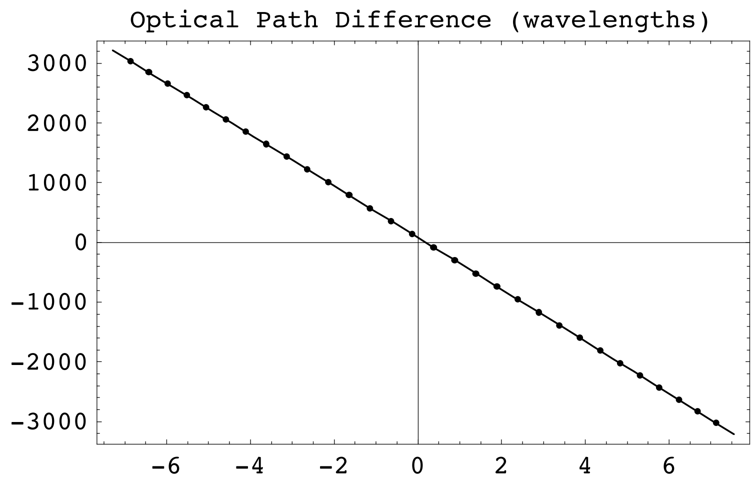





Optical Path Difference (wavelengths)
6000
4000
2000
0
-2000
-4000
-6000
-7.5
-5
-2.5
0
2.5
5
7.5

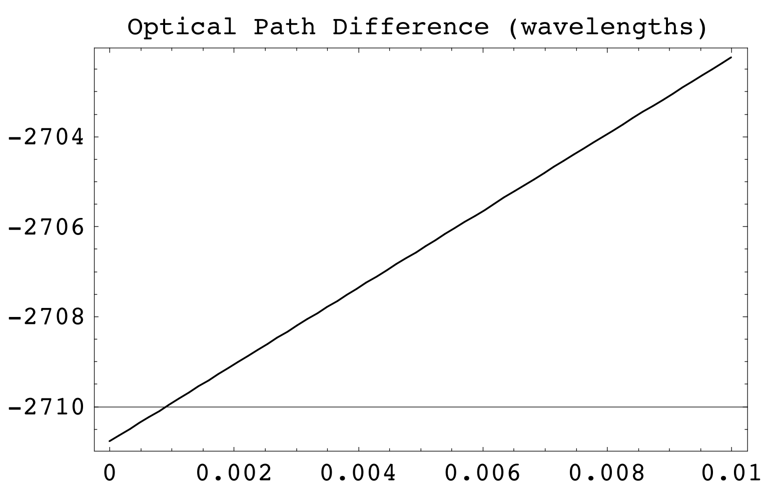

Optical Path Length (absolute distance)
571
570.5
570
569.5
569
568.5
568
-6
-4
-2
0
2
4
6

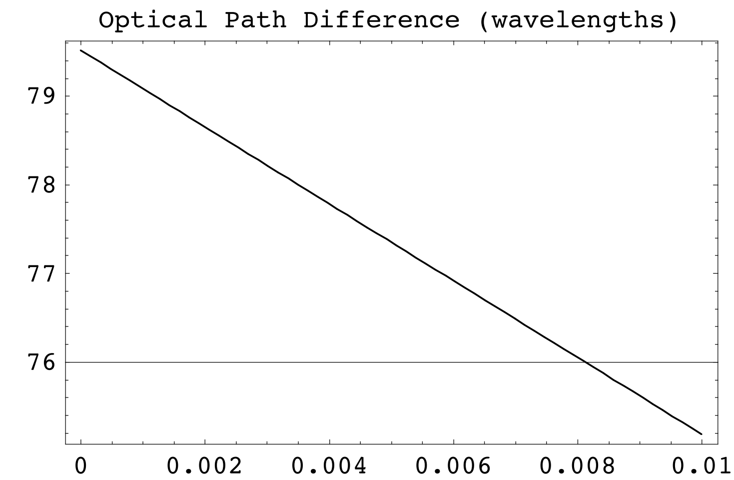

WaveFront Intensity
58
56
54
52
50
-6
-4
-2
0
2
4
6

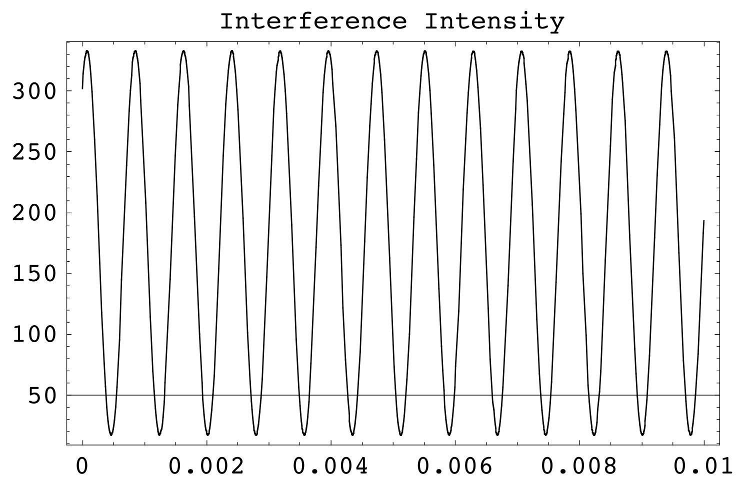



```
Out[20]= {{OpticalLengthFunction → InterpolatingFunction[{{-7.25666, 8.9558}}, <>],
  OpticalPathDifference → InterpolatingFunction[{{-7.25666, 8.9558}}, <>],
  OpticalPathRange → {923.057, 930.48},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
  RayBoundary → {-7.25666, 8.9558}, WaveFrontIntensity →
   InterpolatingFunction[{{-7.25666, 8.9558}}, <>], WaveLength → 0.532},
 {OpticalLengthFunction → InterpolatingFunction[{{-7.2916, 7.55523}}, <>],
  OpticalPathDifference → InterpolatingFunction[{{-7.2916, 7.55523}}, <>],
  OpticalPathRange → {567.928, 571.349},
  OutputGraphics → {OpticalLengthFunction → (- Graphics -),
    WaveFrontIntensity → (- Graphics -), OpticalPathDifference → (- Graphics -)},
  RayBoundary → {-7.2916, 7.55523}, WaveFrontIntensity →
   InterpolatingFunction[{{-7.2916, 7.55523}}, <>], WaveLength → 0.532}}
```

```
In[21]:= FindInterference[combinedsys,PlotDomain->{0,.01}];
```







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Example 5

In[275]:=

```
?GaussianBeam
```

```
GaussianBeam[spotsize, divergence, options] and GaussianBeam[
  complexbeamparameter, options] is a light source that takes either the
  output beam spotsize radius (specified at 1/e of the amplitude peak
  in the starting plane) and far-field beam divergence (specified as
  the full angle in radians) or the complex beam parameter as input and
  creates an extended point source ray model of a Gaussian laser beam.

In addition, GaussianBeam uses the FullForm option to determine whether a
  uniform grid of rays are created or two distinct fans of rays are instead
  constructed. For FullForm->False, GaussianBeam creates either a single
  two-dimensional fan of rays in the x-y plane or two cross-hatched fans
  of rays that lie in the x-y and x-z planes, depending on whether the
  spotsize and divergence or complexbeamparameter contains single numbers
  or sets of two numbers. For FullForm->True, GaussianBeam attempts to
  construct a uniform three-dimensional grid of rays to model the spot size
  and beam divergence behavior. In some instances, however, when the two
  tranverse beam waists occur at different locations along the optical axis,
  then GaussianBeam cannot presently construct a full three-dimensional
  model. In this event, it reverts to the FullForm->False behavior.

Each input parameter may also take a symbolic setting in addition to a numeric
  value. In such an event, a two-element list of terms, {symbolic,numeric}, is
  given for each symbolic parameter setting. In addition to geometric ray tracing,
  GaussianBeam is also used in symbolic Gaussian beam propagation calculations.

See also: FarField, SpotSizeFactor, BeamWaist, SymbolicBeamWaist, BeamSpotSize,
  SymbolicBeamSpotSize, BeamCurvature, SymbolicBeamCurvature, BeamDivergence,
  SymbolicBeamDivergence, BeamScaleLength, SymbolicBeamScaleLength,
  ComplexBeamParameter, and SymbolicComplexBeamParameter.
```

In[541]:=

```
system =
    {
        Move[GaussianBeam[1.5,.001],-50],
        PlanoConcaveLens[-30,20,2,"L1",CurvatureDirection->Back],
        Move[PlanoConvexLens[160,30,6.5,"L2"],130],
        Move[BeamSplitter[{70,30},50,10],175,-45],
        Move[Mirror[50,10],250],
        Move[Mirror[50,10],{175,50},90.01],
        Move[BeamSplitter[{70,30},50,10,""],175,-45,GraphicDesign->Off],
        Move[Screen[50],{175,-50},90]
    };
```

In[542]:=

```
AnalyzeSystem[system,PlotType->TopView];
```

In[543]:=

```
wavefronts = FindWaveFronts[system]
```

In[544]:=

```
FindInterference[wavefronts];
```

In[545]:=
```
FindInterference[system]
```

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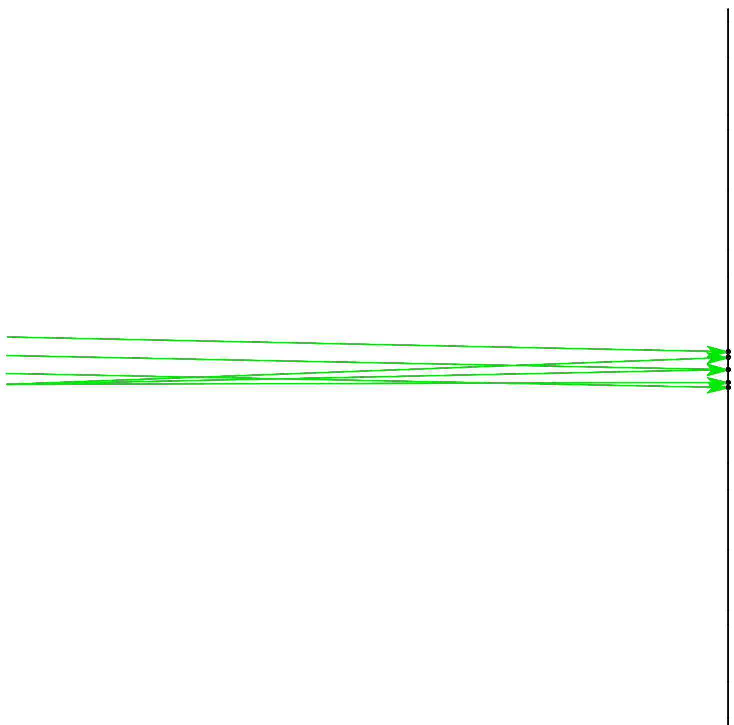
6.3 Two-Dimensional Wavefronts

Example 1

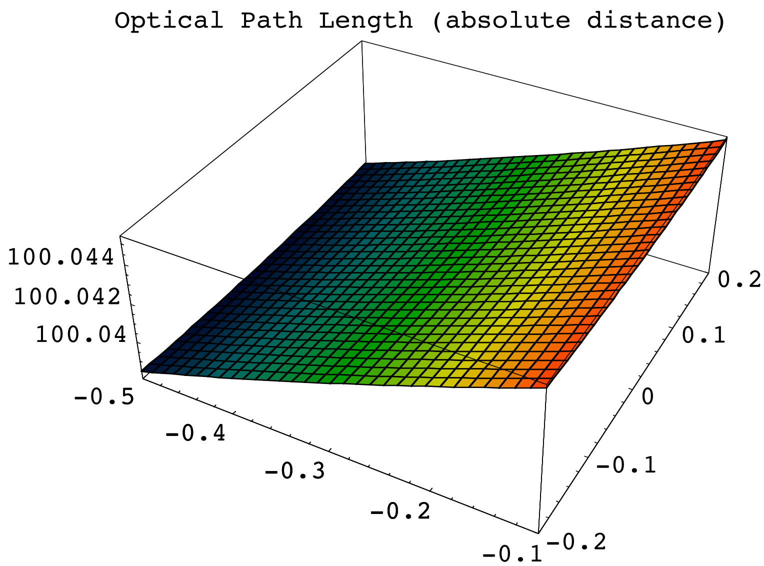
In[17]:=
```
sys = {
  MoveDirected[PointOfRays[{2,2}],{0,-2},{100,0},SideOfObject->After],
  MoveDirected[GridOfRays[{5,5}],{0,2},{100,0},SideOfObject->After],
  Move[Screen[100],100]};
```

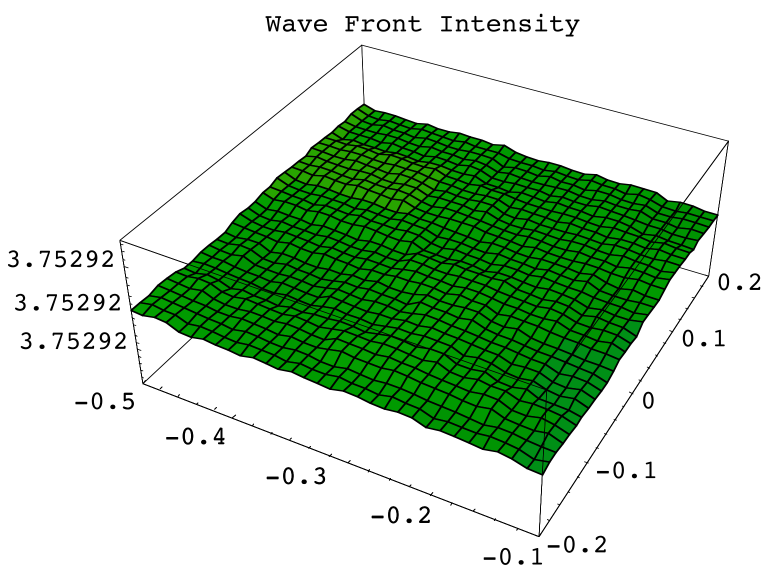
In[18]:=
```
DrawSystem[sys,PlotType->TopView];
```

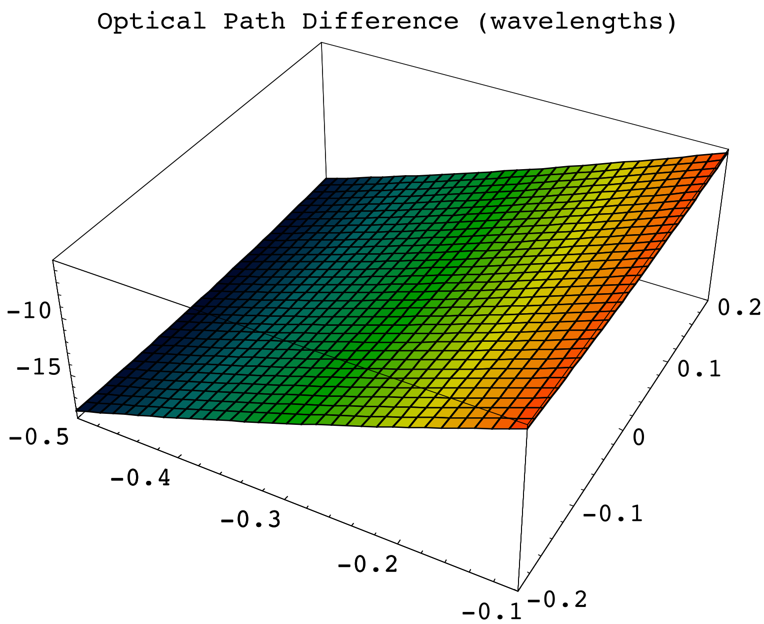


In[19]:=
```
wavefront = FindWaveFronts[sys,PlotDomain->{{-.5,-.1},{-.2,.2}}]
```

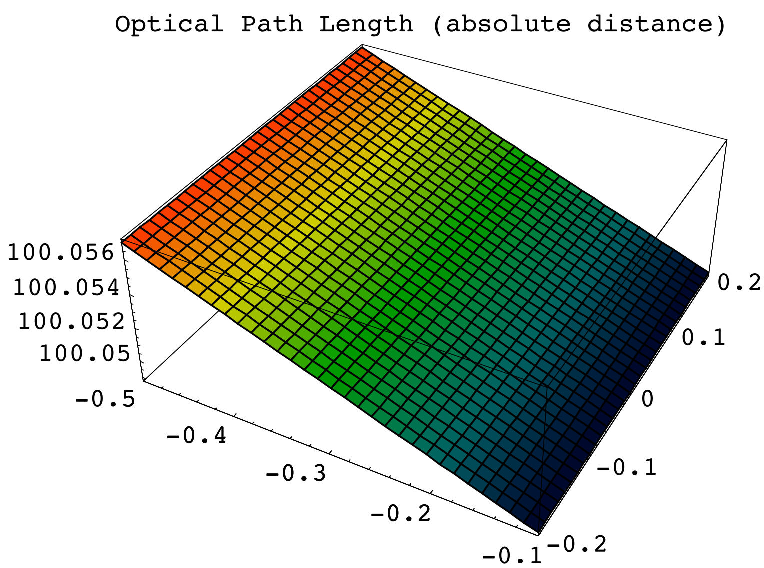
Surface Location :
{ComponentNumber → 1, SurfaceNumber → 1, WaveFrontID → 1, Source → 1}

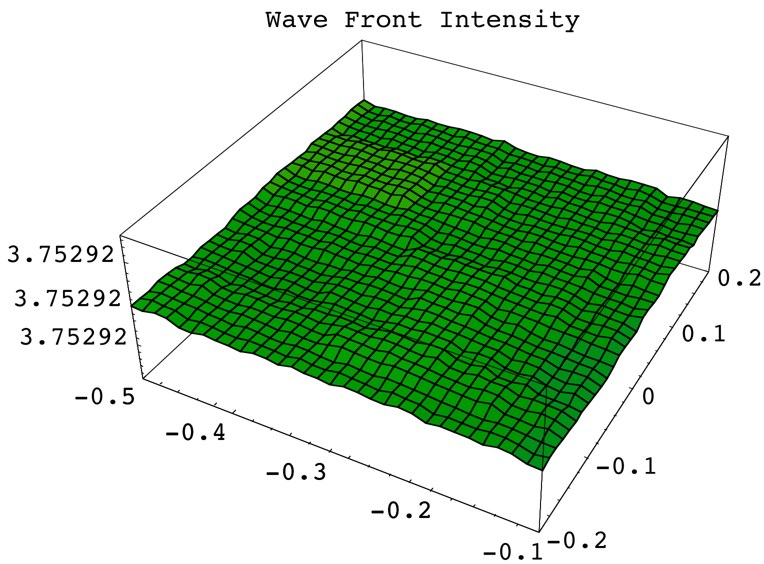

Optical Path Length (absolute distance)
100.044
100.042
100.04
-0.5
-0.4
-0.3
-0.2
-0.1
-0.2
-0.1
0
0.1
0.2

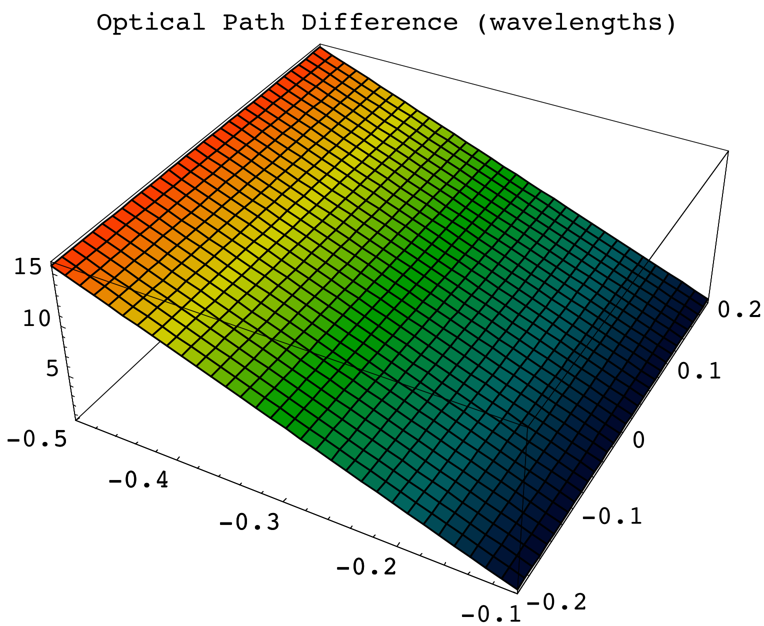

Wave Front Intensity
3.75292
3.75292
3.75292
-0.5
-0.4
-0.3
-0.2
-0.1
-0.2
-0.1
0
0.1
0.2



Surface Location :
{ComponentNumber → 1, SurfaceNumber → 1, WaveFrontID → 1, Source → 2}



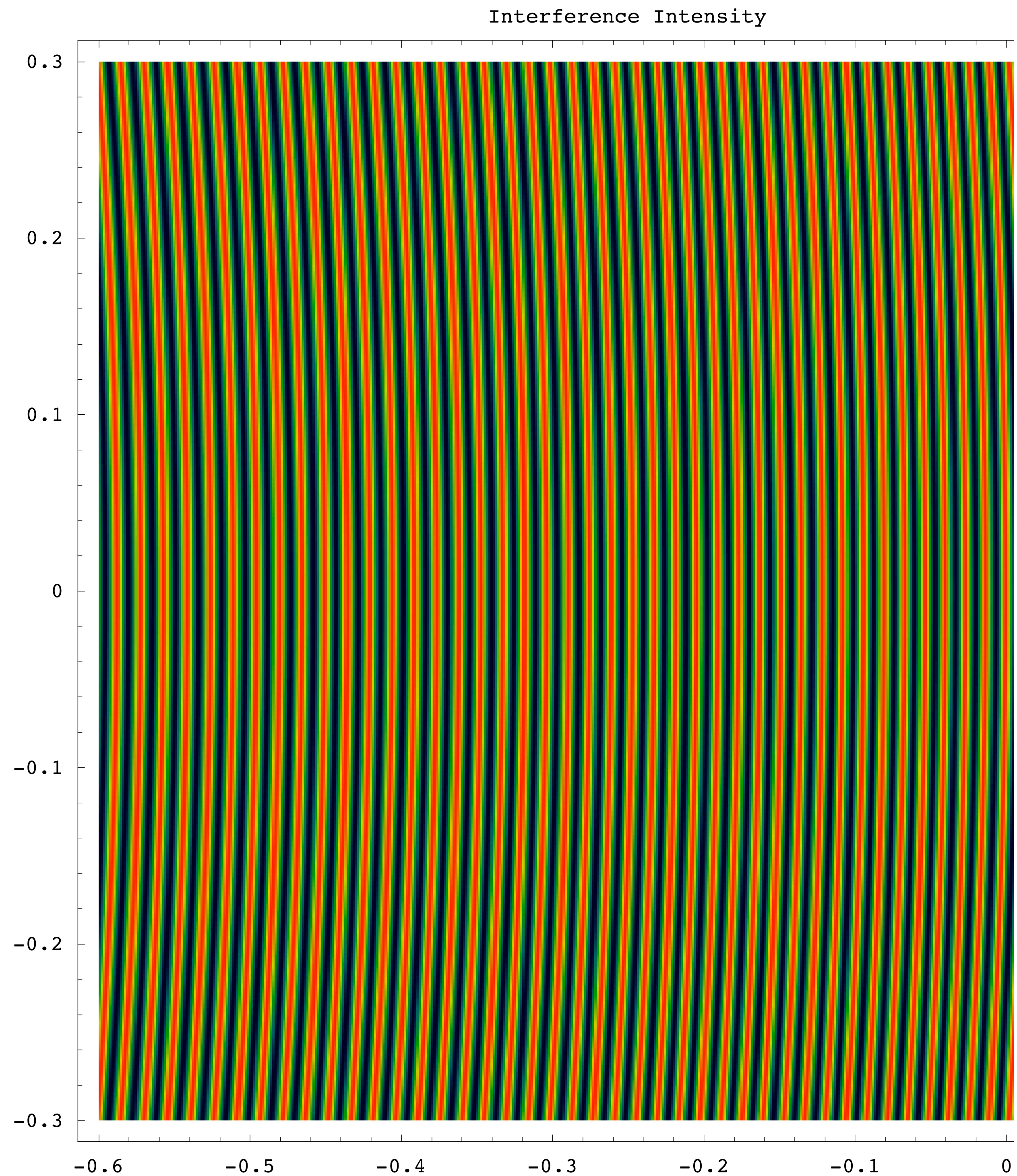





```
Out[19]= {{ComponentNumber → 1, Energy → 100.,
    Offset → 100.048, OpticalLengthFunction → CompiledFunction[
      If[#1 == 0 && #2 == 0, 100.047, 100.062 + 0.0199973 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] -
        9.98984 × 10^-7 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) + 0.0152494 (-1 + 0.327788 (#1^2 + #2^2)) -
        5.0179 × 10^-6 Cos[ArcTan[#1, #2]] (-0.809677 Sqrt[#1^2 + #2^2] + 0.199052 (#1^2 + #2^2)^(3/2)) +
        1.39325 × 10^-9 Cos[2 ArcTan[#1, #2]] (-0.491682 (#1^2 + #2^2) + 0.107445 (#1^2 + #2^2)^2) -
        7.74051 × 10^-7 (1 - 0.983364 (#1^2 + #2^2) + 0.161168 (#1^2 + #2^2)^2) +
        6.88032 × 10^-10 Cos[ArcTan[#1, #2]]
          (1.21451 Sqrt[#1^2 + #2^2] - 0.796207 (#1^2 + #2^2)^(3/2) + 0.108745 (#1^2 + #2^2)^(5/2))],
      -CompiledCode-], OpticalPathDifference → CompiledFunction[
```

```
    If[#1 == 0 && #2 == 0, -2.17721, 26.4886 + 37.589 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] -
      0.00187779 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) +
      1.87611×10^-7 Cos[3 ArcTan[#1, #2]] (#1^2 + #2^2)^(3/2) + 28.6644 (-1 + 0.327788 (#1^2 + #2^2)) -
      0.00943214 Cos[ArcTan[#1, #2]] (-0.809677 Sqrt[#1^2 + #2^2] + 0.199052 (#1^2 + #2^2)^(3/2)) +
      2.6187×10^-6 Cos[2 ArcTan[#1, #2]] (-0.491682 (#1^2 + #2^2) + 0.107445 (#1^2 + #2^2)^2) -
      0.00145498 (1 - 0.983364 (#1^2 + #2^2) + 0.161168 (#1^2 + #2^2)^2) - 7.6866×10^-10
       Cos[3 ArcTan[#1, #2]] (-0.265402 (#1^2 + #2^2)^(3/2) + 0.0543723 (#1^2 + #2^2)^(5/2)) +
      1.29321×10^-6 Cos[ArcTan[#1, #2]] (1.21451 Sqrt[#1^2 + #2^2] - 0.796207 (#1^2 + #2^2)^(3/2) +
         0.108745 (#1^2 + #2^2)^(5/2)) - 5.52714×10^-10 Cos[2 ArcTan[#1, #2]]
       (0.983364 (#1^2 + #2^2) - 0.537225 (#1^2 + #2^2)^2 + 0.066036 (#1^2 + #2^2)^3) +
      1.32838×10^-7 (-1 + 1.96673 (#1^2 + #2^2) - 0.805838 (#1^2 + #2^2)^2 + 0.088048 (#1^2 + #2^2)^3)],
    -CompiledCode-], OpticalPathRange -> {100.027, 100.112},
  OutputGraphics -> {OpticalLengthFunction -> (- SurfaceGraphics -),
    WaveFrontIntensity -> (- SurfaceGraphics -),
    OpticalPathDifference -> (- SurfaceGraphics -)},
  PlotDomain -> {{-0.5, -0.1}, {-0.2, 0.2}},
  RayBoundary ->
   {{-1.74586, 1.74708}, {-1.7462, 1.7462}},
  RefractiveIndex -> 1.00027,
  ResidualFitError -> 1.96537×10^-10,
  Source -> 1,
  SpatialScale -> 1,
  SurfaceNumber -> 1,
  WaveFrontID -> 1,
  WaveFrontIntensity ->
   InterpolatingFunction[{{-3.35596, 3.35596}, {-3.35546, 3.35546}}, <>],
  WaveLength -> 0.532, WaveTilt -> WaveTilt},
 {ComponentNumber -> 1, Energy -> 100.,
  Offset -> 100.049,
  OpticalLengthFunction ->
   CompiledFunction[If[#1 == 0 && #2 == 0, 100.047,
     100.047 - 0.0200014 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] ], -CompiledCode-],
  OpticalPathDifference -> CompiledFunction[If[#1 == 0 && #2 == 0, -3.03259,
     -3.03259 - 37.5966 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] ], -CompiledCode-],
  OpticalPathRange -> {99.9969, 100.097}, OutputGraphics ->
   {OpticalLengthFunction -> (- SurfaceGraphics -),
    WaveFrontIntensity -> (- SurfaceGraphics -),
    OpticalPathDifference -> (- SurfaceGraphics -)},
  PlotDomain -> {{-0.5, -0.1}, {-0.2, 0.2}},
  RayBoundary -> {{-2.5005, 2.5005}, {-2.5, 2.5}},
  RefractiveIndex -> 1.00027,
  ResidualFitError -> 1.57566×10^-11,
  Source -> 2, SpatialScale -> 1,
  SurfaceNumber -> 1, WaveFrontID -> 1,
  WaveFrontIntensity ->
   InterpolatingFunction[{{-3.35596, 3.35596}, {-3.35546, 3.35546}}, <>],
  WaveLength -> 0.532, WaveTilt -> WaveTilt}}
```

```
In[34]:= FindInterference[wavefront,PlotDomain->{{-.6,.1},{-.3,.3}}, PlotPoints->500]
```




```
Out[34]= {InterferenceFunction → CompiledFunction[-intensity data-],
  OutputGraphics → InterferenceFunction → (- DensityGraphics -),
  ResidualFitError → {1.96537 × 10^-10, 1.57566 × 10^-11}, WaveLength → 0.532,
  SpatialScale → 1, OpticalPathRange → {99.9969, 100.112},
  RayBoundary → {{-1.74586, 1.74708}, {-1.7462, 1.7462}}}
```

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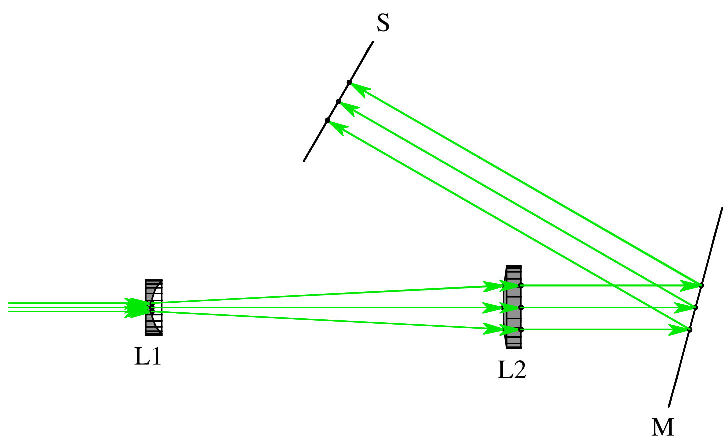
Example 2

```
In[21]:= L1=PlanoConcaveLens[-30,20,2,"L1",CurvatureDirection->Back];
         L2=PlanoConvexLens[160,30,6.5,"L2"];

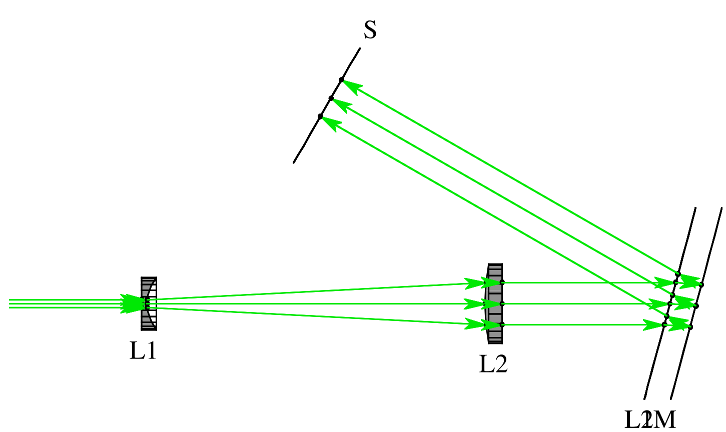
In[23]:= sys1 = {
         Move[GridOfRays[{3,3}],-50],
         L1,
         Move[L2,130],
         MoveReflected[Mirror[75],{130,0},{200,0},{70,75}],
         MoveDirected[Screen[50],{70,75},{200,0}]};

In[24]:= DrawSystem[sys1,PlotType->TopView,FormatType->OutputForm];
```



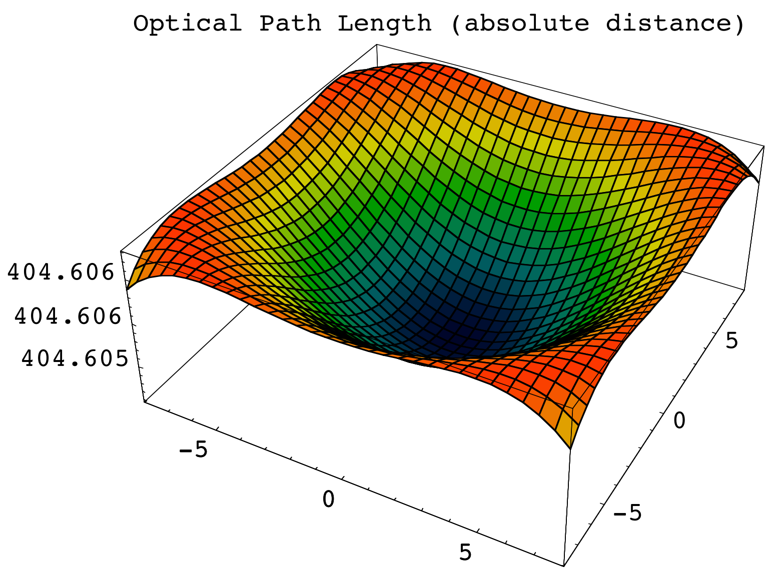

```
In[25]:= sys2 = {
         Move[GridOfRays[{3,3}],-50],
         L1,
         Move[L2,130],
         MoveReflected[LensSurface[75],{130,0},{200,0},{70,75}],
         Move[MoveReflected[Mirror[75],{130,0},{200,0},{70,75}],10],
         MoveReflected[LensSurface[75],{130,0},{200,0},{70,75}],
         MoveDirected[Screen[50],{70,75},{200,0}]};

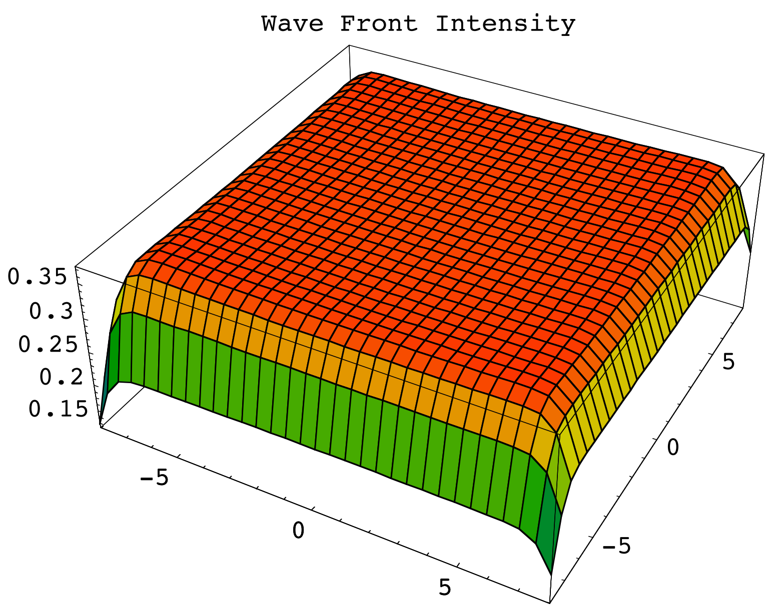
In[26]:= DrawSystem[sys2,PlotType->TopView,FormatType->OutputForm];
```

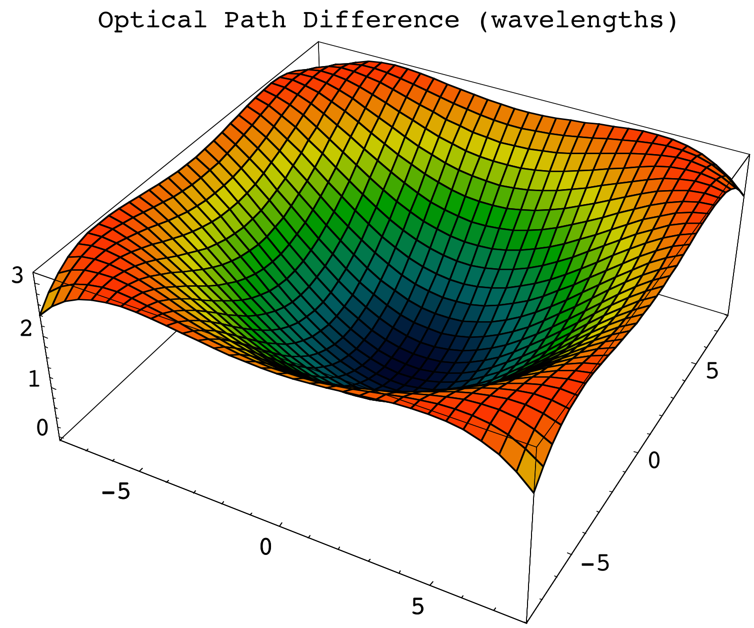



```
In[27]:= wavefront1 = FindWaveFronts[sys1,ZernikeFit->True]//Timing
```

```
Surface Location :
 {ComponentNumber → 4, SurfaceNumber → 1, WaveFrontID → 1, Source → 1}
```



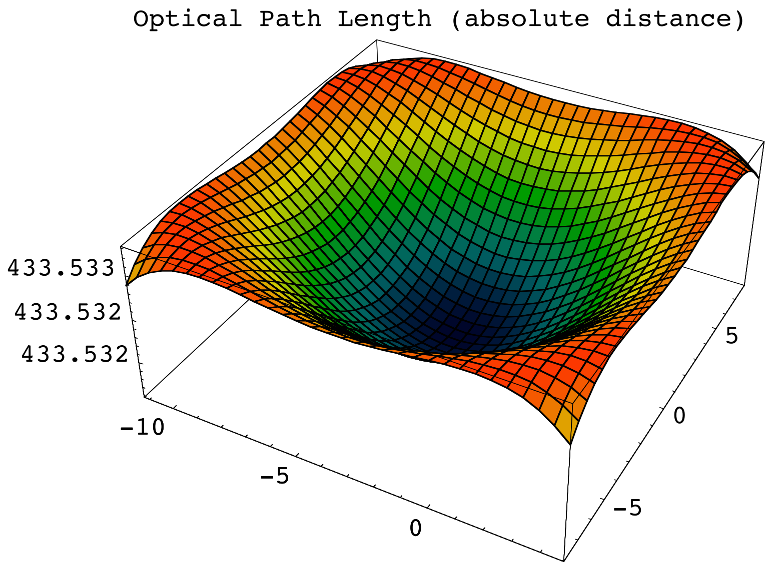


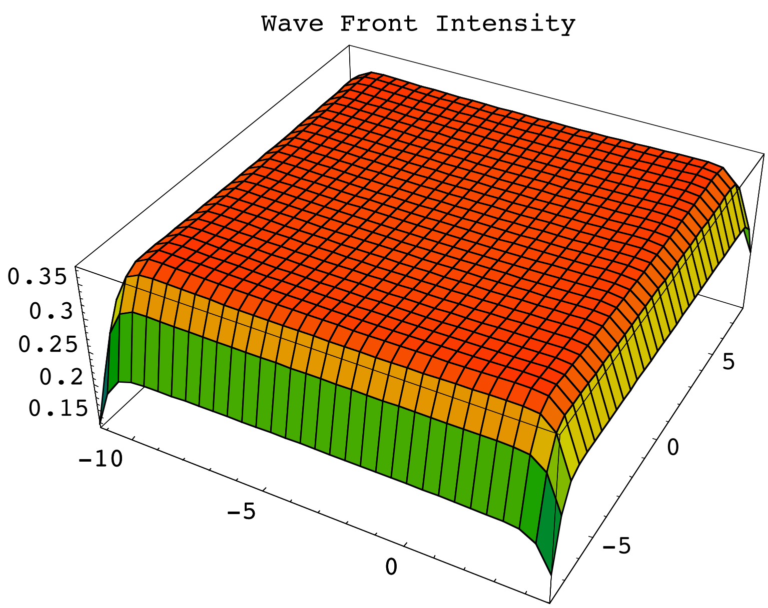


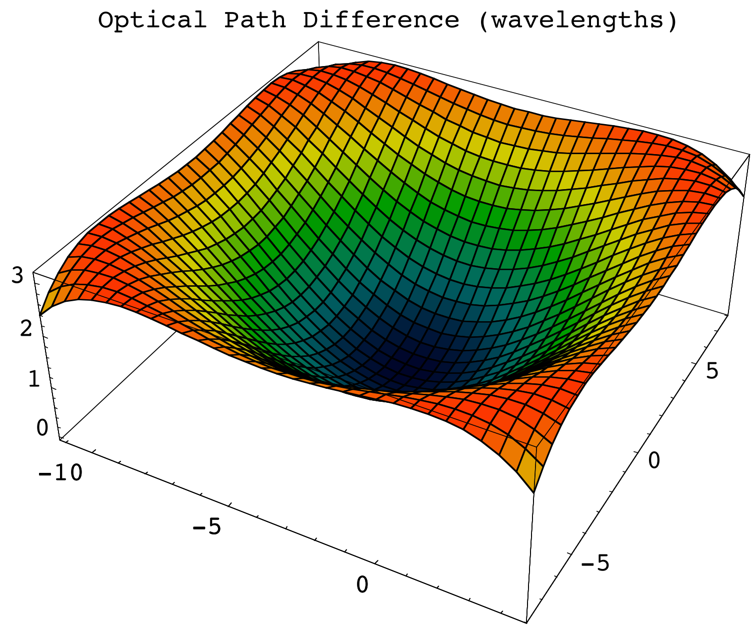

```
Out[27]=  {15.6 Second, {{ComponentNumber → 4, Energy → 100.,
     Offset → 404.605, OpticalLengthFunction → CompiledFunction[
       If[#1 == 0 && #2 == 0, 404.605, 404.606 + 0.000649723 (-1 + 0.015586 (#1^2 + #2^2)) -
         0.000601516 (1 - 0.0467581 (#1^2 + #2^2) + 0.000364386 (#1^2 + #2^2)^2) + 2.00872 × 10^-7
          Cos[4 ArcTan[#1, #2]] (-0.000303655 (#1^2 + #2^2)^2 + 2.83966 × 10^-6 (#1^2 + #2^2)^3) -
         0.000010641 (-1 + 0.0935161 (#1^2 + #2^2) - 0.00182193 (#1^2 + #2^2)^2 +
            9.46555 × 10^-6 (#1^2 + #2^2)^3)], -CompiledCode-],
     OpticalPathDifference → CompiledFunction[If[#1 == 0 && #2 == 0, -0.00912271, 2.32283 +
         2.59445 × 10^-8 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 + 1.22128 (-1 + 0.015586 (#1^2 + #2^2)) -
         1.13067 (1 - 0.0467581 (#1^2 + #2^2) + 0.000364386 (#1^2 + #2^2)^2) +
         0.000377581 Cos[4 ArcTan[#1, #2]]
          (-0.000303655 (#1^2 + #2^2)^2 + 2.83966 × 10^-6 (#1^2 + #2^2)^3) - 0.0200019
          (-1 + 0.0935161 (#1^2 + #2^2) - 0.00182193 (#1^2 + #2^2)^2 + 9.46555 × 10^-6 (#1^2 + #2^2)^3)],
       -CompiledCode-], OpticalPathRange → {404.605, 404.606},
     OutputGraphics → {OpticalLengthFunction → (- SurfaceGraphics -),
       WaveFrontIntensity → (- SurfaceGraphics -),
       OpticalPathDifference → (- SurfaceGraphics -)},
     RayBoundary → {{-8.01, 8.01}, {-8.01, 8.01}},
     RefractiveIndex → 1.00027,
     ResidualFitError → 0.000124497,
     Source → 1,
     SpatialScale → 1,
     SurfaceNumber → 1,
     WaveFrontID → 1,
     WaveFrontIntensity →
      InterpolatingFunction[{{-10.7505, 10.7505}, {-10.7505, 10.7505}}, <>],
     WaveLength → 0.532, WaveTilt → WaveTilt,
     ZernikeFit → True}}}

In[28]:= wavefront2 = FindWaveFronts[sys2]//Timing
```

```
Surface Location :
 {ComponentNumber → 6, SurfaceNumber → 1, WaveFrontID → 1, Source → 1}
```



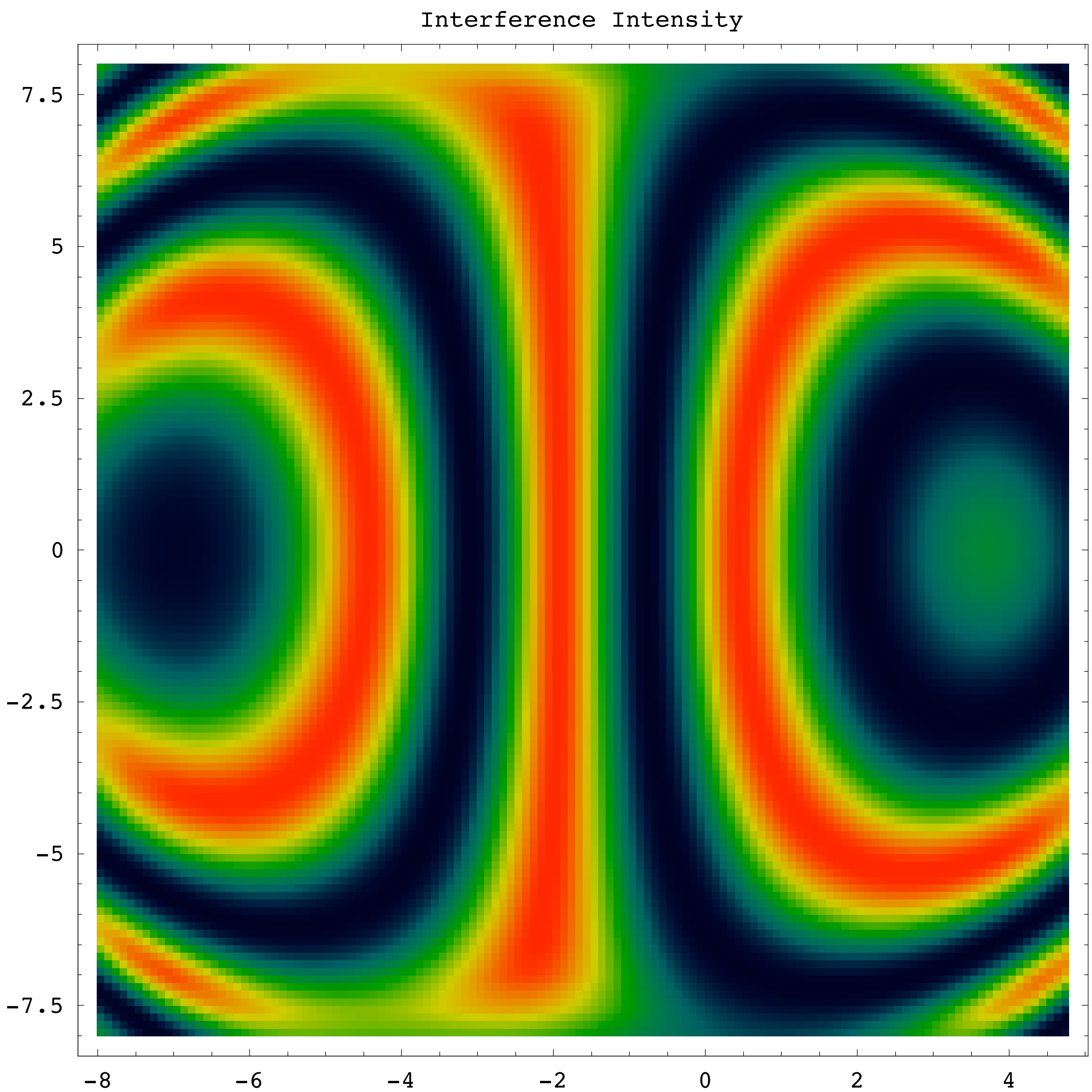





```
Out[28]= {17.85 Second, {{ComponentNumber → 6, Energy → 100., Offset → 433.531,
    OpticalLengthFunction → CompiledFunction[If[#1 == 0 && #2 == 0, 433.531,
      433.532 - 0.000157762 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] - 5.13798×10^-6
        Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) - 5.92895×10^-9 Cos[3 ArcTan[#1, #2]] (#1^2 + #2^2)^(3/2) -
       0.00136863 (-1 + 0.0105011 (#1^2 + #2^2)) - 0.00271231 Cos[ArcTan[#1, #2]]
        (-0.144921 Sqrt[#1^2 + #2^2] + 0.00114138 (#1^2 + #2^2)^(3/2)) - 0.0000588622
        Cos[2 ArcTan[#1, #2]] (-0.0157517 (#1^2 + #2^2) + 0.000110273 (#1^2 + #2^2)^2) -
       0.00144447 (1 - 0.0315033 (#1^2 + #2^2) + 0.00016541 (#1^2 + #2^2)^2) + 1.07305×10^-6
        Cos[3 ArcTan[#1, #2]] (-0.00152184 (#1^2 + #2^2)^(3/2) + 9.98811×10^-6 (#1^2 + #2^2)^(5/2)) -
       0.00010194 Cos[ArcTan[#1, #2]] (0.217382 Sqrt[#1^2 + #2^2] -
          0.00456551 (#1^2 + #2^2)^(3/2) + 0.0000199762 (#1^2 + #2^2)^(5/2)) + 7.23666×10^-7
        Cos[4 ArcTan[#1, #2]] (-0.000137842 (#1^2 + #2^2)^2 + 8.68495×10^-7 (#1^2 + #2^2)^3) +
       4.16756×10^-8 Cos[2 ArcTan[#1, #2]] (0.0315033 (#1^2 + #2^2) -
          0.000551367 (#1^2 + #2^2)^2 + 2.17124×10^-6 (#1^2 + #2^2)^3) - 0.0000363751
        (-1 + 0.0630067 (#1^2 + #2^2) - 0.00082705 (#1^2 + #2^2)^2 + 2.89498×10^-6 (#1^2 + #2^2)^3)],
     -CompiledCode-], OpticalPathDifference → CompiledFunction[
     If[#1 == 0 && #2 == 0, 0.67783, 0.752007 - 0.296544 Cos[ArcTan[#1, #2]] Sqrt[#1^2 + #2^2] -
       0.00965786 Cos[2 ArcTan[#1, #2]] (#1^2 + #2^2) -
       0.0000111447 Cos[3 ArcTan[#1, #2]] (#1^2 + #2^2)^(3/2) +
       1.53072×10^-7 Cos[4 ArcTan[#1, #2]] (#1^2 + #2^2)^2 + 3.80542×10^-9
        Cos[5 ArcTan[#1, #2]] (#1^2 + #2^2)^(5/2) - 2.57262 (-1 + 0.0105011 (#1^2 + #2^2)) -
       5.09833 Cos[ArcTan[#1, #2]] (-0.144921 Sqrt[#1^2 + #2^2] + 0.00114138 (#1^2 + #2^2)^(3/2)) -
       0.110643 Cos[2 ArcTan[#1, #2]] (-0.0157517 (#1^2 + #2^2) + 0.000110273 (#1^2 + #2^2)^2) -
       2.71517 (1 - 0.0315033 (#1^2 + #2^2) + 0.00016541 (#1^2 + #2^2)^2) + 0.00201701
```

```
    Cos[3 ArcTan[#1, #2]] (-0.00152184 (#1^2 + #2^2)^(3/2) + 9.98811×10^-6 (#1^2 + #2^2)^(5/2)) -
   0.191616 Cos[ArcTan[#1, #2]] (0.217382 Sqrt[#1^2 + #2^2] - 0.00456551 (#1^2 + #2^2)^(3/2) +
      0.0000199762 (#1^2 + #2^2)^(5/2)) + 0.00136027 Cos[4 ArcTan[#1, #2]]
    (-0.000137842 (#1^2 + #2^2)^2 + 8.68495×10^-7 (#1^2 + #2^2)^3) +
   0.0000783413 Cos[2 ArcTan[#1, #2]] (0.0315033 (#1^2 + #2^2) -
      0.000551367 (#1^2 + #2^2)^2 + 2.17124×10^-6 (#1^2 + #2^2)^3) - 0.0683743
    (-1 + 0.0630067 (#1^2 + #2^2) - 0.00082705 (#1^2 + #2^2)^2 + 2.89498×10^-6 (#1^2 + #2^2)^3)],
  -CompiledCode-], OpticalPathRange → {433.531, 433.533},
OutputGraphics → {OpticalLengthFunction → (- SurfaceGraphics -),
  WaveFrontIntensity → (- SurfaceGraphics -),
  OpticalPathDifference → (- SurfaceGraphics -)},
RayBoundary → {{-11.2367, 4.7871}, {-8.01199, 8.01199}},
RefractiveIndex →
 1.00027,
ResidualFitError → 0.000136554,
Source → 1,
SpatialScale → 1,
SurfaceNumber → 1,
WaveFrontID → 1,
WaveFrontIntensity →
 InterpolatingFunction[{{-13.9779, 7.52824}, {-10.7531, 10.7531}}, <>],
WaveLength → 0.532,
WaveTilt → WaveTilt}}}
```

In[29]:= **FindInterference[Join[wavefront1[[2]],wavefront2[[2]]]]//Timing**




```
Out[29]= {2.52 Second, {InterferenceFunction → CompiledFunction[-intensity data-],
   OutputGraphics → InterferenceFunction → (- DensityGraphics -),
   ResidualFitError → {0.000124497, 0.000136554}, WaveLength → 0.532, SpatialScale → 1,
   OpticalPathRange → {404.605, 433.533}, RayBoundary → {{-8.01, 4.7871}, {-8.01, 8.01}}}}
```

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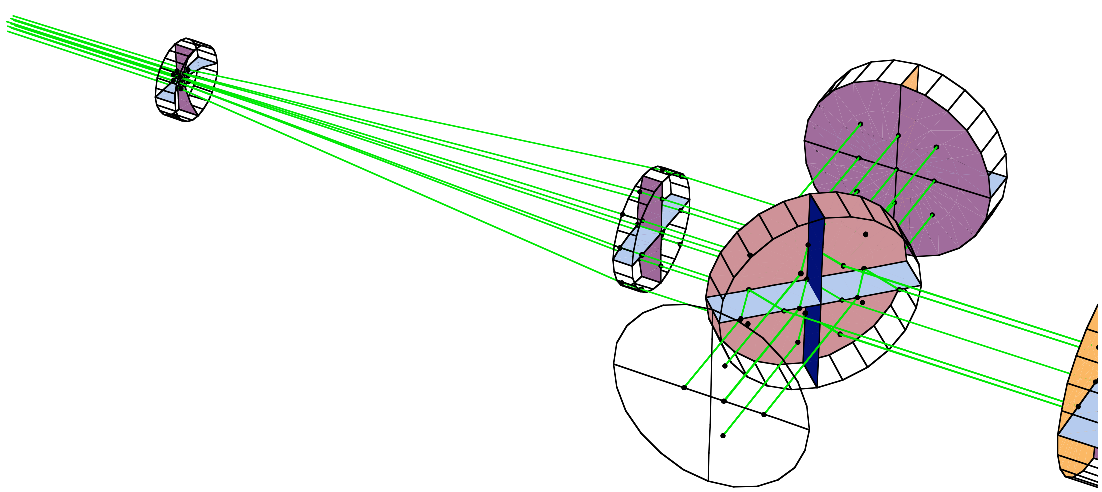
Example 3

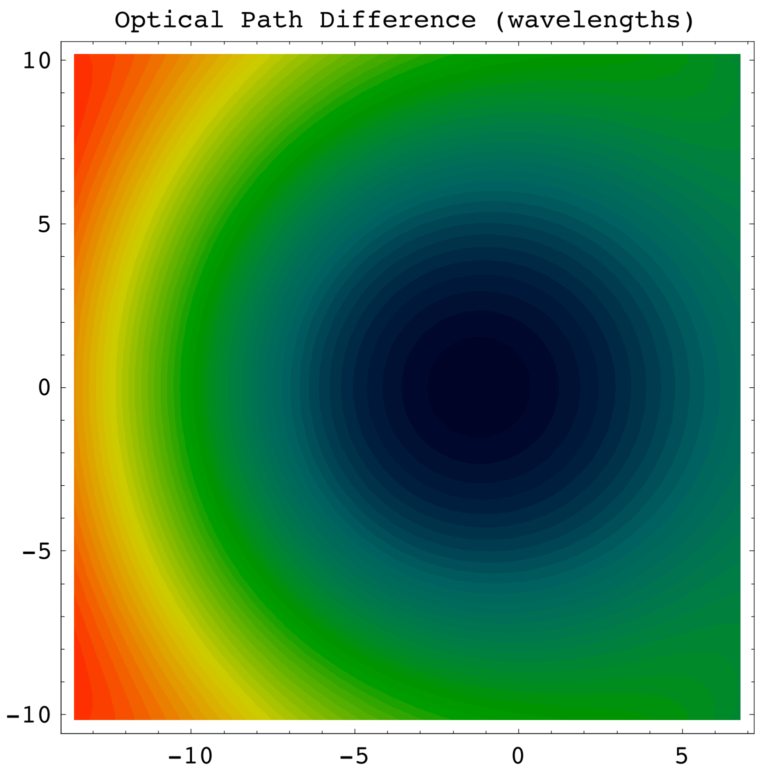
In[30]:=
```
system =
    {
        Move[GaussianBeam[1.5,.01,FullForm->True],-50],
        PlanoConcaveLens[-30,20,2,"L1",CurvatureDirection->Back],
        Move[PlanoConvexLens[160,30,6.5,"L2"],130],
        Move[BeamSplitter[{70,30},50,10],175,-45],
        Move[Mirror[50,10],250],
        Move[Mirror[50,10],{175,50},90.01],
        Move[BeamSplitter[{70,30},50,10,""],175,-45,GraphicDesign->Off],
        Move[Screen[50],{175,-50},90]
    };
```

In[31]:=
```
AnalyzeSystem[system];
```

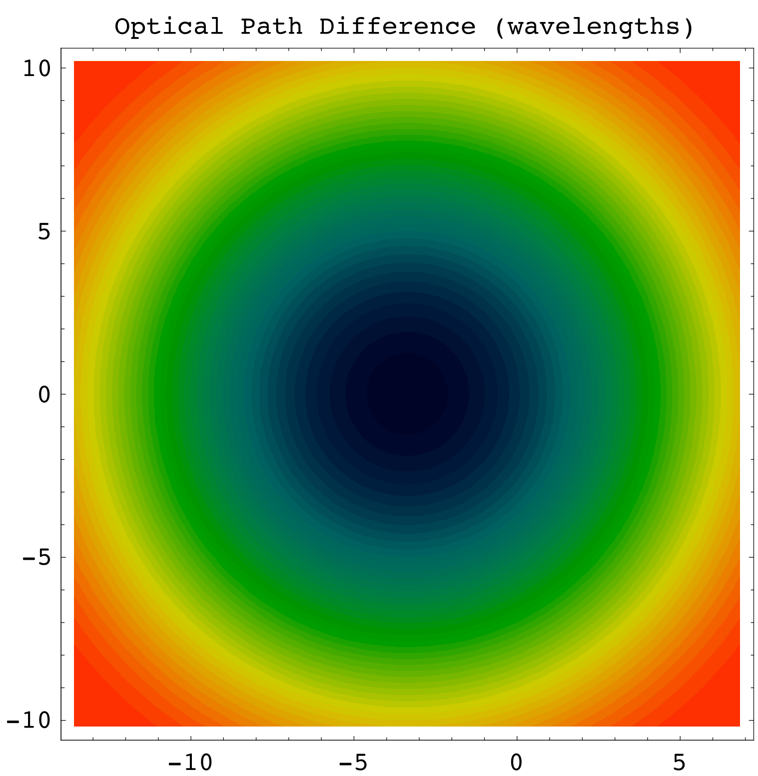


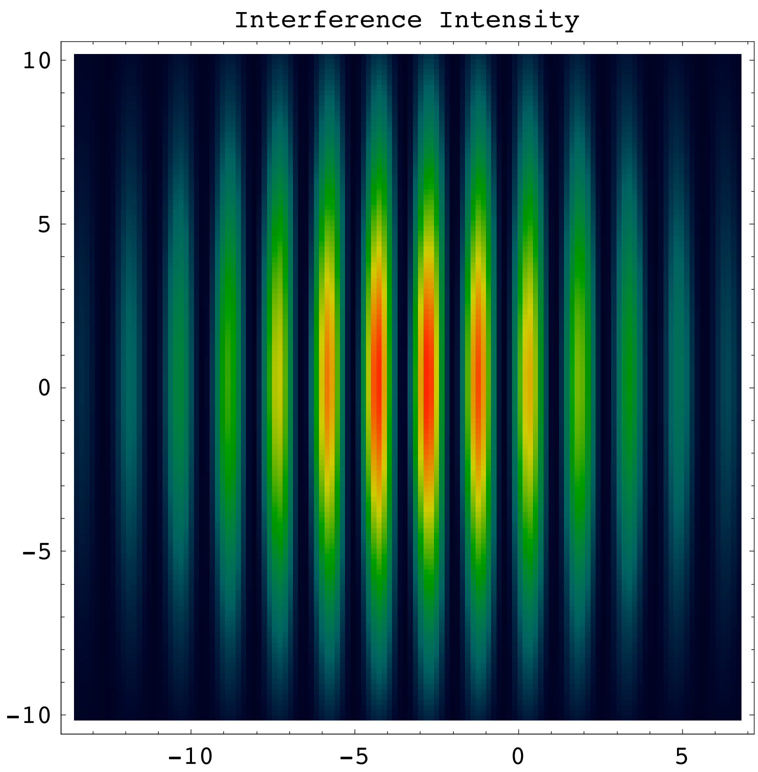
In[32]:=
```
interference = FindInterference[system]
```

Surface Location :
{ComponentNumber → 7, SurfaceNumber → 1, WaveFrontID → 1, Source → 1}



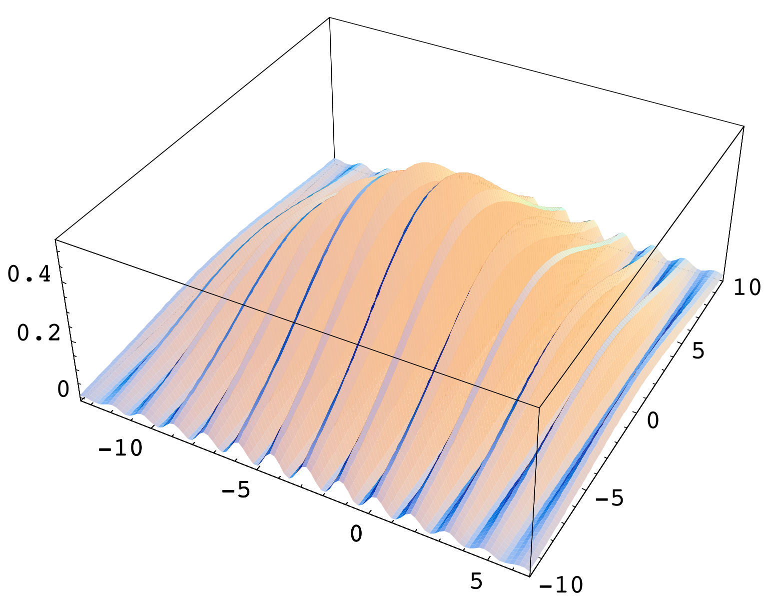
Surface Location :
{ComponentNumber → 7, SurfaceNumber → 1, WaveFrontID → 2, Source → 1}






```
Out[32]= {InterferenceFunction → CompiledFunction[-intensity data-],
  OutputGraphics → InterferenceFunction → (- DensityGraphics -),
  ResidualFitError → {0.000682003, 0.000962582}, WaveLength → 0.532,
  SpatialScale → 1, OpticalPathRange → {385.89, 448.662},
  RayBoundary → {{-13.5559, 6.78875}, {-10.1758, 10.1758}}}
```

```
In[33]:= Plot3D[Evaluate[(InterferenceFunction/.interference)][x,y],{x,-13.5,6.7},{y,-10.,10.}
  ,PlotPoints->100,Mesh->False];
```



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6.4 Wavefront Propagation with Gaussian Wavelets

Introduction to Wavelet Analysis

Wavelet analysis is a very new concept that is still in its infancy. This section introduces the idea and uses it in several examples. At the moment, there are two special functions used here for wavelet analysis: a new light source, called **LineOfWavelets** that constructs a set of **GaussianBeam** wavelet functions, and a special function, called **FindField** that calculates the interference between the different Gaussian wavelets. Both **LineOfWavelets** and **FindField** will eventually become built-in functions of Wavica, along with several other functions that includes a point source function called **WedgeOfWavelets**, but for the moment, they are defined here externally until their final formats are fully worked out. The permanent versions of these two functions may be altered and their present formats may not get supported in the future. Presently, these functions must be taken as "experimental". Currently, all calculations are only carried out in the two-dimensions of the optical and transverse axes. In the near future, there will be additional support for full three-dimensional calculations with the **GridOfWavelets** and **PointOfWavelets** source functions to be developed as well.

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Define LineOfWavelets (Evaluate This Subsection)

In this section, we define **LineOfWavelets**. You must evaluate the following two subsections before running any wavelet analysis examples.

In[517]:=
```
Clear[LineOfWavelets];
```

In[518]:=
```
Options[LineOfWavelets]:={
    PaddingFactor->1,
    NumberOfRays->11,
    WaveLength->.532,
    IntrinsicMedium->Vacuum};
```

In[519]:=

```
LineOfWavelets[linewidthin_,opts___]:=
Block[{divergence, y, options, beamspotsize, paddingfactor, numberofrays, wavelength,
beamoffset, intrinsicmedium, refractiveindex, linewidth},
    options = Flatten[{opts,Options[LineOfWavelets]}];
    {paddingfactor, numberofrays, wavelength, intrinsicmedium} =
        {PaddingFactor, NumberOfRays, WaveLength, IntrinsicMedium}/.options;
    If[paddingfactor<1&&numberofrays>1,
        Print["Warning, LineOfWavelets will not be uniform."];
        Print["Set PaddingFactor greater or equal to 1 for uniform illumination."]
    ];
    If[numberofrays>1,
        linewidth = linewidthin;
        beamoffset = linewidth/(numberofrays-1)
    ,
        beamoffset = linewidthin;
        linewidth = 0
    ];
    beamspotsize = beamoffset*paddingfactor;
    refractiveindex = ModelRefractiveIndex[intrinsicmedium][options];
    divergence = 2*(wavelength*10^-3)/(Pi*refractiveindex*beamspotsize)+10^-10;
    Table[
        Move[GaussianBeam[beamspotsize,divergence,options],{0,y}],
        {y,-linewidth/2,linewidth/2,beamoffset}]
];
```

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Define FindField (Evaluate This Subsection)

In this section, we define **FindField**. **FindField** returns a symbolic function that represents either the field intensity (for **IntensityTransform->True**) or the complex optical field (**IntensityTransform->False**). You must evaluate this subsection (as well as the **LineOfWavelets** subsection) before running any wavelet analysis examples.

In[520]:=

```
Clear[FindField];
```

In[521]:=

```
Options[FindField]:=
    {IntensityTransform->True,
    CoherentSource->True,
    NormalizeOutput->True,
    Compiled->True};
```

In[522]:=

```
FindField[opticalsystem_,opts___] :=
Block[{rp, sp, xc, yc, xp, yp, tx, ty, sc, zo, wo, W, R, field, k, wl, ol, minOL,
gaussianresults, ds, scale, intensity, results, sourcewaist, width, centers, widths,
options, intensitytransform, coherentsource, normalizeoutput, compiled},
    options = Flatten[{opts,Options[FindField]}];
        {intensitytransform, coherentsource, normalizeoutput, compiled} =
            {IntensityTransform, CoherentSource, NormalizeOutput, Compiled}/.options;
    gaussianresults = GaussianTrace[opticalsystem, ABCDConstruction->Horizontal,
        NumericalResults->True, ReportedSurfaces->{-1}];
    If[Head[gaussianresults[[1]]]=!=List,gaussianresults={gaussianresults}];
    Clear[xp,yp];
    centers = widths = {};
    minOL = Sort[OpticalLength/.gaussianresults][[1]];
```

```
    scale = 1/Length[gaussianresults];
    field = Map[
        (
        {zo,wo,sc,wl,ol,intensity} =
{BeamScaleLength,BeamWaist,WaistDistance,WaveLength*10^-3,OpticalLength,Intensity/100}/
.#;
        sc = -sc;
        {tx,ty} = (ABCDRotationMatrix/.#)[[1]][[{1,2}]];
        {xc,yc} = (ABCDCenterPoint/.#)[[{1,2}]];
        centers = {centers,yc};
        k = 2 Pi/wl;
        ds = Dot[{xp,yp-yc},{tx,ty}];
        rpSq = (xp)^2+(yp-yc)^2-ds^2;
        ol = ol - minOL + ds;
        sp = sc + ds;
        W = wo*Sqrt[1+(sp/zo)^2];
        width = W/.{xp->0,yp->0};
        widths = {widths,width};
        R = sp*(1+(zo/sp)^2);
        energy =
NIntegrate[((Exp[-rpSq/W^2])^2/W)/.xp->0,{yp,yc-3*width,yc+3*width},MaxRecursion->12];
        Sqrt[intensity/W]*
Exp[-rpSq/W^2]*Exp[-I(k*ol-ArcTan[zo,sp])]*Exp[-I*k*rpSq/(2*R)]/Sqrt[energy]
        )&
    ,
        gaussianresults
    ];
    If[intensitytransform === True,
        If[coherentsource === True,
            field = Abs[Apply[Plus,field]]^2
        ,
            field = Apply[Plus,Map[(Abs[#]^2)&,field]]
        ];
        centers = Sort[Flatten[centers]][[{1,-1}]];
        width = Sort[Flatten[widths]][[-1]];
        If[normalizeoutput=!=False,
            field =
field/NIntegrate[field/.xp->0,{yp,-3*width+centers[[1]],3*width+centers[[2]]}]
        ];
    ,
        If[normalizeoutput=!=False,
            field = scale*Apply[Plus,field]
        ,
            field = Apply[Plus,field]
        ]
    ];
    field = field/.{xp->#1,yp->#2};
    If[compiled===True,
        Apply[Compile,{{#1,#2}, field}]
    ,
        Apply[Function,{{#1,#2}, field}]
    ]
]
```

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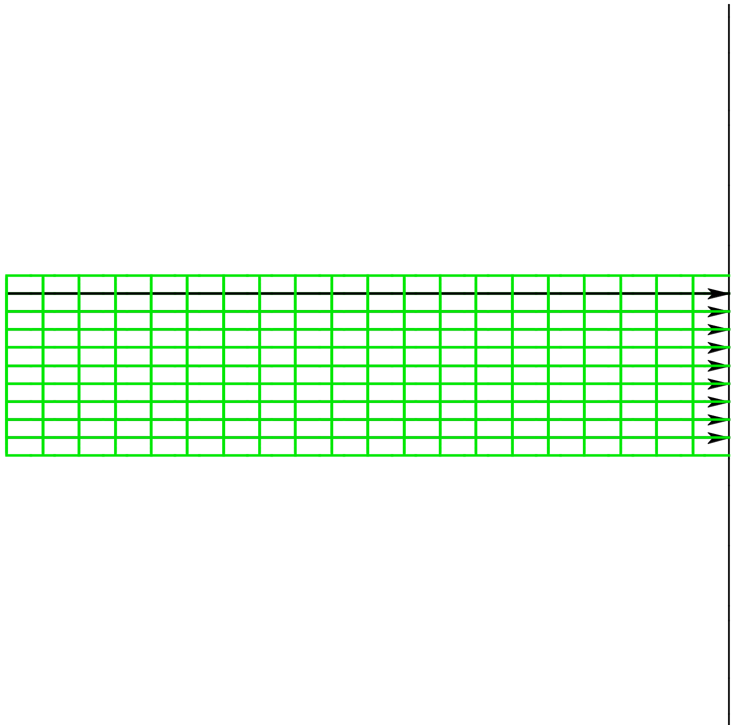
Collimated Beam of Wavelets

```
In[46]:= waveletSystem = {
           LineOfWavelets[10,NumberOfRays->9,PaddingFactor->1],
           Move[Screen[50],50]}
```

```
Out[46]= {{Move[GaussianBeam[5/4, 0.000270945,
             IntrinsicMedium → Vacuum, NumberOfRays → 9, PaddingFactor → 1], {0, -5.}],
          Move[GaussianBeam[5/4, 0.000270945, IntrinsicMedium → Vacuum,
             NumberOfRays → 9, PaddingFactor → 1], {0, -3.75}],
          Move[GaussianBeam[5/4, 0.000270945, IntrinsicMedium → Vacuum,
             NumberOfRays → 9, PaddingFactor → 1], {0, -2.5}],
          Move[GaussianBeam[5/4, 0.000270945, IntrinsicMedium → Vacuum, NumberOfRays → 9,
             PaddingFactor → 1], {0, -1.25}], GaussianBeam[5/4, 0.000270945,
            IntrinsicMedium → Vacuum, NumberOfRays → 9, PaddingFactor → 1],
          Move[GaussianBeam[5/4, 0.000270945, IntrinsicMedium → Vacuum,
             NumberOfRays → 9, PaddingFactor → 1], {0, 1.25}],
          Move[GaussianBeam[5/4, 0.000270945, IntrinsicMedium → Vacuum,
             NumberOfRays → 9, PaddingFactor → 1], {0, 2.5}],
          Move[GaussianBeam[5/4, 0.000270945, IntrinsicMedium → Vacuum,
             NumberOfRays → 9, PaddingFactor → 1], {0, 3.75}],
          Move[GaussianBeam[5/4, 0.000270945, IntrinsicMedium → Vacuum,
             NumberOfRays → 9, PaddingFactor → 1], {0, 5.}]}, Move[Screen[50], 50.]}
```

```
In[47]:= ShowSystem[waveletSystem,ShowGaussian->True,PlotType->TopView];
```



```
In[48]:= intensity = FindField[waveletSystem]
```

Out[48]= CompiledFunction[0.0482256,

$$\text{Abs}\Bigg[\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,(-5+\#2)^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,(-5+\#2)^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}+$$

$$\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,\left(-\frac{15}{4}+\#2\right)^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,\left(-\frac{15}{4}+\#2\right)^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}+$$

$$\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,\left(-\frac{5}{2}+\#2\right)^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,\left(-\frac{5}{2}+\#2\right)^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}+$$

$$\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,\left(-\frac{5}{4}+\#2\right)^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,\left(-\frac{5}{4}+\#2\right)^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}+$$

$$\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,\#2^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,\#2^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}+$$

$$\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,\left(\frac{5}{4}+\#2\right)^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,\left(\frac{5}{4}+\#2\right)^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}+$$

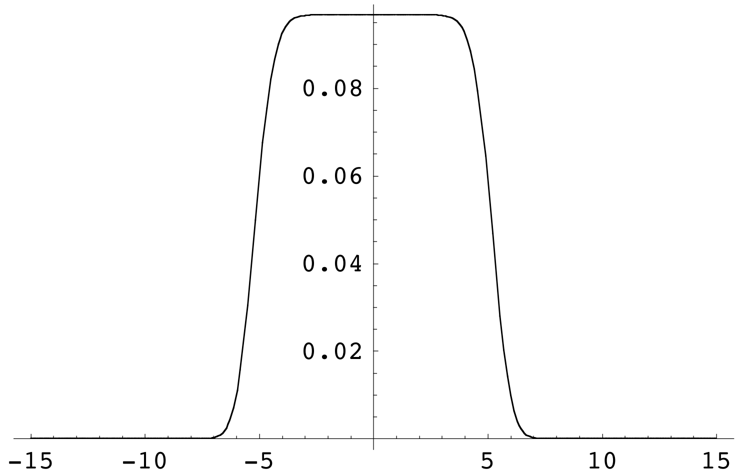
$$\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,\left(\frac{5}{2}+\#2\right)^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,\left(\frac{5}{2}+\#2\right)^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}+$$

$$\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,\left(\frac{15}{4}+\#2\right)^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,\left(\frac{15}{4}+\#2\right)^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}+$$

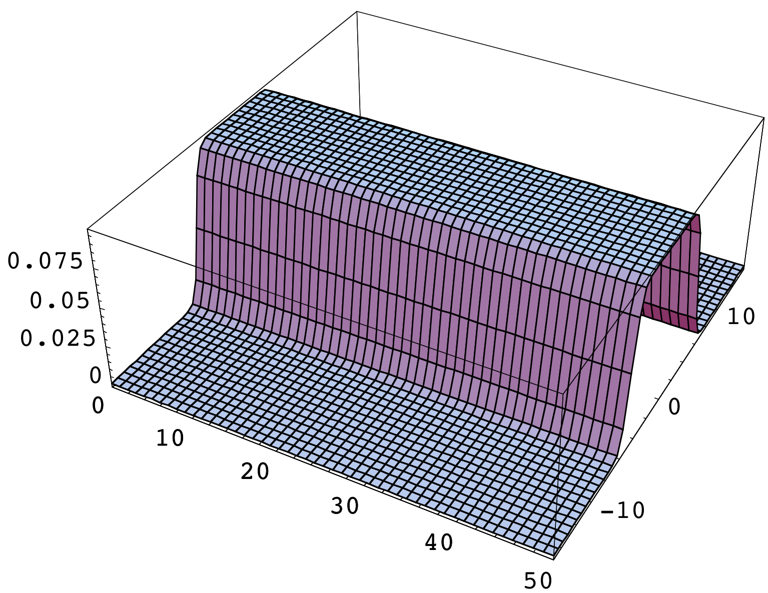
$$\frac{0.798942\, e^{-i\,(-\text{ArcTan}[0.000108378\,(57.9274+\#1)]+11810.5\,\#1)-\frac{5905.25\,i\,(5+\#2)^2}{(57.9274+\#1)\left(1+\frac{8.51365\times10^{7}}{(57.9274+\#1)^2}\right)}-\frac{0.64\,(5+\#2)^2}{1+1.17458\times10^{-8}\,(57.9274+\#1)^2}}}{(1+1.17458\times10^{-8}\,(57.9274+\#1)^2)^{1/4}}\Bigg]\hat{}$$

2, -CompiledCode-]

```
In[49]:= Plot[intensity[0,y],{y,-15,15},PlotRange->All];
```




```
In[50]:= Plot3D[intensity[x,y],{x,0,50},{y,-15,15},PlotRange->All,PlotPoints->50];
```



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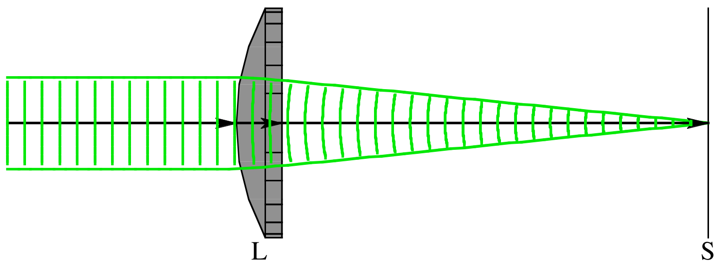
One Wavelet with Lens

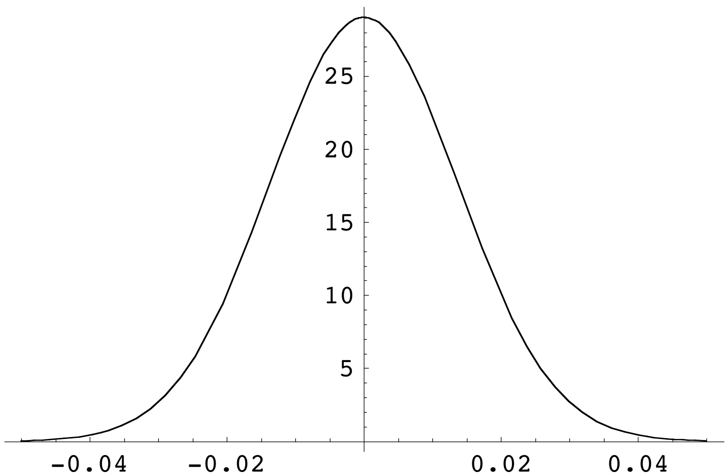
```
In[51]:= waveletSystem = {
            Move[LineOfWavelets[10,NumberOfRays->1],-50],
            PlanoConvexLens[100,50,10],
            Move[Screen[50],103]};
```

```
In[52]:= ShowSystem[waveletSystem,ShowGaussian->True,PlotType->TopView];
```

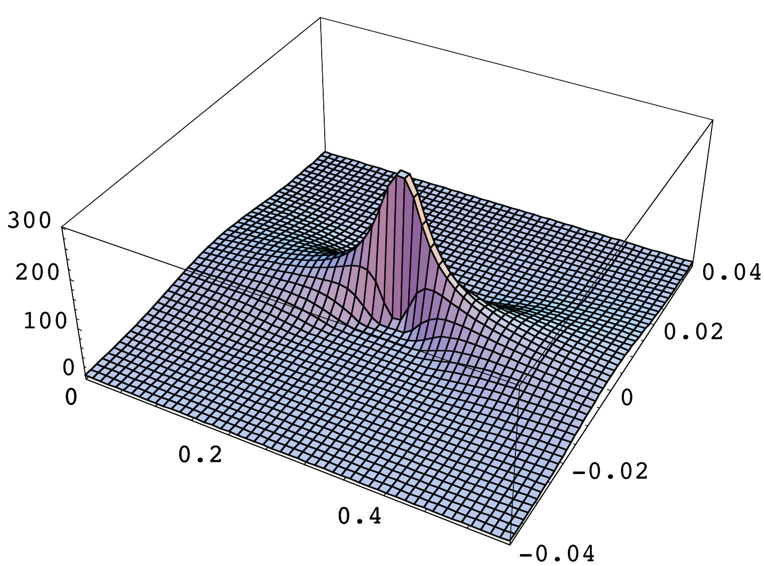



```
In[53]:= intensity = FindField[waveletSystem];
```

```
In[54]:= Plot[intensity[0,y],{y,-.05,.05},PlotRange->All];
```




```
In[55]:= Plot3D[intensity[x,y],{x,0,.55},{y,-.04,.04},PlotRange->All,PlotPoints->50];
```



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Two Wavelets with Lens

In[523]:=
```
waveletSystem = {
    Move[LineOfWavelets[10,NumberOfRays->2,PaddingFactor->.1],-50],
    PlanoConvexLens[100,50,10],
    Move[Screen[50],103]};
```

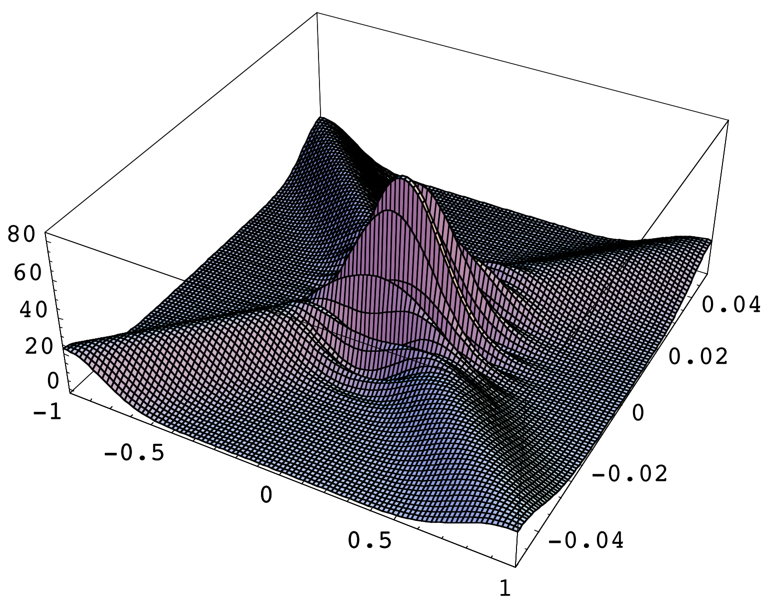
In[524]:=
```
ShowSystem[waveletSystem,ShowGaussian->True,PlotType->TopView];
```

In[525]:=
```
Timing[intensity = FindField[waveletSystem]][[1]]/60/Second*"Minute"
```

In[526]:=
```
Plot[intensity[0,y],{y,-.07,.07},PlotRange->All];
```

In[527]:=
```
Plot[intensity[-10,y],{y,-1,1},PlotRange->All];
```

In[61]:=
```
Plot3D[intensity[x,y],{x,-1,1},{y,-.05,.05},PlotRange->All,PlotPoints->100];
```



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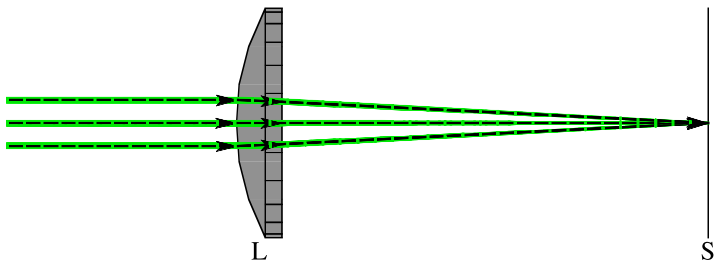
Three Wavelets with Lens

In[62]:=
```
waveletSystem = {
    Move[LineOfWavelets[10,NumberOfRays->3,PaddingFactor->.1],-50],
    PlanoConvexLens[100,50,10],
    Move[Screen[50],103]};
```

```
Warning, LineOfWavelets will not be uniform.

Set PaddingFactor greater or equal to 1 for uniform illumination.
```

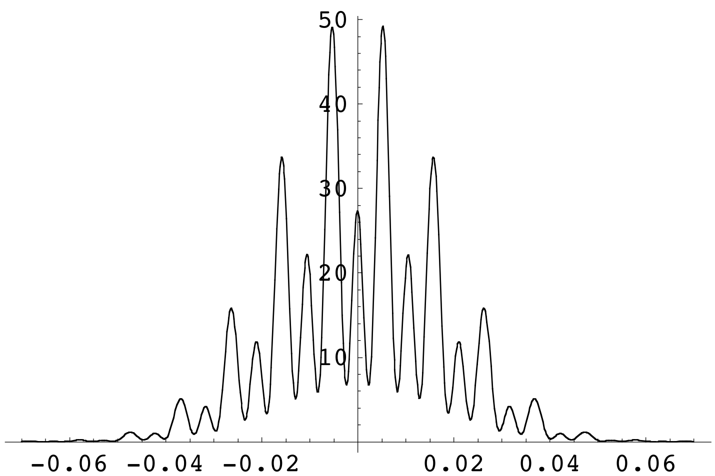
```
In[63]:= ShowSystem[waveletSystem,ShowGaussian->True,PlotType->TopView];
```



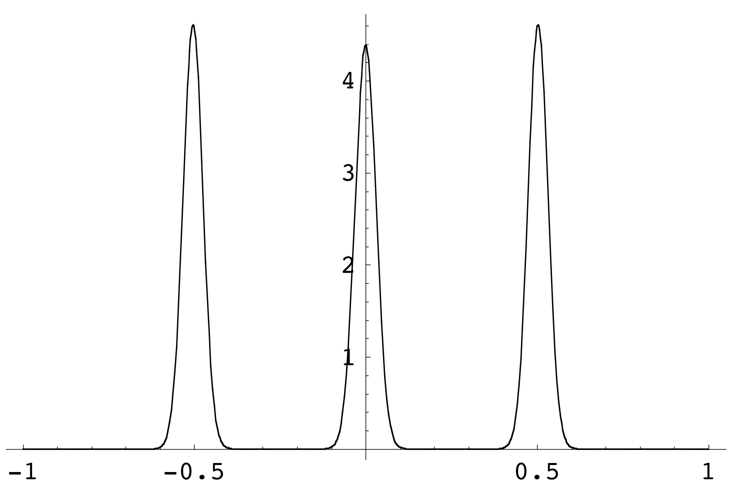

```
In[64]:= Timing[intensity = FindField[waveletSystem]][[1]]/60/Second*"Minute"

Out[64]= 0.685278 Minute

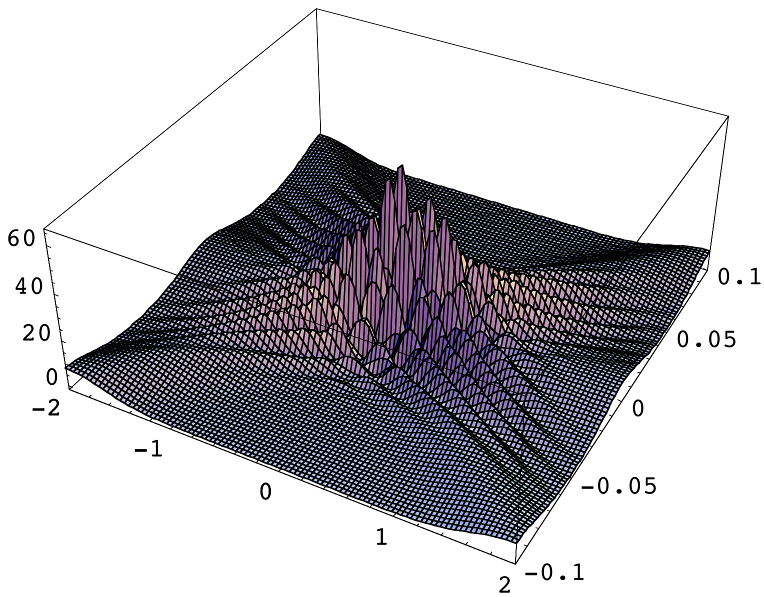
In[65]:= Plot[intensity[0,y],{y,-.07,.07},PlotRange->All];
```




```
In[66]:= Plot[intensity[-10,y],{y,-1,1},PlotRange->All];
```



```
In[67]:= Plot3D[intensity[x,y],{x,-2,2},{y,-.1,.1},PlotRange->All,PlotPoints->100];
```



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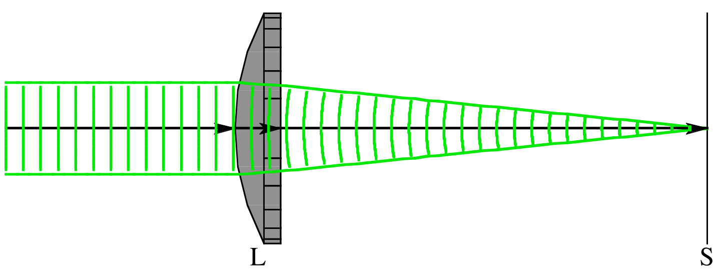
Wavelet analysis of an Imaging System

In[243]:=
In this section, we calculate the wavefront propagation for a plane wave traveling through a lens. Here is an overall schematic of the system.

```
In[68]:= ShowSystem[{
         Move[LineOfWavelets[10,NumberOfRays->1],-50],
         PlanoConvexLens[100,50,10],
         Move[Screen[50],103]},PlotType->TopView,ShowGaussian->True];
```



In[243]:=
Next, we define the system using 21 Gaussian wavelets to approximate a collimated beam of light. Because each Gaussian wavelet follows the symbolically calculated solution for the system, relatively few numbers of wavelets are required to accurately model the system's behavior.

```
In[69]:= waveletSystem = {
         Move[LineOfWavelets[10,NumberOfRays->21],-50],
         PlanoConvexLens[100,50,10],
         Move[Screen[50],103]};
```

In[243]:=

Finally, we use **FindField** to construct a symbolic solution of the field intensity. This calculation will take about 10 minutes on many computers.

```
In[70]:= Timing[intensity = FindField[waveletSystem]][[1]]/60/Second*"Minutes"
         ByteCount[intensity]
```

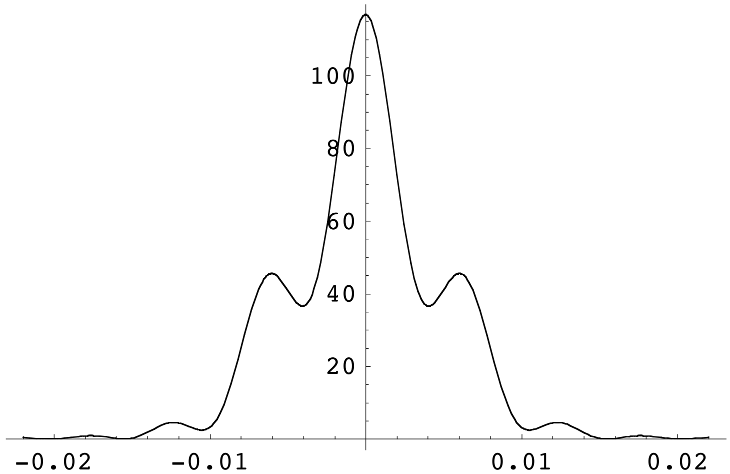
```
Out[70]= 17.8156 Minutes
```

```
Out[71]= 291720
```

In[243]:=

Here is plot of the intensity near the plane of focus. The vertical axis is intensity. The horizontal axis is measured in millimeters.

```
In[72]:= Plot[intensity[0,y],{y,-.022,.022},PlotRange->All,PlotPoints->100];
```



In[243]:=

The vertical axis is normalized to have a unity area. We can check this result with **NIntegrate**. In fact, you could also use the **Integrate** to do this symbolically, but the calculation time would be prohibitive.

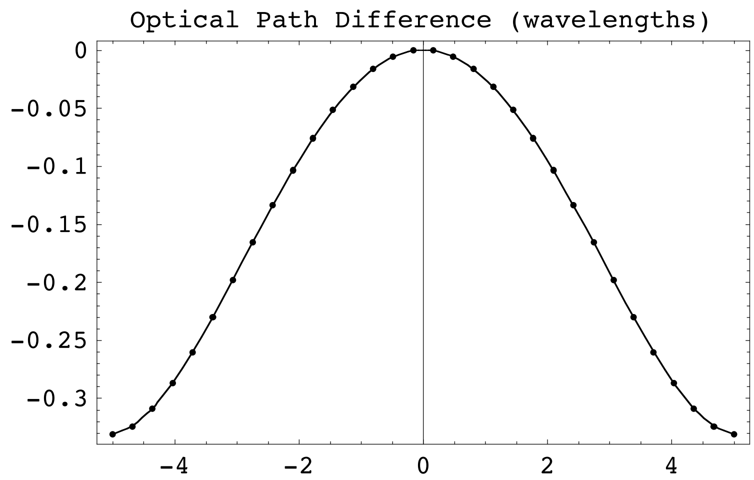
```
In[73]:= NIntegrate[intensity[0,y],{y,-.1,.1}]
```

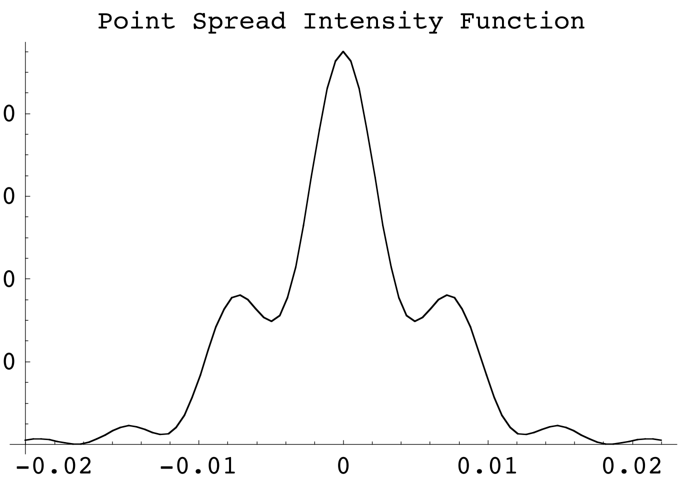
```
Out[73]= 1.
```

In[243]:=

We now compare this plot with the point spread function that is calculated numerically for the same surface location. Although not identical, the two results are consistent with each other.

```
In[74]:= psf = PointSpreadFunction/.PSF[{
              Move[LineOfRays[10],-50],
              PlanoConvexLens[100,50,10],
              Move[Screen[50],103]},NumberOfPoints->256,PaddingFactor->10];
```





In[243]:=

NIntegrate is used again to check its area.

In[75]:= **NIntegrate[psf[y],{y,-.1,.1}]**

```
InterpolatingFunction::dmval :
 Input value {0.0984085} lies outside the range of data in
   the interpolating function. Extrapolation will be used.

InterpolatingFunction::dmval :
 Input value {-0.0984085} lies outside the range of data
   in the interpolating function. Extrapolation will be used.

InterpolatingFunction::dmval :
 Input value {0.090618} lies outside the range of data in
   the interpolating function. Extrapolation will be used.

General::stop : Further output of
   InterpolatingFunction::dmval will be suppressed during this calculation.

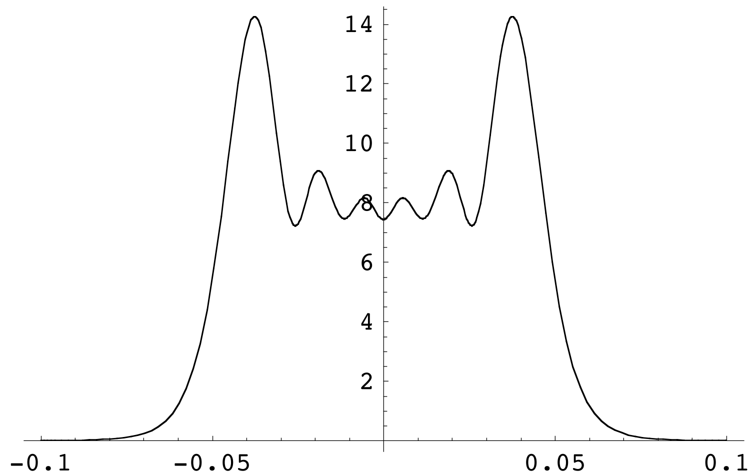
NIntegrate::ncvb : NIntegrate failed to converge to prescribed
   accuracy after 7 recursive bisections in y near y = -0.00859375.
```

Out[75]= 0.983565

In[243]:=

Here is an symbolic intensity plot at a different plane near the focus.

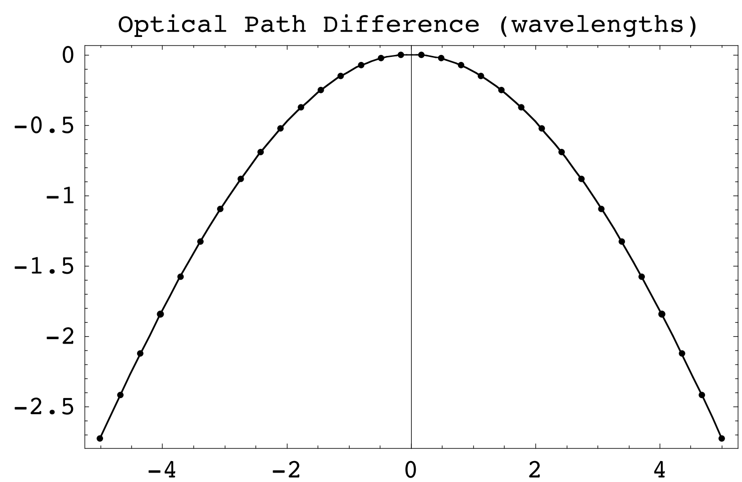
```
In[76]:= Plot[intensity[-1,y],{y,-.1,.1},PlotRange->All,PlotPoints->100];
```

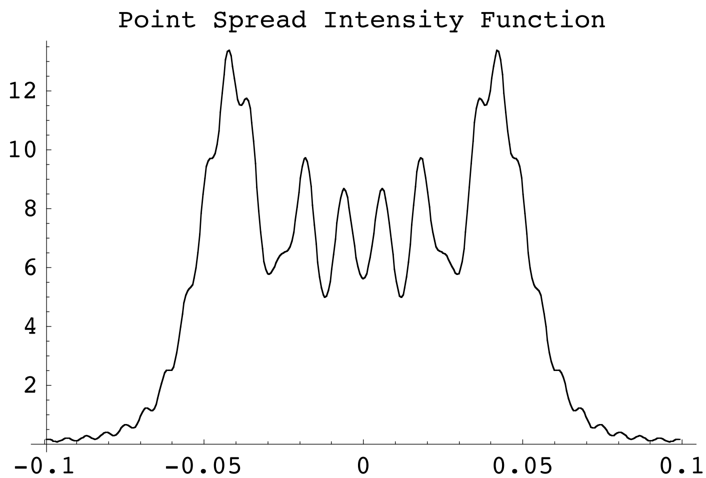



```
In[243]:=
```

Again, the numeric point spread function is consistent in form. Note that the numeric PSF shows eveidence of high frequency ringing that is absent from the symbolic solution. This is due the hard edge of the pupil function in the numeric PSF.

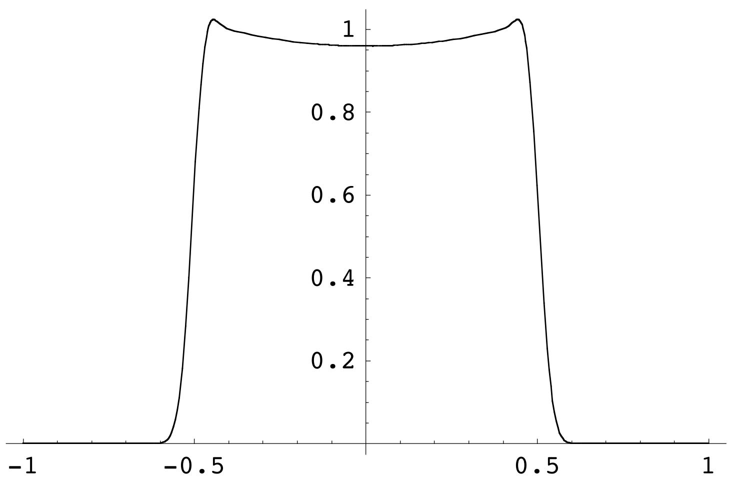
```
In[77]:= psf = PointSpreadFunction/.PSF[{
        Move[LineOfRays[10],-50],
        PlanoConvexLens[100,50,10],
        Move[Screen[50],102]},NumberOfPoints->512,PaddingFactor->10];
```





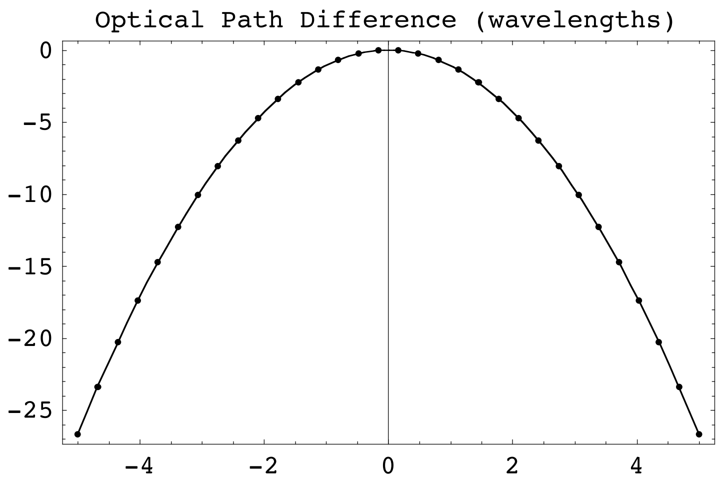
In[243]:=
Again we try a symbolic intensity plot but, this time we move further away from the plane of focus.

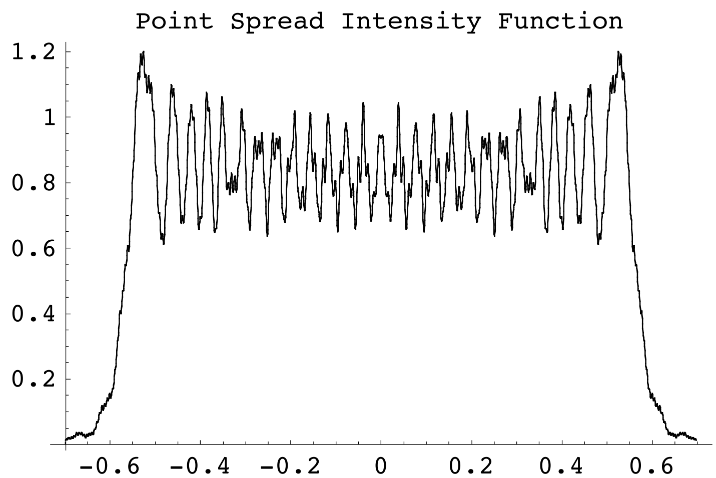
```
In[78]:= Plot[intensity[-10,y],{y,-1,1},PlotRange->All,PlotPoints->100];
```



In[243]:=
The numeric PSF is again consistent, but with a great deal of high frequency ringing present.

```
In[79]:= psf = PointSpreadFunction/.PSF[{
            Move[LineOfRays[10],-50],
            PlanoConvexLens[100,50,10],
            Move[Screen[50],93]},NumberOfPoints->4096,PaddingFactor->10];
```

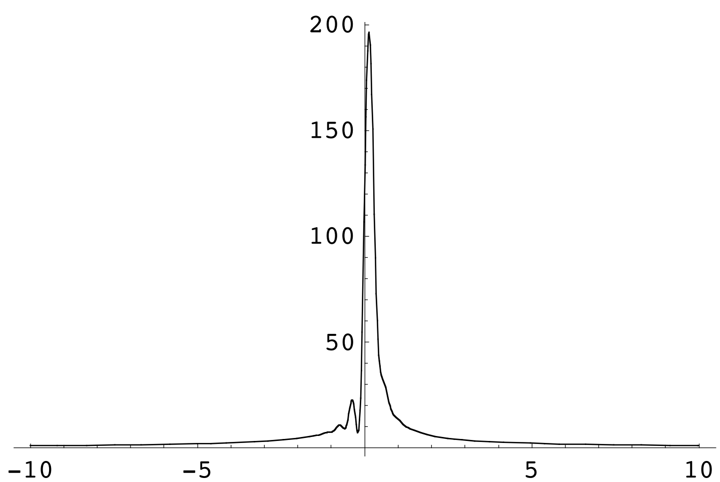




In[243]:=

We can find the plane of peak intensity by making a symbolic plot along the optical axis of the system.

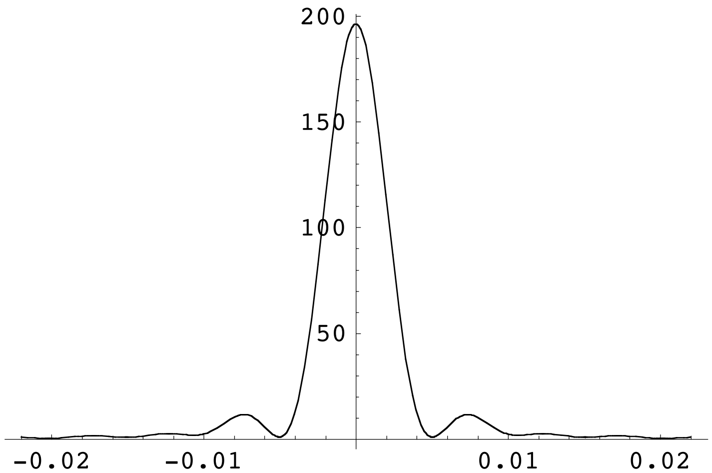
In[80]:= **Plot[intensity[x,0],{x,-10,10},PlotRange->All];**



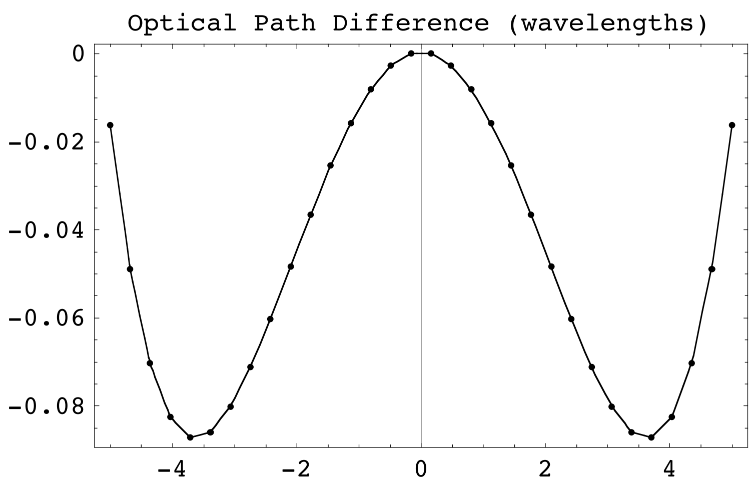
In[243]:=

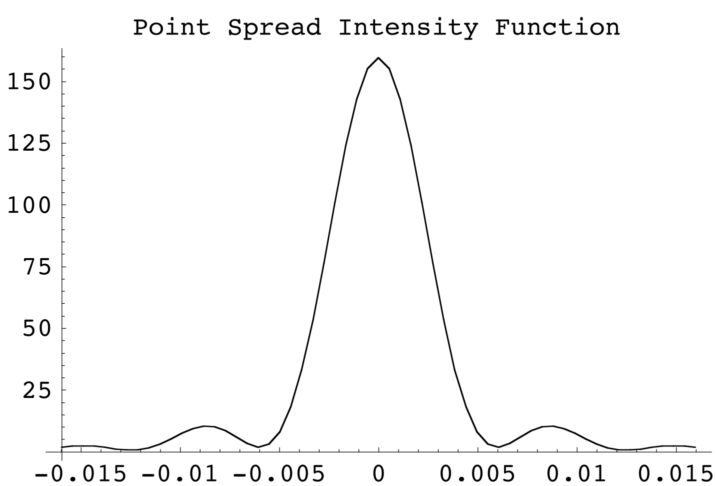
We can now compare plots of the symbolic and numeric intensity profiles at the plane of peak intensity. Again, the overall shapes are in agreement with each other.

```
In[81]:= Plot[intensity[.133,y],{y,-.022,.022},PlotRange->All,PlotPoints->100];
```




```
In[82]:= psf = PointSpreadFunction/.PSF[{
         Move[LineOfRays[10],-50],
         PlanoConvexLens[100,50,10],
         Move[Screen[50],103.133]},NumberOfPoints->1024,PaddingFactor->10];
```

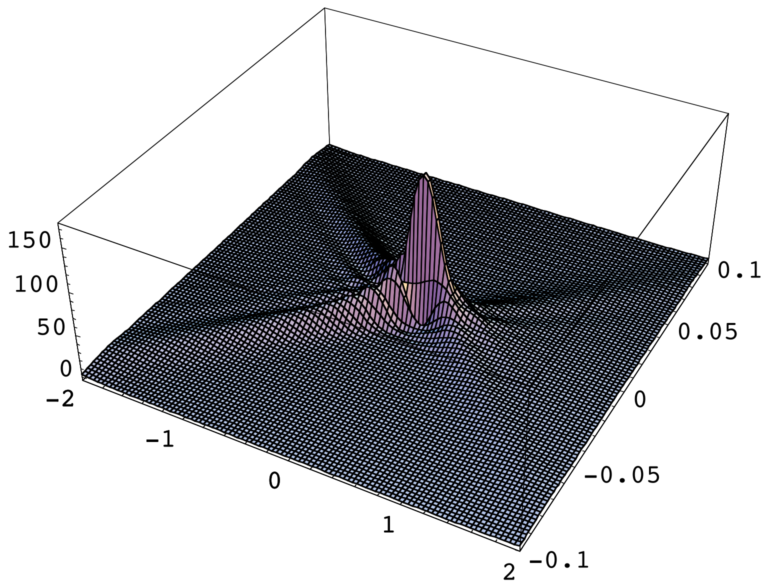




In[243]:=

Finally, we make a three-dimensional plot of symbolic intensity between the optical and transverse axes. The peak indicates the paraxial focus of the system.

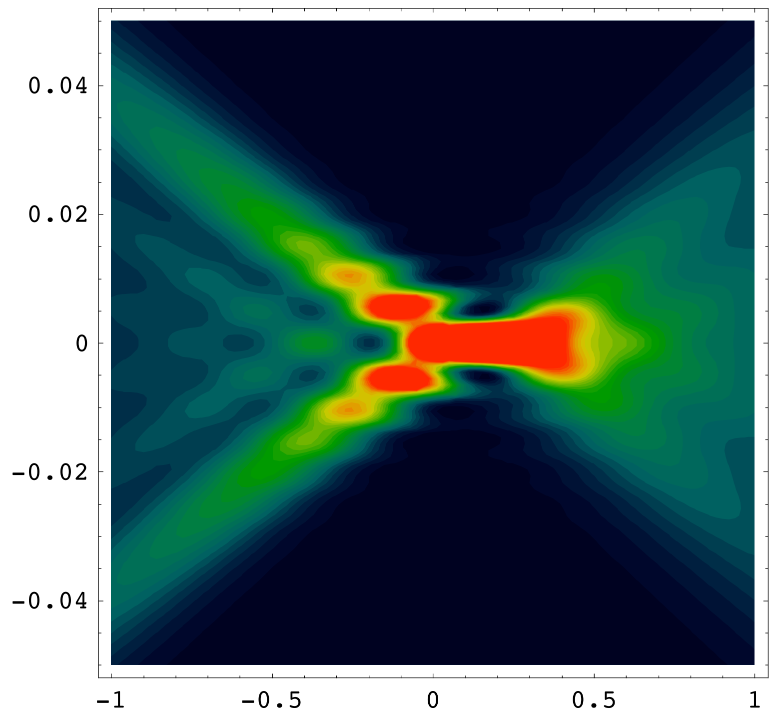
```
In[83]:= Plot3D[intensity[x,y],{x,-2,2},{y,-.1,.1},PlotRange->All,PlotPoints->100];
```



In[243]:=

Here is a similar intensity plot using the **ContourPlot** function. The horizontal dimension is measured along the optical axis of the system. The plot has been a thresholded in order to observe the low-intensity details.

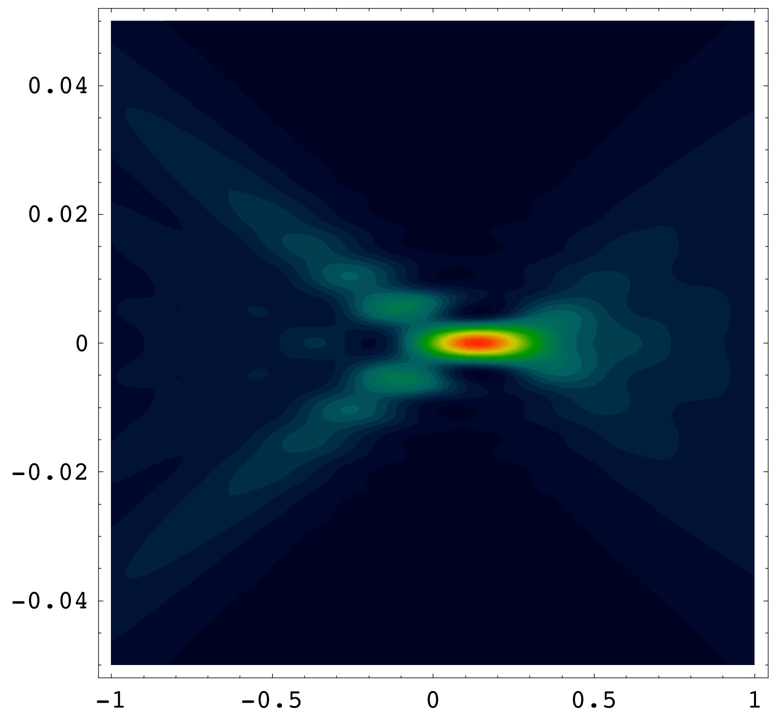
In[84]:=
```
ContourPlot[intensity[x,y], {x,-1, 1}, {y,-.05,.05}, PlotPoints->100,
  PlotRange->{0,50}, Contours->30, ContourLines->False,
  ColorFunction->Function[Hue[.65-#*(.65),1,#*.9+.1]]];
```



In[243]:=

Here is the same plot with all intensity values included.

```
In[85]:= ContourPlot[intensity[x,y], {x,-1, 1}, {y,-.05,.05}, PlotPoints->200,
         PlotRange->All, Contours->30, ContourLines->False,
         ColorFunction->Function[Hue[.65-#*(.65),1,#*.9+.1]]];
```



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